МАТЕМАТИЧНЕ
ТА КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ

МАТЕМАТИЧЕСКОЕ
И КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ

MATHEMATICAL
AND COMPUTER MODELING

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METHOD OF NUMERICAL ANALYSIS OF THE PROBLEM OF MASS TRANSFER OF A CYLINDRICAL BODY WITH THE UNIFORM TRANSLATIONAL FLOW

Context. The problem of mass transfer of a cylindrical body with a uniform translational flow of a viscous incompressible fluid is examined in the paper.

Objective. The purpose of this work is to develop a new method for numerical analysis of the problem of mass transfer of a cylindrical body with a uniform translational flow, which based on the joint application of the *R*-functions structural method and the Galerkin projection method.

Method. In general case, the problem of stationary mass transfer of a cylindrical body with a viscous incompressible fluid flow is reduced to the solution of the equation of hydrodynamic flow passing a surface and an equation for concentration with corresponding boundary conditions on the surface of the body and far away from it. The geometry of the area, and also the boundary conditions (including the condition at infinity) may be taken into account precisely by using the constructive apparatus of the *R*-functions theory by V. L. Rvachev, the Academician of Ukrainian National Academy of Sciences. In this study, a complete structure of the solution of a linear boundary value problem for the concentration that exactly satisfies the boundary conditions on the boundary and condition at infinity is constructed on the basis of the *R*-functions theory methods, and this made it possible to lead the tasks in the infinite domain to tasks in the finite domain. To solve the linear problem for concentration the numerical algorithm on the basis of Galerkin method is developed.

Results. The computational experiment for the problem of the flow past circular and elliptical cylinders at various Reynolds and Peclet numbers was carried out.

Conclusions. The conducted experiments have confirmed the efficiency of the proposed method of numerical analysis of the problem of mass transfer of a cylindrical body with a uniform translational flow, based on the joint application of the *R*-functions structural method and Galerkin projection method. The prospects for the further research may be to use the developed method for the implementation of iterative methods for solving the task of nonlinear mass transfer, semi-discrete and projection methods for solving the non-stationary tasks, as well as in solving the tasks of optimal management of relevant technological processes.

Keywords: flow task, stationary flow, viscous fluid, stream function, mass transfer, concentration function, *R*-functions method, Galerkin method.

NOMENCLATURE

$c = c(r, \varphi)$ – a function of concentration;

c_0 – the constant concentration given on the boundary $\partial \Omega$ of a streamlined body;

$\partial \Omega$ – the boundary of a streamlined body;

$l = 1, 2$ – the number of equation;

n – the external normal to $\partial \Omega$;

Pe – Peclet number;

r, φ, z – variables of the cylindrical coordinate system;

Re – Reynolds number;

u_l – the solutions of auxiliary problems with homogeneous boundary conditions;

U_∞ – unperturbed fluid velocity at infinity;

$\nu = \text{Re}^{-1}$ – the coefficient of viscosity;

$\{\varphi_k (r, \varphi)\}$ – a complete system of particular solutions of equation $\Delta u = 0$ with respect to the exterior of a cylinder of finite radius;

$\{\tau_j(r,\varphi)\}$ – a complete system of particular solutions

of equation $\Delta u = 0$ relative to domain $\{\omega(r,\varphi) < M\}$;

$\{\phi_i(r,\varphi)\}$ – a sequence of functions that is complete with respect to the whole plane;

Φ_1, Φ_2 – undefined components of the solution structure;

$\psi = \psi(r,\varphi)$ – the stream function;

ω – a sufficiently smooth function built by using the constructive apparatus of the R-functions theory and describing the geometry of the domain Ω;

$\omega = 0$ – a normalized equation of $\partial\Omega$;

Ω – the flow domain;

Ω_n – the bounded domain $\{0 < \omega(r,\varphi) < M_n\}$.

INTRODUCTION

The problems of calculating viscous flows, which are complicated by mass transfer, are widely used in heat power engineering, chemical and food technologies, geo- and astrophysical studies, and environmental protection. Many processes of chemical technology are associated with fluid movement in process equipment. In the preparation of reagents and in the isolation of reaction products, such operations as leaching, absorption, extraction and distillation play an important role. The laws of hydrodynamics, heat and mass transfer are essential for the entire technological process. Processes of heat and mass transfer are also ones of the most important in the energy sector, as well as in a number of the technological processes in the metallurgical and other industries. In addition, the problems of mass transfer of bodies with a uniform viscous flow underlie the calculation of many technological processes, which are associated with dissolution, extraction, evaporation, precipitation of colloids, etc. Therefore, the development of new, as well as the improvement of existing methods of mathematical modeling and numerical analysis of external stationary problems of hydrodynamics of a viscous incompressible fluid, which take into consideration the mass transfer, is an actual scientific issue.

The object of this study is the stationary hydrodynamic process of flow past bodies by a viscous incompressible fluid, complicated by mass transfer and described by a system of equations with respect to the stream function and concentration.

The subject of this study is a mathematical model of the stationary task of flow past body by a viscous incompressible fluid with allowance for mass transfer, and a method for its numerical analysis.

The purpose of the work is to develop a new method for mathematical modeling of the mass transfer of a cylindrical body with a uniform translational flow which based on the R-functions method and Galerkin method.

1 PROBLEM STATEMENT

At small Peclet numbers, to describe the mass transfer process the Oseen approximation (equation 1) is used [1]:

$$\Delta c = \mathrm{Pe}\,U_\infty\left(\cos\varphi\frac{\partial c}{\partial r} - \frac{\sin\varphi}{r}\frac{\partial c}{\partial\varphi}\right) \text{ in } \Omega. \qquad (1)$$

In the general case, the mass transfer process is described by the equation for the concentration (equation 2) in the form [1, 2]:

$$\Delta c = \frac{\mathrm{Pe}}{r}\left(\frac{\partial\psi}{\partial\varphi}\frac{\partial c}{\partial r} - \frac{\partial\psi}{\partial r}\frac{\partial c}{\partial\varphi}\right) \text{ in } \Omega. \qquad (2)$$

In equations (1) and (2) $\Delta c = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial c}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 c}{\partial\varphi^2}$.

Equations (1) and (2) in a rectangular coordinate system have the following form:

$$\Delta c = \mathrm{Pe}\,U_\infty\frac{\partial c}{\partial x} \text{ in } \Omega, \qquad (3)$$

$$\Delta c = \mathrm{Pe}\left(\frac{\partial\psi}{\partial y}\frac{\partial c}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial c}{\partial y}\right) \text{ in } \Omega, \qquad (4)$$

where $\Delta c = \dfrac{\partial^2 c}{\partial x^2} + \dfrac{\partial^2 c}{\partial y^2}$.

Equations (1) and (2) should be supplemented by the boundary condition on $\partial\Omega$ and the condition at infinity [1]:

$$c\big|_{\partial\Omega} = c_0, \qquad (5)$$

$$\lim_{r\to+\infty} c = 0. \qquad (6)$$

The stream function $\psi(r,\varphi)$ can be found, for example, as the solution of the following nonlinear task of flow past a cylindrical body by a viscous incompressible fluid [3–6]:

$$\nu\Delta^2\psi = \frac{1}{r}\frac{\partial\psi}{\partial\varphi}\frac{\partial(\Delta\psi)}{\partial r} - \frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial(\Delta\psi)}{\partial\varphi} \text{ in } \Omega, \qquad (7)$$

$$\psi\big|_{\partial\Omega} = 0, \quad \frac{\partial\psi}{\partial\mathbf{n}}\bigg|_{\partial\Omega} = 0, \qquad (8)$$

$$\lim_{r\to+\infty} r^{-1}\psi = U_\infty\sin\varphi, \qquad (9)$$

where $\Delta\psi = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial\psi}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2\psi}{\partial\varphi^2}$, $\Delta^2\psi = \Delta(\Delta\psi)$.

The task (1), (5), (6) does not depend on the stream function $\psi(r,\varphi)$, and the solution of the task (2), (5), (6) consists of two steps:

a) the search of the stream function as a solution of the task (7)–(9);

b) the solution of the task for concentration.

2 REVIEW OF THE LITERATURE

Various tasks which are arising in the study of external viscous fluid flows can be investigated theoretically or by means of physical experiment. At present, mathematical modeling and computational experiment are increasingly

being used in the study of hydrodynamic problems. Basically, finite difference method, finite element method, boundary integral element method and others are used for the numerical analysis of such problems. These methods are easy to implement, but do not have the necessary property of universality: when moving to a new area (especially non-classical geometry), it is necessary to generate a new grid, and often to replace complex sections of the boundary with simple ones, composed of, for example, straight line segments. The use of the *R*-functions structural-variational method [7, 8] by V. L. Rvachev, the Academician of Ukrainian National Academy of Sciences, is an alternative to the existing methods of numerical computation of hydrodynamic problems.

The *R*-functions method in computational hydrodynamics was applied in [9–14]. The task of viscous fluid external flow around bodies of revolution in a spherical coordinate system, which is complicated by mass transfer, was solved in [15, 16] with using the *R*-functions method.

In this study, we propose to apply the *R*-functions and Galerkin method for mathematical modeling the problem of mass transfer of a cylindrical body with a uniform translational flow.

3 MATERIALS AND METHODS

The method for solving the task for the stream function, based on the application of *R*-functions, successive approximations and Galerkin method, is described in [17, 18]. Substituting the stream function $\psi(r, \varphi)$ so obtained into equation (2), let us solve the problems (1), (5), (6) and (2), (5), (6) by the *R*-functions method. To do this, using the constructive means of the *R*-functions theory [7, 8] let us construct the structure of the boundary value task solution, i.e. the functions bundle that exactly satisfies the boundary condition and the condition at infinity.

Let us consider a sufficiently smooth function [19]

$$f_M(x) = \begin{cases} 1 - \exp\dfrac{Mx}{x - M}, & 0 \le x < M; \\ 1, & x \ge M \ (M = \mathrm{const} > 0). \end{cases} \quad (10)$$

The function (10) $f_M(x) \in C^\infty[0, +\infty)$ and satisfies the conditions:

a) $f_M(0) = 0$; b) $f_M'(0) = 1$;

c) $f_M'(x) \ge 0 \ \forall x \ge 0$; d) $f_M(x) \equiv 1 \ \forall x \ge M$.

Let us denote $\omega_M = f_M(\omega)$, then

1) $\omega_M > 0$ in Ω; 2) $\omega_M \big|_{\partial\Omega} = 0$;

3) $\dfrac{\partial \omega_M}{\partial \mathbf{n}} \bigg|_{\partial\Omega} = -1$; 4) $\omega_M \equiv 1$, if $\omega_M \ge M$.

In the problems (3), (5), (6), and (4)–(6) we make the replacement

$$c = c_0(1 - \omega_M) + u_l.$$

The choice of such a substitution is due to the fact that the function $c_0(1 - \omega_M)$ satisfies the boundary condition (5) and the condition at infinity (6).

Then for u_l, $l = 1, 2$, we obtain the tasks

$$-\Delta u_l + B_l u_l = F_l \ \text{in} \ \Omega, \quad (11)$$

$$u_l \big|_{\partial\Omega} = 0, \quad (12)$$

$$\lim_{r \to +\infty} u_l = 0, \quad (13)$$

where

$$B_1 = \mathrm{Pe}U_\infty \frac{\partial u_1}{\partial x}, \ B_2 = \mathrm{Pe}\left(\frac{\partial \psi}{\partial y} \frac{\partial u_2}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial u_2}{\partial y} \right),$$

$$F_1 = \Delta(c_0(1 - \omega_M)) - \mathrm{Pe}U_\infty \frac{\partial(c_0(1 - \omega_M))}{\partial x},$$

$$F_2 = \Delta(c_0(1 - \omega_M)) -$$

$$- \mathrm{Pe}\left(\frac{\partial \psi}{\partial y} \frac{\partial(c_0(1 - \omega_M))}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial(c_0(1 - \omega_M))}{\partial y} \right).$$

Note that $F_l \equiv 0$, $l = 1, 2$, in the domain $\{\omega(x, y) \ge M\}$.

Let us find the generalized solution u of tasks (11)–(13) as the limit, when $n \to \infty$, of solutions u_n of equations (11), which are considered in a sequence of domains $\{\Omega_n\}$, that is a monotonic exhaustion of an infinite area Ω.

In domains Ω_n we will consider the boundary value tasks

$$-\Delta u_{n,l} + B_l u_{n,l} = F_l \ \text{in} \ \Omega_n, \quad (14)$$

$$u_{n,l} \big|_{\partial\Omega} = 0, \quad (15)$$

where functions $u_{n,l}$ are continued by zero outside of Ω_n.

An approximate solution of tasks (14) – (15) for each $n = 1, 2, ...$ according to the Bubnov-Galerkin method, will be sought in the form:

$$u_{n,l,N} = \sum_{j=1}^{N} c_{n,l,j} \phi_j. \quad (16)$$

The numbers $c_{n,l,j}$, $j = 1, ..., N$, are found from the system of linear algebraic equations

$$\sum_{j=1}^{N} c_{n,l,j}\{[\phi_j, \phi_i] + (K_l \phi_j, \phi_i)\} = (F_l, \phi_i), \quad (17)$$

where

$$i = \overline{1, N}, \ l = 1, 2,$$

$$(F_1, \phi_i) = -\int_{\Omega_n} c_0 \nabla(1 - \omega_M) \cdot \nabla \phi_i \, d\Omega +$$

$$+ \int_{\Omega_n} c_0 \mathrm{Pe}U_\infty \cdot (1 - \omega_M) \frac{\partial \phi_i}{\partial x} d\Omega,$$

$$(F_2, \phi_i) = - \int\limits_{\Omega_n} c_0 \nabla(1 - \omega_M) \cdot \nabla \phi_i d\Omega +$$

$$+ \int\limits_{\Omega_n} c_0 \text{Pe}(1 - \omega_M) \left(\frac{\partial \psi}{\partial y} \frac{\partial \phi_i}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \phi_i}{\partial y} \right) d\Omega.$$

To construct the coordinate sequence, the complete system of particular solutions of the Laplace equation [20] and the *R*-functions method [7, 8] will be used.

We have proved that for any choice of sufficiently smooth functions Φ_1, Φ_2 and at the requirement that $\Phi_1 \to 0$ as $r \to +\infty$ a function of the form

$$u = \omega_M \Phi_1 + \omega_M (1 - \omega_M) \Phi_2$$

exactly satisfies the boundary conditions (12) and (13), that is, it is the structure of the solution of the boundary value problem (11)–(13).

Approximations of the functions Φ_1 и Φ_2 in the domain Ω_n will be seek in the form

$$\Phi_1 = \sum_{k=1}^{m_1} \alpha_k \cdot \varphi_k, \quad \Phi_2 = \sum_{j=1}^{m_2} \beta_j \cdot \tau_j,$$

where $\{\varphi_k(r, \varphi)\} = \left\{ r^{-n} \begin{matrix} \cos n\varphi \\ \sin n\varphi \end{matrix}, n = 1,2,... \right\}$,

$$\{\tau_j(r, \varphi)\} = \left\{ 1, r^k \begin{matrix} \cos k\varphi \\ \sin k\varphi \end{matrix}, k = 1,2,... \right\}.$$

Then the sequence of functions ,which is complete with relatively to the whole plane, has the form:

$$\{\phi_i(r, \varphi)\} = \{\omega_M(r, \varphi)\varphi_k(r, \varphi),$$

$$\omega_M(r, \varphi)(1 - \omega_M(r, \varphi))\tau_j(r, \varphi)\}. \quad (18)$$

The values of the coefficients α_k ($k = 1,2,...,m_1$) and β_j ($j = 1,2,...,m_2$) in accordance with the Bubnov-Galerkin method will be found from the condition of residual orthogonality to the first N ($N = m_1 + m_2$) elements of the sequence (18), which leads to a system of linear algebraic equations in the form (17).

We have proved the convergence of the Galerkin approximations $u_{n,l,N}$ of the form (16) at $N \to \infty$, $M_n \to \infty$ in the energy norm to the unique generalized solutions of tasks (11)–(13).

4 EXPERIMENTS

A computational experiment was carried out for the tasks of flow past a circular cylinder $x^2 + y^2 = 1$ and an elliptical cylinder $\frac{x^2}{4} + \frac{y^2}{1} = 1$ at $c_0 = 1$, $M = 5, 10$, $m_1 = 10$, $m_2 = 11$ and at various Reynolds and Peclet numbers. The double integrals in the systems for determination $c_{n,l,j}$ were taken approximately by the Gauss formula with 50 nodes for each variable.

5 RESULTS

The concentration lines for a circular cylinder are shown in Fig. 1–3. Fig. 4–6 show the concentration lines for elliptical cylinder.
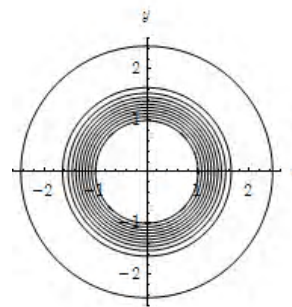

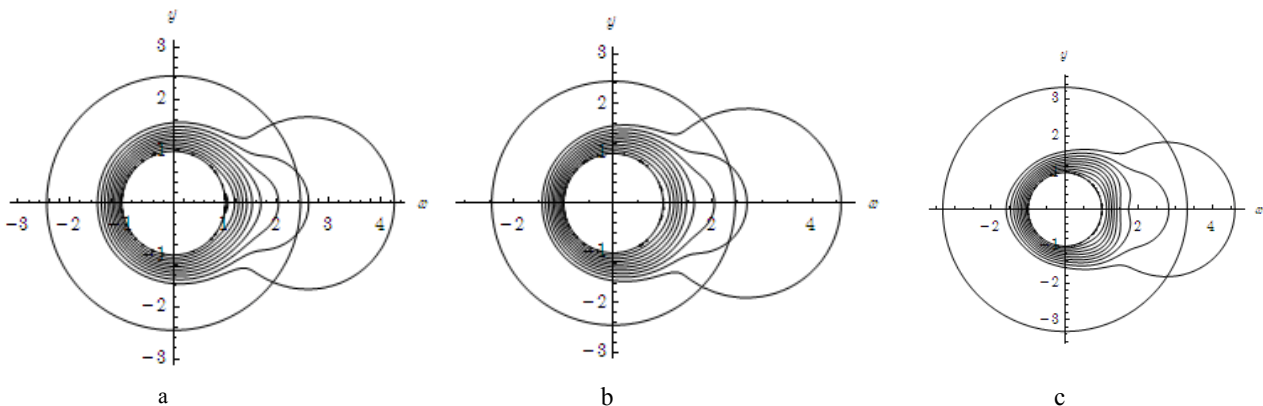Figure 1 – The concentration lines for a circular cylinder

at $M = 5$ and $\text{Pe} = 0,01$

| a | b | c |

Figure 2 – The concentration lines for a circular cylinder at $\text{Pe} = 10$:

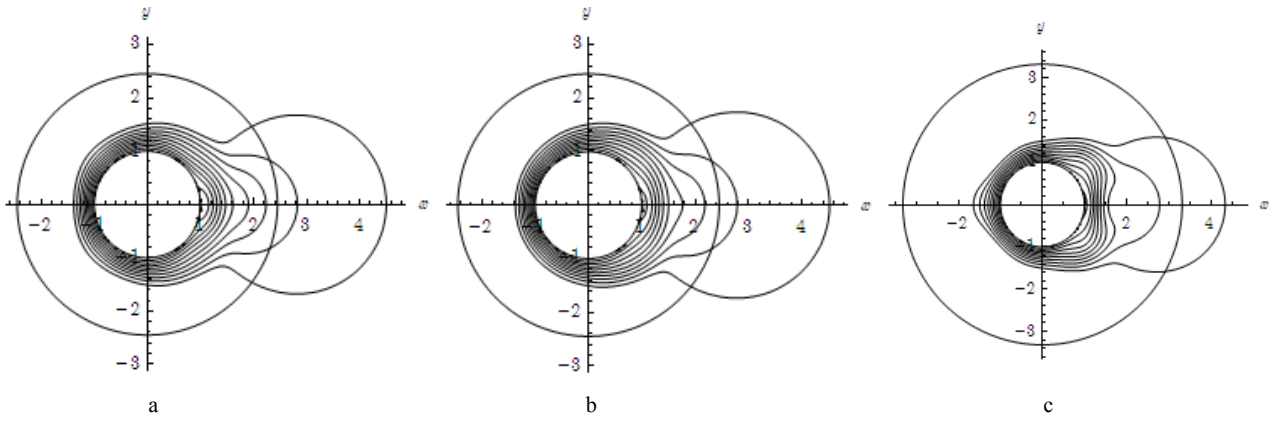a – $\text{Re} = 0$, b – $\text{Re} = 5$, c – $\text{Re} = 15$

Figure 3 – The concentration lines for a circular cylinder at $Pe = 20$:

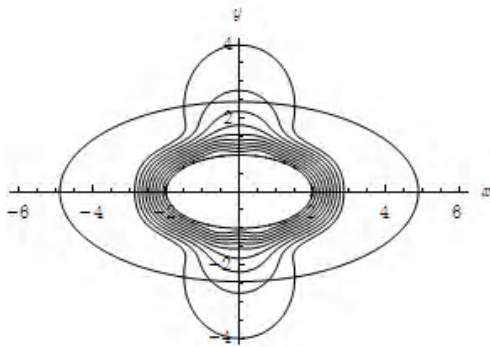$a - Re = 0$, $b - Re = 5$, $c - Re = 15$

Figure 4 – The concentration lines for elliptical cylinder

at $M = 5$ and $Pe = 0,01$

6 DISCUSSION

At small Reynolds and Peclet numbers the substance is transported uniformly, dissolving into the liquid. As the Reynolds and Peclet numbers increase, the particles of substance begin to move with the flow. These results are consistent with the physics of the process. The efficiency of the proposed method for a spherical coordinate system was verified on the problem of flow past a sphere [13], for which an exact solution is known [20].

CONCLUSIONS

The numerical method for calculating the mass transfer of a cylindrical body with a uniform translational flow, which based on the joint application of the *R*-functions method and Galerkin method, is proposed for the first time in this
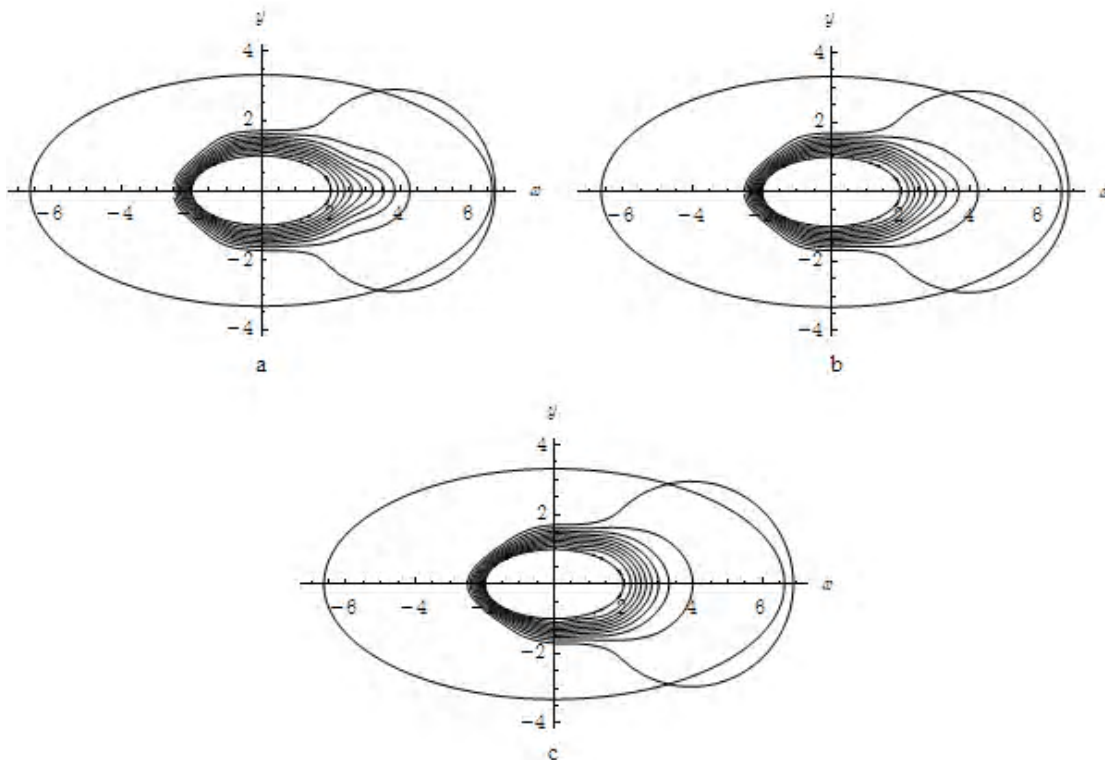

Figure 5 – The concentration lines for elliptical cylinder at $Pe = 10$:

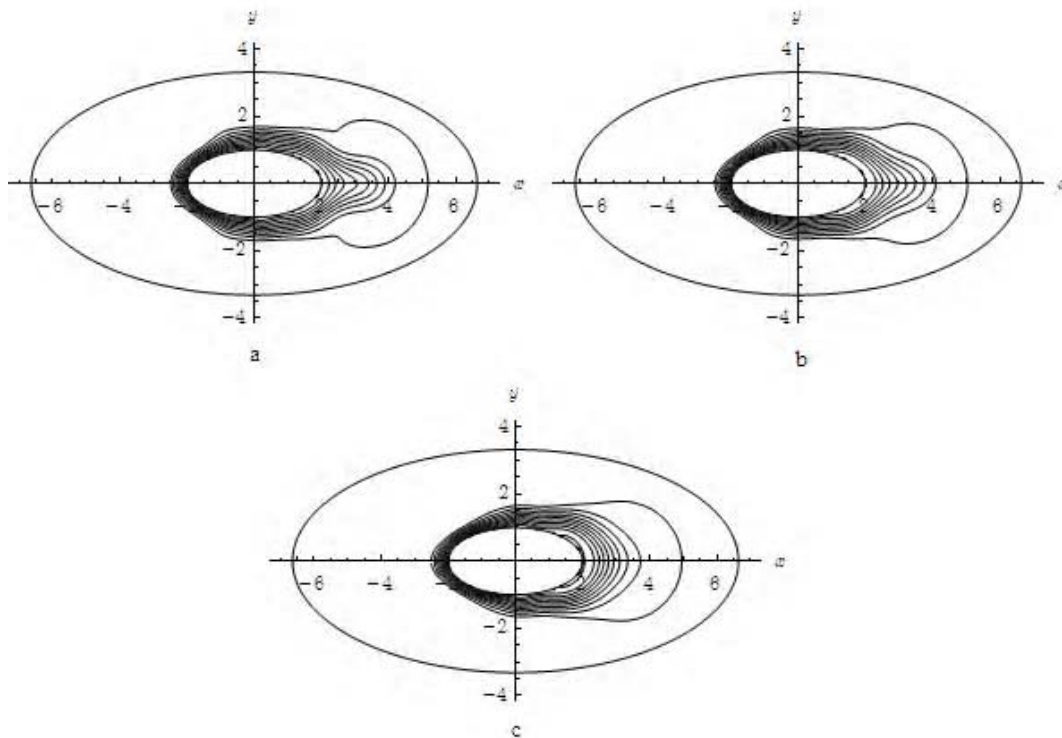$a - Re = 5$, $b - Re = 15$, $c - Re = 30$

Figure 6 – The concentration lines for elliptical cylinder at $\mathrm{Pe} = 20$:

$\mathrm{a} - \mathrm{Re} = 5$, $\mathrm{b} - \mathrm{Re} = 15$, $\mathrm{c} - \mathrm{Re} = 30$

study. By using the R-functions method the structure of the solution of the problems of flow past bodies with allowance for mass transfer, which precisely satisfying the boundary condition on the boundary and the condition at infinity, was constructed, and this made it possible to lead tasks in the infinite domain to tasks in the finite domain. To approximate the uncertain components of the solution structure, the Galerkin method was applied. The stationary problem of flow past a cylindrical body in a cylindrical coordinate system for a circular and elliptical cylinders has been solved numerically for various Reynolds and Peclet numbers.

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МЕТОД ЧИСЕЛЬНОГО АНАЛІЗУ ЗАДАЧІ МАСООБМІНУ ЦИЛІНДРИЧНОГО ТІЛА З РІВНОМІРНИМ ПОСТУПАЛЬНИМ ПОТОКОМ

Актуальність. У статті розглядається задача масообміну циліндричного тіла з рівномірним поступальним потоком в'язкої нестисливої рідини.

Мета роботи – розробка нового методу чисельного аналізу задачі масообміну циліндричного тіла з рівномірним поступальним потоком, заснованого на сумісному застосуванні структурного методу R-функцій і проекційного методу Гальоркіна.

Метод. У загальному випадку задача про стаціонарний масообмін циліндричного тіла з потоком в'язкої нестисливої рідини зводиться до розв'язання рівняння гідродинамічного обтікання поверхні і рівняння для концентрації з відповідними крайовими умовами на поверхні тіла і далеко від нього. Точно врахувати геометрію області, а також крайові умови (в т.ч. і умову на нескінченності), можна, скориставшись конструктивним апаратом теорії R функцій акад. НАН України В. Л. Рвачова. В роботі на основі методів теорії R-функцій побудована повна структура розв'язку лінійної крайової задачі для концентрації, яка точно задовольняє крайовим умовам на межі тіла і умові на нескінченності, що дозволило звести задачі в нескінченній області до задач в скінченній області. Для розв'язання лінійної задачі для концентрації розроблено чисельний алгоритм на основі методу Гальоркіна.

Результати. Обчислювальний експеримент проведений для задачі обтікання кругового і еліптичного циліндрів для різних чисел Рейнольдса і Пекле.

Висновки. Проведені експерименти підтвердили працездатність запропонованого методу чисельного аналізу задачі масообміну циліндричного тіла з рівномірним поступальним потоком, заснованого на сумісному використанні структурного методу R-функцій і проекційного методу Гальоркіна. Перспективи подальших досліджень можуть полягати у використанні розробленого методу при реалізації ітераційних методів розв'язання задачі нелінійного масообміну, полудіскретних і проекційних методів розв'язання нестаціонарних задач, а також при розв'язанні задач оптимального управління відповідними технологічними процесами.

Ключові слова: задача обтікання, стаціонарна течія, в'язка рідина, функція течії, масообмін, функція концентрації, метод R-функцій, метод Гальоркіна.

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МЕТОД ЧИСЛЕННОГО АНАЛИЗА ЗАДАЧИ МАССООБМЕНА ЦИЛИНДРИЧЕСКОГО ТЕЛА С РАВНОМЕРНЫМ ПОСТУПАТЕЛЬНЫМ ПОТОКОМ

Актуальность. В статье рассматривается задача массообмена цилиндрического тела с равномерным поступательным потоком вязкой несжимаемой жидкости.

Цель работы – разработка нового метода численного анализа задачи массообмена цилиндрического тела с равномерным поступательным потоком, основанного на совместном применении структурного метода R-функций и проекционного метода Галеркина.

Метод. В общем случае задача о стационарном массообмене цилиндрического тела с потоком вязкой несжимаемой жидкости сводится к решению уравнения гидродинамического обтекания поверхности и уравнения для концентрации с соответствующими краевыми условиями на поверхности тела и вдали от него. Точно учесть геометрию области, а также краевые условия (в т.ч. и условие на бесконечности), можно, воспользовавшись конструктивным аппаратом теории R-функций акад. НАН Украины В. Л. Рвачева. В работе на основании методов теории R-функций построена полная структура решения линейной краевой задачи для концентрации, точно удовлетворяющая краевым условиям на границе и условию на бесконечности, что позволило свести задачи в бесконечной области к задачам в конечной области. Для решения линейной задачи для концентрации разработан численный алгоритм на основании метода Галеркина.

Результаты. Вычислительный эксперимент проведен для задачи обтекания кругового и эллиптического цилиндров при различных числах Рейнольдса и Пекле.

Выводы. Проведенные эксперименты подтвердили работоспособность предложенного метода численного анализа задачи массо-обмена цилиндрического тела с равномерным поступательным потоком, основанного на совместном применении структурного мето-да *R*-функций и проекционного метода Галеркина. Перспективы дальнейших исследований могут заключаться в использовании разра-ботанного метода при реализации итерационных методов решения задачи нелинейного массообмена, полудискретных и проекцион-ных методов решения нестационарных задач, а также при решении задач оптимального управления соответствующими технологическими процессами.

Ключевые слова: задача обтекания, стационарное течение, вязкая жидкость, функция тока, массообмен, функция концентрации, метод *R*-функций, метод Галеркина.

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