

THE NON-LINEAR REGRESSION MODEL TO ESTIMATE THE SOFTWARE SIZE OF OPEN SOURCE JAVA-BASED SYSTEMS

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ABSTRACT

Context. The problem of estimating the software size in the early stage of a software project is important, since the information obtained from estimating the software size is used for predicting the software development effort, including open-source Java-based information systems. The object of the study is the process of estimating the software size of open-source Java-based information systems. The subject of the study is the regression models for estimating the software size of open-source Java-based information systems.

Objective. The goal of the work is the creation of the non-linear regression model for estimating the software size of open-source Java-based information systems on the basis of the Johnson multivariate normalizing transformation.

Method. The model, confidence and prediction intervals of multiply non-linear regression for estimating the software size of open-source Java-based information systems are constructed on the basis of the Johnson multivariate normalizing transformation for non-Gaussian data with the help of appropriate techniques. The techniques to build the models, equations, confidence and prediction intervals of non-linear regressions are based on the multiple non-linear regression analysis using the multivariate normalizing transformations. The appropriate techniques are considered. The techniques allow to take into account the correlation between random variables in the case of normalization of multivariate non-Gaussian data. In general, this leads to a reduction of the mean magnitude of relative error, the widths of the confidence and prediction intervals in comparison with the linear models or nonlinear models constructed using univariate normalizing transformations.

Results. Comparison of the constructed model with the linear model and non-linear regression models based on the decimal logarithm and the Johnson univariate transformation has been performed.

Conclusions. The non-linear regression model to estimate the software size of open-source Java-based information systems is constructed on the basis of the Johnson multivariate transformation for S_B family. This model, in comparison with other regression models (both linear and non-linear), has a larger multiple coefficient of determination, a larger value of percentage of prediction and a smaller value of the mean magnitude of relative error. The prospects for further research may include the application of other multivariate normalizing transformations and data sets to construct the non-linear regression model for estimating the software size of open-source Java-based information systems.

KEYWORDS: software size estimation, Java-based information system, non-linear regression model, univariate normalizing transformation, non-Gaussian data.

ABBREVIATIONS

HTML is hypertext markup language;
JSP is Java server pages;
KLOC is the thousand lines of code;
LB is lower bound;
MD is Mahalanobis distance;
MMR is a magnitude of relative error;
MMRE is a mean magnitude of relative error;
PHP is hypertext preprocessor;
PRED is percentage of prediction;
SQL is structured query language;
UB is upper bound;
VBA is visual Basic for application.

NOMENCLATURE

$\hat{\mathbf{b}}$ is estimator for vector of linear regression equation parameters, $\mathbf{b} = \{b_1, b_2, \dots, b_k\}^T$;
 \hat{b}_i is estimator for the i -th parameter of linear regression equation;
 k is a number of independent variables (regressors);
 N is a number of data points;

$N(0,1)$ is a Gaussian distribution with zero mathematical expectation and unit variance;

\mathbf{P} is a non-Gaussian random vector,

$$\mathbf{P} = \{Y, X_1, X_2, \dots, X_k\}^T;$$

R^2 is a multiple coefficient of determination;

$$\mathbf{S}_N$$
 is a sample covariance matrix, $\mathbf{S}_N = [S_{ij}]$;

\mathbf{T} is a Gaussian random vector,

$$\mathbf{T} = \{Z_Y, Z_1, Z_2, \dots, Z_k\}^T;$$

$t_{\alpha/2, v}$ is a quantile of student's t -distribution with v degrees of freedom and $\alpha/2$ significance level;

X_1 is a total number of classes;

X_2 is a total number of relationships;

X_3 is an average number of attributes per class,

Y is an actual software size in KLOC;

\mathbf{Z}_X^+ is a matrix of centered regressors that contains the values $Z_{1_i} - \bar{Z}_1, Z_{2_i} - \bar{Z}_2, \dots, Z_{k_i} - \bar{Z}_k$;

\mathbf{z}_X is a vector with components Z_i ;

- $(\mathbf{z}_X)^T$ is a transpose of \mathbf{z}_X ;
- \bar{Z}_Y is a sample mean of the values of the variable Z_Y ;
- \hat{Z}_Y is a prediction linear regression equation result;
- α is a significance level;
- β_2 is a multivariate kurtosis;
- γ is a vector of parameters of the Johnson multivariate translation, $\gamma = (\gamma_Y, \gamma_1, \gamma_2, \dots, \gamma_k)^T$;
- ε is a Gaussian random variable which defines residuals, $\varepsilon \sim N(0,1)$;
- η is a vector of parameters of the Johnson multivariate translation, $\eta = diag(\eta_Y, \eta_1, \dots, \eta_k)$;
- λ is a vector of parameters of the Johnson multivariate translation, $\lambda = diag(\lambda_Y, \lambda_1, \dots, \lambda_k)$;
- v is a number of degrees of freedom;
- Σ is a covariance matrix, $\Sigma = [\Sigma_{ij}]$;
- φ is a vector of parameters of the Johnson multivariate translation, $\varphi = (\varphi_Y, \varphi_1, \varphi_2, \dots, \varphi_k)^T$;
- ψ is a vector of multivariate normalizing transformation, $\psi = \{\psi_Y, \psi_1, \psi_2, \dots, \psi_k\}^T$.

INTRODUCTION

Java is a programming language and computing platform first released by Sun Microsystems in 1995 (<https://www.java.com>). Now Java is used practically everywhere from laptops to datacenters, game consoles to supercomputers, cell phones to the Internet, including information systems. Software size is one of the most important internal metrics of software including software of open-source Java-based information systems.

The information obtained from estimating the software size are useful for predicting the software development effort by such well-known model as COCOMO II. This leads to the need to develop appropriate models to estimate the software size [1–4].

The paper [2] proposed the linear regression equations for estimating the software size of some programming languages including Java. The proposed equation is constructed by multiple linear regression analysis on the basis of the metrics that can be measured from conceptual data model based a class diagram. However, there are four basic assumptions that justify the use of linear regression models, one of which is normality of the error distribution. But this assumption is valid only in particular cases. This leads to the need to use the non-linear regression models including for estimating the software size of Java-based open-source information systems.

The object of study is the process of estimating the software size of open-source Java-based information systems.

The subject of study is the non-linear regression models to estimate the software size of open-source Java-based information systems.

The known regression equation for estimating the software size of open-source Java-based information systems [2] is linear and generally have large widths of confidence and prediction intervals.

The purpose of the work is to construct the non-linear regression model for estimating the software size of open-source Java-based information systems. The software size prediction results by constructed model should be better in comparison with other regression models, both linear and nonlinear, primarily on such standard evaluations as the multiple coefficient of determination and mean magnitude of relative error.

1 PROBLEM STATEMENT

Suppose given the original sample as the four-dimensional non-Gaussian data set: actual software size in the thousand lines of code (KLOC) Y , the total number of classes X_1 , the total number of relationships X_2 and the average number of attributes per class X_3 in conceptual data model from N information systems developed using the Java programming language with JSP, HTML and SQL. Suppose that there are bijective multivariate normalizing transformation of non-Gaussian random vector $\mathbf{P} = \{Y, X_1, X_2, \dots, X_k\}^T$ to Gaussian random vector $\mathbf{T} = \{Z_Y, Z_1, Z_2, \dots, Z_k\}^T$ is given by

$$\mathbf{T} = \psi(\mathbf{P}) \quad (1)$$

and the inverse transformation for (1)

$$\mathbf{P} = \psi^{-1}(\mathbf{T}). \quad (2)$$

It is required to build the non-linear regression model in the form $Y = Y(X_1, X_2, X_3, \varepsilon)$ on the basis of the transformations (1) and (2).

2 REVIEW OF THE LITERATURE

In paper [2] the linear regression equation for estimating the software size of open-source Java-based information systems was proposed in the form

$$\hat{Y} = \hat{b}_0 + \hat{b}_1 X_1 + \hat{b}_2 X_2 + \hat{b}_3 X_3, \quad (3)$$

where $\hat{b}_0 = -10.121$, $\hat{b}_1 = 1.201$, $\hat{b}_2 = 1.439$ and $\hat{b}_3 = 0.726$.

A normalizing transformation is often a good way to build the models, equations, confidence and prediction intervals of non-linear regressions [5–8]. According to [7] transformations are made for essentially four purposes, two of which are: firstly, to obtain approximate normality for the distribution of the error term (residuals) or the

dependent random variable, secondly, to transform the response and/or the predictor in such a way that the strength of the linear relationship between new variables (normalized variables) is better than the linear relationship between dependent and independent random variables.

Well-known techniques for building the equations, confidence and prediction intervals of multivariate non-linear regressions are based on the univariate normalizing transformations (such as, the decimal logarithm, Box-Cox transformation), which do not take into account the correlation between random variables in the case of normalization of multivariate non-Gaussian data. Application of such univariate normalizing transformations for building the non-linear regression models does not always lead to good normality and linear relationship between normalized variables. This leads to the need to use the multivariate normalizing transformations.

In [9] the techniques to build the equations, confidence and prediction intervals of non-linear regressions for multivariate non-Gaussian data on the basis of the bijective multivariate normalizing transformations were proposed. The techniques consist of three steps. In the first step, a set of multivariate non-Gaussian data is normalized using a bijective multivariate normalizing transformation. In the second step, the equation, confidence and prediction intervals of linear regression for the normalized data are built. In the third step, the equations, confidence and prediction intervals of non-linear regressions for multivariate non-Gaussian data are constructed on the basis of the equation, confidence and prediction intervals of linear regression for the normalized data and the multivariate normalizing transformation. Note there is no the error term in non-linear regression equation. The absence of the error term in non-linear regression equation does not allow modeling the random dependent variable for its prediction. This leads to the need to develop the non-linear regression model for estimating the software size of open-source Java-based information systems.

3 MATERIALS AND METHODS

After normalizing the non-Gaussian data by the transformation (1) the linear regression model is built for normalized data. The linear regression model for normalized data according to (1) will have the form

$$Z_Y = \hat{Z}_Y + \varepsilon = \bar{Z}_Y + (\mathbf{Z}_X^+)^T \hat{\mathbf{b}} + \varepsilon. \quad (4)$$

After that the non-linear regression model is built on the basis of the linear regression model (4) for the normalized data and the transformations (1) and (2). The non-linear regression model will have the form

$$Y = \psi_Y^{-1} [\bar{Z}_Y + (\mathbf{Z}_X^+)^T \hat{\mathbf{b}} + \varepsilon]. \quad (5)$$

The technique to build a confidence interval of non-linear regression is based on transformations (1) and (2), and a confidence interval of linear regression for normalized data

$$\hat{Z}_Y \pm t_{\alpha/2,v} S_{Z_Y} \left\{ \frac{1}{N} + (\mathbf{Z}_X^+)^T \left[(\mathbf{Z}_X^+)^T \mathbf{Z}_X^+ \right]^{-1} (\mathbf{Z}_X^+) \right\}^{1/2},$$

$$\text{where } S_{Z_Y}^2 = \frac{1}{v} \sum_{i=1}^N (Z_{Y_i} - \hat{Z}_{Y_i})^2, \quad v = N - k - 1; \quad (\mathbf{Z}_X^+)^T \mathbf{Z}_X^+$$

is the $k \times k$ matrix

$$(\mathbf{Z}_X^+)^T \mathbf{Z}_X^+ = \begin{pmatrix} S_{Z_1 Z_1} & S_{Z_1 Z_2} & \dots & S_{Z_1 Z_k} \\ S_{Z_2 Z_1} & S_{Z_2 Z_2} & \dots & S_{Z_2 Z_k} \\ \dots & \dots & \dots & \dots \\ S_{Z_k Z_1} & S_{Z_k Z_2} & \dots & S_{Z_k Z_k} \end{pmatrix},$$

$$\text{where } S_{Z_q Z_r} = \sum_{i=1}^N [Z_{q_i} - \bar{Z}_q] [Z_{r_i} - \bar{Z}_r], \quad q, r = 1, 2, \dots, k.$$

The confidence interval for non-linear regression is built on the basis of the interval (6) and inverse transformation (2)

$$\psi_Y^{-1} \left(\hat{Z}_Y \pm t_{\alpha/2,v} S_{Z_Y} \left\{ \frac{1}{N} + (\mathbf{Z}_X^+)^T \left[(\mathbf{Z}_X^+)^T \mathbf{Z}_X^+ \right]^{-1} (\mathbf{Z}_X^+) \right\}^{1/2} \right).$$

The technique to build a prediction interval is based on multivariate transformation (1), the inverse transformation (2) and a prediction interval for normalized data

$$\hat{Z}_Y \pm t_{\alpha/2,v} S_{Z_Y} \left\{ 1 + \frac{1}{N} + (\mathbf{Z}_X^+)^T \left[(\mathbf{Z}_X^+)^T \mathbf{Z}_X^+ \right]^{-1} (\mathbf{Z}_X^+) \right\}^{1/2} \quad (7)$$

The prediction interval for non-linear regression is built on the basis of the interval (7) and inverse transformation (2)

$$\psi_Y^{-1} \left(\hat{Z}_Y \pm t_{\alpha/2,v} S_{Z_Y} \left\{ 1 + \frac{1}{N} + (\mathbf{Z}_X^+)^T \left[(\mathbf{Z}_X^+)^T \mathbf{Z}_X^+ \right]^{-1} (\mathbf{Z}_X^+) \right\}^{1/2} \right).$$

For normalizing the multivariate non-Gaussian data, we use the Johnson translation system. In our case the Johnson normalizing translation is given by [10]

$$\mathbf{T} = \boldsymbol{\gamma} + \boldsymbol{\eta} \mathbf{h} \lambda^{-1} (\mathbf{P} - \boldsymbol{\varphi}), \quad (8)$$

where $\mathbf{h}[(y_Y, y_1, \dots, y_k)] = \{h_Y(y_Y), h_1(y_1), \dots, h_k(y_k)\}^T$; $h_i(\cdot)$ is one of the translation functions

$$h = \begin{cases} \ln(y), & \text{for } S_L \text{ (log normal) family;} \\ \ln[y/(1-y)], & \text{for } S_B \text{ (bounded) family;} \\ \text{Arsh}(y), & \text{for } S_U \text{ (unbounded) family;} \\ y & \text{for } S_N \text{ (normal) family.} \end{cases} \quad (9)$$

Here $y = (X - \varphi)/\lambda$; $\text{Arsh}(y) = \ln\left(y + \sqrt{y^2 + 1}\right)$. In our

case X equals Y , X_1 , X_2 or X_3 respectively.

The equation, confidence and prediction intervals of non-linear regression to estimate the software size of open-source Java-based systems are constructed on the basis of the Johnson multivariate normalizing transformation for the four-dimensional non-Gaussian data set: actual software size in the thousand lines of code (KLOC) Y , the total number of classes X_1 , the total number of relationships X_2 and the average number of attributes per class X_3 in conceptual data model from 30 information systems developed using the Java programming language with JSP, HTML and SQL. Table 1 contains the data from [2] on four metrics of software for 30 open-source Java-based systems.

Table 1 – The data set and squared MDs

No	Y	X_1	X_2	X_3	Squared MD	
					univariate	multivariate
1	11.717	8	6	4.25	0.99	0.99
2	47.52	23	19	9.565	1.65	1.50
3	84.01	26	40	11.462	10.13	6.55
4	26.999	15	14	8.933	1.33	1.89
5	41.72	20	15	5.9	0.25	0.50
6	13.015	5	6	12.4	5.89	6.64
7	30.402	18	7	6.611	1.59	3.10
8	29.159	23	10	6.957	2.26	3.13
9	53.443	28	25	4.179	3.53	4.02
10	18.694	13	9	6.615	0.39	0.89
11	26.384	16	6	5.125	2.23	4.06
12	38.721	19	16	6.579	0.19	0.19
13	75.643	26	30	6.154	1.87	2.74
14	46.72	21	24	6.048	1.31	1.66
15	6.413	7	5	4.143	2.41	6.07
16	79.534	20	37	4.85	7.06	8.20
17	36.343	18	17	5.333	0.35	0.47
18	59.684	22	31	6.182	2.49	2.62
19	50.454	15	20	11.6	2.51	3.35
20	3.055	4	1	7	10.83	7.10
21	63.257	34	17	3.971	9.16	8.29
22	91.28	35	28	13.571	17.73	11.22
23	32.707	11	17	7.545	0.98	1.54
24	11	5	5	3.6	6.15	6.36
25	5.543	6	4	3.833	2.54	5.16
26	22.686	12	11	6.667	0.11	0.22
27	3.911	3	2	6.667	7.26	5.52
28	20.841	14	7	3	8.17	7.21
29	9.269	6	5	3.5	3.23	2.82
30	7.732	7	2	11.143	5.42	5.98

For detecting the outliers in the data from Table 1 we use the technique based on multivariate normalizing transformations and the squared Mahalanobis distance (MD) [11]. There are no outliers in the data from Table 1 for 0.005 significance level and the Johnson multivariate transformation (8) for S_B family. The same result was obtained in [12] for the transformation (8) for S_U family. In [2] it was also assumed that the data contains no outliers. The values of squared MD for normalized data by the Johnson univariate transformation (9) for S_B family from Table 1 indicate the data of system 22 is multivariate outlier, since for this data row the squared MD equals to 17.73 is greater than the value of the quantile of the Chi-Square distribution, which equals to 14.86 for 0.005 significance level. Although note that without using normalization, the data of system 11 is multivariate outlier, since for this data row the squared MD equals to 15.44.

Parameters of the multivariate transformation (9) for S_B family were estimated by the maximum likelihood method. Estimators for parameters of the transformation (9) are: $\hat{\gamma}_Y = 9.63091$, $\hat{\gamma}_1 = 15.5355$, $\hat{\gamma}_2 = 25.4294$, $\hat{\gamma}_3 = 0.72801$, $\hat{\eta}_Y = 1.05243$, $\hat{\eta}_1 = 1.58306$, $\hat{\eta}_2 = 2.54714$, $\hat{\eta}_3 = 0.54312$, $\hat{\varphi}_Y = -1.4568$, $\hat{\varphi}_1 = -1.8884$, $\hat{\varphi}_2 = -6.9746$, $\hat{\varphi}_3 = 3.2925$, $\hat{\lambda}_Y = 153102.605$, $\hat{\lambda}_1 = 243051.0$, $\hat{\lambda}_2 = 311229.5$ and $\hat{\lambda}_3 = 13.90$. The sample covariance matrix \mathbf{S}_N of the \mathbf{T} is used as the approximate moment-matching estimator of Σ

$$\mathbf{S}_N = \begin{pmatrix} 1.0000 & 0.9514 & 0.9333 & 0.1574 \\ 0.9514 & 1.0000 & 0.9006 & 0.1345 \\ 0.9333 & 0.9006 & 1.0000 & 0.0554 \\ 0.1574 & 0.1345 & 0.0554 & 1.0000 \end{pmatrix}.$$

After normalizing the non-Gaussian data by the multivariate transformation (9) for S_B family the linear regression model (3) is built for normalized data

$$Z_Y = \hat{Z}_Y + \varepsilon = \hat{b}_0 + \hat{b}_1 Z_1 + \hat{b}_2 Z_2 + \hat{b}_3 Z_3 + \varepsilon. \quad (10)$$

Parameters of the linear regression model (10) were estimated by the least square method. Estimators for parameters of the equation (10) are such: $\hat{b}_0 = 1.02 \cdot 10^{-5}$, $\hat{b}_1 = 0.56085$, $\hat{b}_2 = 0.42491$, $\hat{b}_3 = 0.05846$.

After that the non-linear regression model (4) is built

$$Y = \hat{\phi}_Y + \hat{\lambda}_Y \left[1 + e^{-(\hat{Z}_Y + \varepsilon - \hat{\gamma}_Y)/\hat{\eta}_Y} \right]^{-1}. \quad (11)$$

where $Z_j = \gamma_j + \eta_j \ln \frac{X_j - \varphi_j}{\varphi_j + \lambda_j - X_j}$, $\varphi_j < X_j < \varphi_j + \lambda_j$, $j = 1, 2, 3$.

The model (11) is the non-linear regression model to estimate the software size of open-source Java-based information systems.

4 EXPERIMENTS

For comparison of the model (11) with other models two non-linear regression models are built on the basis of the data from Table 1 and two univariate normalizing transformations: the decimal logarithm transformation and the Johnson transformation.

The non-linear regression model is constructed on the basis of the linear regression model (4) for the normalized data and the decimal logarithm transformation

$$Y = 10^{\varepsilon + \hat{b}_0} X_1^{\hat{b}_1} X_2^{\hat{b}_2} X_3^{\hat{b}_3}. \quad (12)$$

where the estimators for parameters of the model (12) are:
 $\hat{b}_0 = -0.04536$, $\hat{b}_1 = 0.64235$, $\hat{b}_2 = 0.56305$ and
 $\hat{b}_3 = 0.18045$.

The non-linear regression model is constructed on the basis of the linear regression model (4) for the normalized data and the Johnson univariate transformation for S_B family. In this case the estimators for parameters of the model (11) are: $\hat{\gamma}_Y = 0.46387$, $\hat{\gamma}_1 = 0.38093$, $\hat{\gamma}_2 = 0.60545$, $\hat{\gamma}_3 = 0.65592$, $\hat{\eta}_Y = 0.50326$,

$$\begin{aligned} \hat{\eta}_1 &= 0.62689, & \hat{\eta}_2 &= 0.62215, & \hat{\eta}_3 &= 0.72789, \\ \hat{\phi}_Y &= 2.817, & \hat{\phi}_1 &= 2.634, & \hat{\phi}_2 &= 0.700, & \hat{\phi}_3 &= 2.839, \\ \hat{\lambda}_Y &= 89.930, & \hat{\lambda}_1 &= 33.711, & \hat{\lambda}_2 &= 41.428, & \hat{\lambda}_3 &= 11.780, \\ \hat{b}_0 &= 0, & \hat{b}_1 &= 0.46976, & \hat{b}_2 &= 0.53539 \text{ and } \hat{b}_3 &= 0.11397. \end{aligned}$$

The computer program implementing the constructed models (11) and (12) was developed to conduct experiments. The program was written in the sci-language for the Scilab system. Scilab (<http://www.scilab.org>) is the free and open source software, the alternative to commercial packages for system modeling and simulation packages such as MATLAB and MATRIXx.

5 RESULTS

If the Gaussian random variable ε equals zero the regression models (11) and (12) are the non-linear regression equations for which the prediction results for values of components of vector $\mathbf{X} = \{X_1, X_2, X_3\}$ from Table 1 and values of MRE are shown in the Table 2. The prediction results by model (11) and values of MRE are shown in the Table 2 for two cases: the Johnson univariate and multivariate normalizing transformations. Table 2 also contains the prediction results by linear regression equation (3) from [2] for values of components of vector \mathbf{X} from Table 1 and MRE values. Note, all prediction results by linear regression equation (3), non-linear regression models (11) and (12) are positive.

Table 2 – The prediction results and confidence intervals of regressions for 30 open-source Java-based systems

No	Linear regression				Non-linear regression							
					univariate normalizing transformation				the Johnson multivariate normalizing transformation			
	the decimal logarithm		the Johnson transformation		the Johnson transformation		the Johnson multivariate normalizing transformation					
	\hat{Y}	RME	LB	UB	\hat{Y}	RME	LB	UB	\hat{Y}	RME	LB	UB
1	11.205	0.0437	8.301	14.109	12.197	0.0410	10.923	13.620	10.671	0.0893	8.989	12.756
2	51.784	0.0897	48.396	55.171	53.248	0.1205	47.404	59.812	55.919	0.1768	49.992	61.568
3	86.989	0.0355	79.427	94.551	90.515	0.0774	77.073	106.302	87.586	0.0426	83.866	89.805
4	34.523	0.2787	31.823	37.223	33.653	0.2465	30.525	37.102	35.152	0.3020	30.502	40.115
5	39.765	0.0469	36.726	42.804	39.052	0.0639	35.787	42.615	40.333	0.0333	35.838	44.986
6	13.516	0.0385	6.843	20.188	10.941	0.1593	8.918	13.424	10.900	0.1625	7.968	15.255
7	26.365	0.1328	21.326	31.404	24.255	0.2022	21.099	27.884	24.630	0.1898	20.414	29.485
8	36.937	0.2668	30.570	43.305	35.027	0.2012	30.424	40.326	37.853	0.2982	31.806	44.302
9	62.516	0.1698	57.570	67.462	60.729	0.1363	52.927	69.681	64.288	0.2029	57.204	70.537
10	23.243	0.2433	20.890	25.595	22.674	0.2129	21.041	24.433	22.251	0.1903	19.396	25.446
11	21.446	0.1872	16.885	26.006	19.692	0.2536	17.110	22.665	18.897	0.2838	15.560	22.881
12	40.496	0.0459	38.185	42.808	39.963	0.0321	36.836	43.355	41.354	0.0680	37.021	45.812
13	68.744	0.0912	64.647	72.842	68.808	0.0904	61.557	76.912	70.832	0.0636	65.599	75.335
14	54.028	0.1564	50.795	57.260	52.739	0.1288	47.735	58.266	55.262	0.1828	49.875	60.436
15	8.487	0.3234	5.441	11.534	10.056	0.5681	8.926	11.329	8.703	0.3571	7.338	10.443
16	70.670	0.1115	61.411	79.928	62.671	0.2120	53.424	73.519	73.437	0.0767	64.730	79.996
17	39.831	0.0960	37.362	42.300	38.454	0.0581	35.184	42.028	39.331	0.0822	34.828	44.018
18	65.401	0.0958	59.939	70.864	63.010	0.0557	55.949	70.961	66.982	0.1223	60.723	72.447
19	45.094	0.1062	39.943	50.244	43.124	0.1453	37.198	49.994	47.605	0.0565	39.805	55.417
20	1.200	0.6071	-2.531	4.932	3.118	0.0206	2.525	3.850	3.430	0.1227	3.180	3.849
21	58.055	0.0822	47.956	68.154	54.860	0.1328	45.880	65.597	70.088	0.1080	58.879	78.528
22	82.053	0.1011	75.043	89.064	92.398	0.0123	77.034	110.827	88.089	0.0350	84.129	90.284
23	33.031	0.0099	28.529	37.532	29.837	0.0878	26.096	34.113	31.165	0.0471	26.305	36.522
24	5.692	0.4826	1.975	9.408	7.899	0.2819	6.666	9.359	6.502	0.4089	5.391	8.063
25	5.622	0.0143	2.367	8.878	7.921	0.4290	6.917	9.071	6.795	0.2259	5.745	8.199
26	24.958	0.1002	22.712	27.204	24.148	0.0644	22.338	26.104	23.874	0.0524	20.862	27.219
27	1.197	0.6938	-2.722	5.117	3.796	0.0295	3.169	4.547	3.548	0.0929	3.253	4.038
28	18.942	0.0911	14.883	23.001	17.897	0.1413	15.224	21.039	13.423	0.3559	9.477	19.245
29	6.820	0.2642	3.356	10.284	8.835	0.0468	7.597	10.274	7.279	0.2147	5.991	9.053
30	9.248	0.1960	3.644	14.851	7.176	0.0719	4.599	11.199	7.022	0.0918	5.480	9.390

MMRE and PRED(0.25) are accepted as standard evaluations of prediction results by regression models and equations. The acceptable values of MMRE and PRED(0.25) are not more than 0.25 and not less than 0.75 respectively. The acceptable value of R^2 is approximately the same as for PRED(0.25). The values of R^2 , MMRE and PRED(0.25) equal respectively 0.9621, 0.1734 and 0.7667 for linear regression equation (3), and equal respectively 0.9541, 0.1441 and 0.8667 for the model (12), and equal respectively 0.9574, 0.1579 and 0.8000 for the model (11) for the Johnson univariate transformation. The values of R^2 , MMRE and PRED(0.25) are better for the model (11) for the Johnson multivariate transformation in comparison with all previous equations, and are 0.9672, 0.1389 and 0.8667 respectively.

The confidence and prediction intervals of non-linear regression are defined for the data from Table 1. Table 2 contains the lower (LB) and upper (UB) bounds of the confidence intervals of linear and non-linear regressions on the basis of univariate and multivariate transformations respectively for 0.05 significance level. Note the lower bounds of the confidence interval of linear regression (3) from [2] are negative for the two rows of data: 20 and 27. All the lower bounds of the confidence interval of non-linear regressions are positive. The widths of the confidence interval of non-linear regression on the basis of the Johnson multivariate transformation are less than for linear regression (3) from [2] for 21 rows of data: 1, 3, 6–8, 10, 11, 13, 15, 16, 18, 20–25, 27–30. Also the widths of the confidence interval of non-linear regression on the basis of the Johnson multivariate transformation are less for more data rows than for non-linear regressions following the univariate transformations, both decimal logarithm and the Johnson. The widths of the confidence interval of non-linear regression on the basis of the Johnson multivariate transformation are less than following the decimal logarithm univariate transformation for 24 rows of data: 2–5, 7–9, 11–14, 16–25, 27, 29 and 30. And ones are less than following the Johnson univariate transformation for 27 rows of data: 1, 2, 4–21, 23–26, 28–30. Approximately the same results are obtained for the prediction intervals of regressions.

Table 3 contains the lower (LB) and upper (UB) bounds of the prediction intervals of linear and non-linear regressions on the basis of univariate and multivariate transformations respectively for 0.05 significance level. Note the lower bounds of the prediction interval of linear regression (3) are negative for the eight rows of data: 1, 15, 20, 24, 25, 27, 29 and 30. All the lower bounds of the prediction interval of non-linear regressions are positive. The widths of the prediction interval of non-linear regression on the basis of the Johnson multivariate transformation are less than for linear regression (3) from [2] for 16 rows of data: 1, 3, 6, 7, 10, 11, 15, 20, 22, 24–30. Also the widths of the prediction interval of non-linear regression on the basis of the Johnson multivariate transformation are less for more data rows than for non-

linear regressions following the univariate transformations, both decimal logarithm and the Johnson. The widths of the prediction interval of non-linear regression on the basis of the Johnson multivariate transformation are less than following the decimal logarithm univariate transformation for 17 rows of data: 2–5, 8, 9, 12–14, 16–22 and 27. And ones are less than following the Johnson univariate transformation for 26 rows of data: 1, 2, 4–19, 21, 23–26, 28–30.

Table 3 – The bounds of the prediction intervals

No	Bounds for linear regression		Bounds for non-linear regression			
			Johnson univariate transformation		Johnson multivariate transformation	
	LB	UB	LB	UB	LB	UB
1	-0.004	22.413	5.679	22.412	7.536	19.277
2	40.441	63.127	32.573	75.467	36.600	68.504
3	73.784	100.194	77.736	91.114	73.863	96.264
4	23.365	45.680	17.436	58.472	21.814	48.847
5	28.521	51.009	20.726	63.360	25.763	54.772
6	0.799	26.232	5.559	24.105	7.061	19.398
7	14.424	38.305	11.701	46.283	14.924	36.873
8	24.378	49.497	18.875	61.455	22.764	51.145
9	50.614	74.418	40.607	80.655	45.398	77.523
10	12.164	34.321	10.693	42.556	14.102	34.502
11	9.699	33.192	9.052	37.791	11.900	30.337
12	29.427	51.566	21.431	64.236	26.458	55.673
13	57.169	80.319	49.028	83.854	52.398	82.637
14	42.730	65.325	32.094	74.958	37.508	69.345
15	-2.759	19.733	4.920	18.108	6.290	15.899
16	56.425	84.914	51.026	85.596	52.840	84.458
17	28.727	50.935	20.068	62.454	25.536	54.455
18	53.275	77.527	43.924	81.998	48.094	79.560
19	33.105	57.082	25.246	70.052	29.796	61.411
20	-10.250	12.651	3.009	4.748	2.516	4.443
21	43.250	72.859	45.506	84.384	44.137	78.697
22	69.156	94.951	78.586	91.334	74.127	97.180
23	21.307	44.755	15.088	54.334	19.782	45.949
24	-5.754	17.138	4.077	13.054	4.988	12.419
25	-5.682	16.927	4.203	13.643	5.062	12.421
26	13.902	36.014	11.471	44.858	15.245	36.852
27	-10.316	12.711	3.048	5.105	2.834	5.520
28	7.380	30.503	6.446	29.640	9.577	26.119
29	-4.546	18.186	4.360	14.963	5.483	13.826
30	-2.942	21.438	4.206	14.773	4.601	11.617

Following [13] multivariate kurtosis β_2 is estimated for the data on metrics of software from Table I and the normalized data on the basis of the decimal logarithm transformation, the Johnson univariate and multivariate transformations for S_B family. The estimator of multivariate kurtosis given by [13]

$$\hat{\beta}_2 = \frac{1}{N} \sum_{i=1}^N \left\{ \mathbf{Z}_i - \bar{\mathbf{Z}} \right\}^T \mathbf{S}_N^{-1} \left\{ \mathbf{Z}_i - \bar{\mathbf{Z}} \right\}^2. \quad (13)$$

In our case, in the formula (13), the vectors \mathbf{Z} and $\bar{\mathbf{Z}}$ should be replaced by the vectors \mathbf{P} and $\bar{\mathbf{P}}$ or \mathbf{T} and $\bar{\mathbf{T}}$, respectively, for the initial (non-Gaussian) or normalized data. It is known that $\beta_2 = m(m+2)$ holds under multivariate normality. The given equality is a necessary

condition for multivariate normality. In our case $\beta_2 = 24$. The estimators of multivariate kurtosis equal 27.17, 22.38, 32.05 and 24.02 for the data from Table 1, the normalized data on the basis of the decimal logarithm transformation, the Johnson univariate and multivariate transformations respectively. The values of these estimators indicate that the necessary condition for multivariate normality is practically performed for the normalized data on the basis of the decimal logarithm transformation and the Johnson multivariate transformation, it does not hold for other data.

6 DISCUSSION

As it evident from the Table 2 and Table 3, the values of lower bounds of the confidence and prediction intervals of linear regression (3) from [2] for estimating the software size of open-source Java-based information systems are negative for some data rows. In our opinion, the presence of negative values may be explained by two reasons. Firstly, for the initial data from Table 1, four basic assumptions that justify the use of linear regression model, one of which is normality of the error distribution, are not valid. Secondly, there is reason to reject the hypothesis that the sample of data from Table 1 comes from a multivariate normal distribution. Note all the lower bounds of the confidence and prediction intervals of non-linear regressions are positive.

Also note that in our case for the data from Table 1, the poor normalization of multivariate non-Gaussian data using the Johnson univariate transformation leads to an increase in the widths of the confidence and prediction intervals of non-linear regression for a larger number of data rows compared to both the Johnson multivariate transformation and the decimal logarithm transformation.

The widths of the confidence and prediction intervals of non-linear regression on the basis of the Johnson multivariate transformation are less for more data rows than for linear regression and non-linear regressions following the univariate transformations, both decimal logarithm and the Johnson. Also the values of R^2 , MMRE and PRED(0.25) are better for the model (11) for the Johnson multivariate transformation in comparison with all previous equations and models, both linear and non-linear, based on univariate transformations. This may be explained best multivariate normalization and the fact that there is no reason to reject the hypothesis that the sample of data, which normalized by the Johnson multivariate transformation for S_B family, comes from a multivariate normal distribution.

CONCLUSIONS

The important problem of increase of confidence of software size estimation for open-source Java-based information systems is solved.

The scientific novelty of obtained results is that the techniques to build the non-linear regression model for multivariate non-Gaussian data on the basis of the multivariate normalizing transformations is firstly

proposed. The non-linear regression model to estimate the software size of open-source Java-based information systems is constructed on the basis of the Johnson multivariate transformation for S_B family. This model, in comparison with other regression models (both linear and non-linear), has a larger multiple coefficient of determination, a smaller value of the mean magnitude of relative error, a larger value of percentage of prediction and smaller widths of the confidence and prediction intervals of non-linear regression.

The practical significance of obtained results is that the software realizing the constructed model is developed in the sci-language for Scilab. The experimental results allow to recommend the constructed model for use in practice.

Prospects for further research may include the application of other multivariate normalizing transformations and data sets to construct the non-linear regression model for estimating the software size of open-source Java-based information systems.

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НЕЛІНІЙНА РЕГРЕСІЙНА МОДЕЛЬ ДЛЯ ОЦІНЮВАННЯ РОЗМІРУ ПРОГРАМНОГО ЗАБЕЗПЕЧЕННЯ СИСТЕМ З ВІДКРИТИМ КОДОМ НА JAVA

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АНОТАЦІЯ

Актуальність. Проблема оцінювання розміру програмного забезпечення на ранній стадії програмного проекту є важливою, оскільки інформація, отримана при оцінюванні розміру програмного забезпечення, використовується для прогнозування трудомісткості по розробці програмного забезпечення, включаючи інформаційні системи на базі Java з відкритим вихідним кодом. Об'єктом дослідження є процес оцінювання розміру програмного забезпечення інформаційних систем з відкритим вихідним кодом на Java. Предметом дослідження є моделі регресії для оцінювання розміру програмного забезпечення інформаційних систем з відкритим вихідним кодом на Java. Мета роботи – створення моделі нелінійної регресії для оцінювання розміру програмного забезпечення інформаційних систем з відкритим вихідним кодом на Java на основі багатовимірного нормалізуючого перетворення Джонсона.

Метод. Моделі, довірчі інтервали та інтервали передбачення багатовимірної нелінійної регресії для оцінювання розміру програмного забезпечення інформаційних систем з відкритим вихідним кодом на Java побудовані на основі багатовимірного нормалізуючого перетворення Джонсона для негаусівських даних за допомогою відповідних методів. Методи побудови моделей, рівнянь, довірчих інтервалів і інтервалів передбачення нелінійних регресій засновані на багатовимірному нелінійному регресійному аналізі з використанням багатовимірних нормалізуючих перетворень. Розглянуто відповідні методи. Ці методи дозволяють враховувати кореляцію між випадковими величинами в разі нормалізації багатовимірних негаусівських даних. Загалом, це призводить до зменшення середньої величини відносної похибки, ширини довірчих інтервалів і інтервалів передбачення в порівнянні з лінійними моделями або нелінійними моделями, побудованими з використанням одновимірних нормалізуючих перетворень.

Результати. Здійснено порівняння побудованої моделі з моделями лінійної регресії та нелінійними регресіями на основі десяткового логарифму та одновимірного перетворення Джонсона.

Висновки. Модель нелінійної регресії для оцінювання розміру програмного забезпечення інформаційних систем з відкритим вихідним кодом на Java побудована на основі багатовимірного перетворення Джонсона для сімейства S_B . Ця модель в порівнянні з іншими регресійними моделі (як лінійними, так і нелінійними) має більший множинний коефіцієнт детермінації і менше значення середньої величини відносної похибки. Перспективи подальших досліджень можуть включати застосування інших багатовимірних нормалізуючих перетворень і наборів даних для побудови моделі нелінійної регресії для оцінювання розміру програмного забезпечення інформаційних систем з відкритим вихідним кодом на Java.

Ключові слова: оцінювання розміру програмного забезпечення, інформаційна система на основі Java, модель нелінійної регресії, одновимірне нормалізуюче перетворення, негаусівські дані.

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НЕЛИНЕЙНА РЕГРЕССИОННАЯ МОДЕЛЬ ДЛЯ ОЦЕНКИ РАЗМЕРА ПРОГРАММНОГО ОБЕСПЕЧЕНИЯ СИСТЕМ С ОТКРЫтыМ КОДОМ НА JAVA

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АННОТАЦИЯ

Актуальность. Проблема оценки размера программного обеспечения на ранней стадии программного проекта важна, поскольку информация, полученная при оценке размера программного обеспечения, используется для прогнозирования трудоемкости по разработке программного обеспечения, включая информационные системы на базе Java с открытым исходным кодом. Объект исследования – процесс оценки размера программного обеспечения информационных систем с

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открытым исходным кодом на Java. Предмет исследования – модели регрессии для оценки размера программного обеспечения информационных систем с открытым исходным кодом на Java. Цель работы – создание модели нелинейной регрессии для оценки размера программного обеспечения информационных систем с открытым исходным кодом на Java на основе многомерного нормализующего преобразования Джонсона.

Метод. Модели, доверительные интервалы и интервалы прогнозирования многомерной нелинейной регрессии для оценки размера программного обеспечения информационных систем с открытым исходным кодом на Java построены на основе многомерного нормализующего преобразования Джонсона для негауссовых данных с помощью соответствующих методов. Методы построения моделей, уравнений, интервалов доверия и прогнозирования нелинейных регрессий основаны на многократном нелинейном регрессионном анализе с использованием многомерных нормализующих преобразований. Рассмотрены соответствующие методы. Методы позволяют учитывать корреляцию между случайными величинами в случае нормализации многомерных негауссовых данных. В общем, это приводит к уменьшению средней величины относительной погрешности, ширины доверительных интервалов и интервалов предсказания по сравнению с линейными моделями или нелинейными моделями, построенными с использованием одномерных нормализующих преобразований.

Результаты. Проведено сравнение построенной модели с линейной моделью и нелинейными регрессионными моделями на основе десятичного логарифма и одномерного преобразования Джонсона.

Выводы. Модель нелинейной регрессии для оценки размера программного обеспечения информационных систем с открытым исходным кодом на Java построена на основе многомерного преобразования Джонсона для семейства S_B . Эта модель по сравнению с другими регрессионными моделями (как линейными, так и нелинейными) имеет больший множественный коэффициент детерминации и меньшее значение средней величины относительной погрешности. Перспективы дальнейших исследований могут включать применение других многомерных нормализующих преобразований и наборов данных для построения модели нелинейной регрессии для оценки размера программного обеспечения информационных систем с открытым исходным кодом на Java.

Ключевые слова: оценка размера программного обеспечения, информационная система на основе Java, модель нелинейной регрессии, одномерное нормализующее преобразование, негауссовские данные.

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