РАДІОЕЛЕКТРОНІКА ТА ТЕЛЕКОМУНІКАЦІЇ

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PRINCIPLES AND METHODS OF THE CALCULATION OF TRANSFER CHARACTERISTICS OF DISK PIEZOELECTRIC TRANSFORMERS

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ABSTRACT

Context. Thanks to its unique properties piezoceramics has applications in various fields of engineering and technology. Disk piezoelectric devices are widely used in the elements of information systems. Research has shown that piezoelectric transformers can compete with traditional electromagnetic transformers on both efficiency and power density. The final goal of mathematical modeling of the vibrating piezoelectric elements physical condition is a qualitative and quantitative description of characteristics and parameters of existing electrical and elastic fields.

Objective. The purpose of this paper is to set out the principles of mathematical models construction that are sufficiently adequate to real devices and occurring physical processes using the simplest example of axially symmetric radial oscillations of the piezoelectric disk.

Method. Mathematical models of piezoelectric transformers working with axially symmetric radial oscillations of piezoceramic disks are constructed with a minimal number of assumptions simplifying the real situation. This allows us to state that the proposed construction scheme delivers mathematical models that are sufficiently adequate to the real objects and physical processes that exist in them.

Results. Main results of this work can be formulated as follows: mathematical model of piezoelectric transformer with ring electrode in the primary electrical circuit is constructed; high sensitivity of frequency characteristic of piezoelectric transformer to the values of the output impedance of the electrical signal source in the primary electrical circuit is demonstrated.

Conclusions. As a result of research of real device's mathematical model a set of geometrical, physical and mechanical and electrical parameters of a real object can be determined which provides realization of technical parameters of piezoelectric functional element specified in technical specifications. The cost of the saved resources is the commercial price of the mathematical model. Prospects for further research can be to build a mathematical model of a piezoelectric transformer with sector electrodes.

KEYWORDS: piezoelectric transformer, axially symmetric oscillations, physical processes, mathematical model.

NOMENCLATURE

U_1 is an amplitude value of electric potential difference;

$i=\sqrt{-1}$ is an imaginary unit;

ω is an angular frequency;

t is a time;

x_k are coordinates of the point, in which it is determined the displacement of the piezoelectric material particles from the equilibrium position;

Π is a set of geometrical and physical and mechanical properties of the piezoelectric transformer;

Z_1 is an electrical impedance of the input electrode 1;

I_1 is an amplitude value of the electric current in the conductor, which connects an input electrode 1 with a source of the electrical signals;

$2d_1$ is a width of the input ring electrode;

$\vec{H}$, $\vec{E}$ are vectors of the conjugate magnetic and electric fields;

$\vec{B}$, $\vec{D}$ are vectors of the magnetic and electric induction of the electromagnetic field components;

u_ρ, u_z are amplitude values of the radial and axial components of the material particles displacement vector of dynamically deformable piezoelectric disk.

INTRODUCTION

Thanks to its unique properties piezoceramics has applications in various fields of engineering and

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technology. The relevance of the use of various functional elements of piezoelectronics in radio electronics, information and power systems is explained by their high reliability and small dimensions, which solves the problem of miniaturization of such systems. Piezoelectric disks with surfaces partially covered electrodes are often used to create various functional piezoelectronic devices. Disk piezoelectric devices are widely used in the elements of information systems. In disk piezoelectric elements with surfaces partially covered by electrodes we can simultaneously excite oscillations of compression-tension and transverse bending vibrations. Manipulating the geometric parameters of electrodes and their location relative to each other, you can have a significant effect on the energy of oscillatory motion particular type of material particles of piezoelectric disk volume. It should be especially noted that this piezoelectric element has compatibility with microsystem technology, so it can be made as microelectromechanical structures (MEMS) [1]. One of the main elements of functional piezoelectronics is piezoelectric transformer (PT). Research has shown that PTs can compete with traditional electromagnetic transformers on both efficiency and power density [2–4]. PTs are therefore an interesting field of research [5]. The favorable attributes of the PT are low weight and size and potentially low cost. One additional important characteristic is the high voltage isolation of the ceramic materials used to build PTs [6]. In addition, a piezoelectric transformer is more suitable for mass production than traditional, coil-based transformers [7].

1 PROBLEM STATEMENT

The operation principle of piezoelectric transformers is generally known [8].

When applying an electrical potential difference $U_1 e^{i\omega t}$ to pair of electrodes that are partially cover the front and bottom surfaces of the piezoelectric plate, harmonic oscillations of material particles are excited in a volume of the plate, which, in general, can be described by the displacement vector of material particles $\vec{u}\left(x_k\right)e^{i\omega t}$. Fluctuations of material particles are accompanied by dynamic deformations $\varepsilon_{mn}\left(x_k\right)e^{i\omega t}$ of infinitely small elements of a piezoelectric volume. Due to the direct piezoelectric effect the harmonically varying in time according to $e^{i\omega t}$ polarization charges with a surface density $q_m\left(x_k\right)e^{i\omega t}$ arise in a deformable piezoelectric. Some of these charges are collected by the second pair of electrodes, which like the first pair, partially covers the surface of the piezoelectric plate. The polarization charge on the second pair of electrodes causes an electric current $i\left(t\right) = I\,e^{i\omega t}$ in the conductor, which connects one of the electrodes of the second pair to the load impedance Z_n. The voltage drop $U_2 e^{i\omega t} = Z_n I\,e^{i\omega t}$ is an output signal of the piezoelectric transformer. Obviously, the transformation ratio $K\left(\omega\,,\Pi\right)$ is equal to the ratio of the output signal to the input one, i.e.

$$K\left(\omega,\Pi\right) = \frac{U_2}{U_1} = \frac{Z_n\,I}{U_1},$$

and is a mathematical model of a piezoelectric transformer [9].

The practical value of the analytical structure $K\left(\omega,\Pi\right)$ that adequately describes the physical processes in the real object is evident.

2 REVIEW OF THE LITERATURE

Many publications have been devoted to the construction and research of mathematical models of piezoelectric transformers. Starting with the monograph [8], the basics of the calculation of piezoelectric transformers' transfer characteristics were considered, for example, in [10–13].

However, in many papers only processes occurring in a piezoelectric disk with a surface, fully covered by electrodes, are described. There are also a number of works of a disparate character devoted to the solution of the problem of electromechanical oscillations of piezoelectric elements with separated electrodes (transformer type). The constructions of piezoelectric transformer of a planar transverse-longitudinal and rod type are considered in [10] and [11], respectively. In [12] the analysis of the dependence of transformation coefficient of disk piezoelectric transformer on the location of secondary electrode, on the width of secondary electrode, and on the value of electrical load applied to secondary electrode was made. In [13] the radial axisymmetric oscillations of thin piezoceramic disk with a surface, partially covered by electrodes, are considered.

In many papers [14–19] the described methods of piezoelectric transformers models constructing are mostly based on the use of equivalent electrical circuits and it does not allow analyzing of stress-strain state of solids with the piezoelectric effects.

Based on the above, it can be argued that currently there are no reliable and valid methods of constructing of mathematical models of piezoelectric transformers, which could be used as a theoretical basis for characteristics and parameters calculating of this class of functional elements of modern piezoelectronics.

The purpose of this paper is to set out the principles of mathematical models construction that are sufficiently adequate to real devices and occurring physical processes using the simplest example of axially symmetric radial oscillations of the piezoelectric disk.

3 MATERIALS AND METHODS

Let us consider the disk with the radius R and the thickness α (Fig. 1) made of piezoelectric ceramics PZT with thickness polarization during its manufacture i.e. along the coordinate axis z of the cylindrical coordinate

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system (ρ, φ, z). Electric polarization direction defines the properties and the matrices construction of piezoceramic disk's material constants.

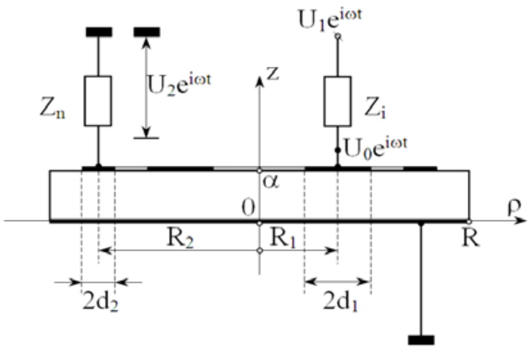

Figure 1 – Diagram of the piezoelectric disk transformer that operates on radial vibrations

The matrix of elastic moduli of piezoceramic disk polarized across the thickness looks like

$$\left|c_{\lambda\beta}^E\right| = \begin{vmatrix} c_{11}^E & c_{12}^E & c_{13}^E & 0 & 0 & 0 \\ & c_{22}^E & c_{23}^E & 0 & 0 & 0 \\ & & c_{33}^E & 0 & 0 & 0 \\ & & & c_{44}^E & 0 & 0 \\ & & & & c_{55}^E & 0 \\ & & & & & c_{66}^E \end{vmatrix}, \quad (1)$$

where $\lambda, \beta = 1; ...; 6$ are Voigt indices; $c_{11}^E = c_{22}^E \neq c_{33}^E$; $c_{12}^E = c_{13}^E = c_{23}^E$; $c_{44}^E = c_{55}^E$; $c_{66}^E = \left(c_{11}^E - c_{12}^E\right)/2$.

The matrix of piezomoduli $e_{k\beta}$ ($k = 1;2;3;\ \beta = 1;2;...;6$) can be written as follows [10]

$$\left|e_{k\beta}\right| = \begin{vmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 \end{vmatrix}, \quad (2)$$

where $e_{31} = e_{32} \neq e_{33}$; $e_{15} = e_{24} = \left(e_{33} - e_{31}\right)/2$.

The matrix of the dielectric permittivity tensor χ_{mn}^ε has diagonal structure and

$$\left|\chi_{mn}^\varepsilon\right| = \begin{vmatrix} \chi_{11}^\varepsilon & 0 & 0 \\ & \chi_{22}^\varepsilon & 0 \\ & & \chi_{33}^\varepsilon \end{vmatrix}, \quad (3)$$

where $\chi_{11}^\varepsilon = \chi_{22}^\varepsilon \neq \chi_{33}^\varepsilon$.

Let us assume that the thickness of the electrodes is negligible in comparison with the disk thickness.

On the ring electrode 1 (its width is equal to $2d_1$ (Fig. 1)) the electrical potential difference $U_1 e^{i\omega t}$ from a source of electrical signals with the output impedance Z_i is applied. Obviously, on the electrode 1 we will have another value of the electrical potential $U_0 e^{i\omega t}$, where $\left|U_0\right| < U_1$, that can be written as follows

$$U_0 = \frac{U_1 Z_1}{Z_i + Z_1}. \quad (4)$$

Electrical impedance Z_1 can be determined from Ohm's law for electrical circuit section

$$Z_1 = U_0 / I_1. \quad (5)$$

If on the surface of the electrode 1 we have harmonically time varying electric charge $q(t) = Q_1 e^{i\omega t}$, the electric current amplitude value is determined as follows [20]

$$I_1 = -i\omega Q_1. \quad (6)$$

The amplitude value of the electric charge Q_1 is determined by the axial component $D_z(\rho, \alpha)$ of the electric induction vector

$$Q_1 = 2\pi \int_{R_1 - d_1}^{R_1 + d_1} \rho\, D_z(\rho, \alpha)\, d\rho. \quad (7)$$

Electrical condition of any material object is determined by Maxwell's equations

$$rot\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \quad (8)$$

$$rot\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (9)$$

where $\vec{J} = r\vec{E}$ is a surface density of the conduction current; r is a specific electric conductivity of the material. Since the piezoelectric ceramic is a fairly good isolator it can be considered that $r \cong 0$. In this case, Maxwell's equation (8) for harmonically varying fields takes the following form

$$rot\,\vec{H} = i\omega \vec{D}. \quad (10)$$

Calculating the divergence of the left and right side of (10), we can come to the following conclusion

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$$div\,\vec{D} = 0\,. \qquad (11)$$

Equation (11) has the meaning of the condition of absence of free carriers of electricity in a volume of the ideal dielectric.

In [21] it is shown that at a frequency range up to 10 MHz, the magnetic component of the electromagnetic field in a deformable piezoelectric ceramics by several orders less than electrical component. It gives the basis for (9)

$$\mathrm{rot}\,\vec{\mathrm{E}} \cong 0\,. \qquad (12)$$

Equation (12) suggests that the electric field in a volume of the deformed piezoceramics is irrotational, i.e. potential and it can be described by a scalar electric potential, and

$$\vec{E} = -\,grad\,\Phi\,. \qquad (13)$$

With the definition (13), known [20, 21] expression for calculating of the m-th electric induction vector component in a volume of a deformable piezoelectric can be written as follows

$$D_m = e_{mkj}\varepsilon_{kj} - \chi^{\varepsilon}_{mn}\left(grad\,\Phi\right)_n\,, \qquad (14)$$

where $e_{mkj} \Leftrightarrow e_{m\beta}$ (β is a Voigt index, by which it is changed a couple of symmetrical tensor indices k, j) is an element of the matrix of piezoelectric constants; ε_{kj} is a component of infinitesimal deformations tensor; χ^{ε}_{mn} is a component of the dielectric permittivity tensor; $\left(grad\,\Phi\right)_n$ is the n-th component of scalar potential gradient vector. When writing the equation (14) in a cylindrical coordinate system we should consider the following correspondence between the symbols (ρ,φ,z) of the coordinate axes of the cylindrical coordinate system and the numbers $k = 1,2,3$ of the coordinate axes x_k of the Cartesian coordinate system: $1 \Leftrightarrow \rho$; $2 \Leftrightarrow \varphi$; $3 \Leftrightarrow z$.

From the general expression (14) the next follows

$$D_\rho = 2\,e_{15}\varepsilon_{\rho z} - \chi^{\varepsilon}_{11}\frac{\partial \Phi^{(1)}}{\partial \rho}\,, \qquad (15)$$

$$D_z = e_{31}\varepsilon_{\rho\rho} + e_{32}\varepsilon_{\varphi\varphi} + e_{33}\varepsilon_{zz} - \chi^{\varepsilon}_{33}\frac{\partial \Phi^{(1)}}{\partial z} =$$

$$= e_{31}\left(\varepsilon_{\rho\rho} + \varepsilon_{\varphi\varphi}\right) + e_{33}\varepsilon_{zz} - \chi^{\varepsilon}_{33}\frac{\partial \Phi^{(1)}}{\partial z}\,, \qquad (16)$$

where $D_\varphi \equiv 0$ because of the axial symmetry of the problem under consideration; $\varepsilon_{\rho z} = \left(\partial u_\rho/\partial z + \partial u_z/\partial \rho\right)/2$ is a shear deformation. In (16) piezoelectric moduli of the same value e_{31} and e_{32} (see comment to the matrix (2)) are written, as is usual in solid mechanics, by the same symbol e_{31}. Components $\varepsilon_{\rho\rho} = \partial u_\rho/\partial \rho$, $\varepsilon_{\varphi\kappa} = u_\rho/\rho$ and $\varepsilon_{zz} = \partial u_z/\partial z$ determine compression and expansion deformations along the coordinate lines of a cylindrical coordinate system. $\Phi^{(1)}$ is an electrical potential in the ring area $\{R_1 - d_1 \le \rho \le R_1 + d_1;\ 0 \le \varphi \le 2\pi;\ 0 \le z \le \alpha\}$ under the electrode 1.

Expressions (15) and (16) substituting into condition (11) gives a second order differential equation in partial derivatives relative to the required scalar potential $\Phi^{(1)}(\rho, z)$ of the electric field in a deformable piezoelectric.

In the particular case of a sufficiently thin disk when $\alpha/R < 1$, it can be argued that in the frequency range in which the length of the elastic wave is larger than the thickness of the piezoelectric disk, electrical and elastic fields in its volume is almost independent of the axial coordinate values z, i.e., practically do not change their values according to thickness of the disk.

If the disk is gently fixed along the surface $\{\rho = R;\ 0 \le \varphi \le 2\pi;\ 0 \le z \le \alpha\}$, the shear deformation becomes zero on this surface and on surfaces $z = 0$ and $z = \alpha$. In addition, on the surface covered by the electrode $z = 0$ the radial component $D_\rho = 0$. The radial component $D_\rho = 0$ on the side surface of the piezoceramic disk [21], on the surface of ring electrode 1 and on the disc symmetry axis, i.e. on the axis Oz. The combination of these facts suggests that in thin piezoceramic disk, in a first approximation, it can be considered that $D_\rho = 0\ \forall(\rho,\varphi,z)\in V$, where V is a volume of the disk. In this case, the vector of electric induction is completely determined by only one non-zero axial component D_z, and the condition (11) takes the form

$$\partial D_z^{(1)}/\partial z = 0\,, \qquad (17)$$

where $D_z^{(1)}$ further underlines the fact that we are talking about electric induction vector at the ring area $\{R_1 - d_1 \le \rho \le R_1 + d_1;\ 0 \le \varphi \le 2\pi;\ 0 \le z \le \alpha\}$ under the electrode 1.

From condition (17) it implies that the axial component $D_z^{(1)}$ is a function of the radial coordinate ρ and is independent of the axial coordinate values z,

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which is in full agreement with the above mentioned adopted suggestion about a weak dependence of the physical characteristics of the fields on the axial coordinate values in the frequency range in which the following inequality holds $\lambda >> \alpha$ (λ is an elastic wave length.

Because of

$$\varepsilon_{\rho\rho} + \varepsilon_{\varphi\varphi} = \partial u_\rho / \partial \rho + u_\rho/\rho = \left[\partial\left(\rho u_\rho\right)/\partial\rho\right]/\rho,$$

definition (16) can be written as follows

$$D_z^{(1)}(\rho) = \frac{e_{31}}{\rho}\frac{\partial}{\partial\rho}\left[\rho u_\rho^{(1)}\right] + e_{33}\frac{\partial u_z^{(1)}}{\partial z} - \chi_{33}^{\varepsilon}\frac{\partial\Phi^{(1)}}{\partial z}, \quad (18)$$

where $u_\rho^{(1)}(\rho, z)$ and $u_z^{(1)}(\rho, z)$ are amplitude values of the components of the material particles displacement vector in the ring area

$$\left\{R_1 - d_1 \le \rho \le R_1 + d_1;\ 0 \le \varphi \le 2\pi;\ 0 \le z \le \alpha\right\}.$$

Integrating with respect to z the left and right side of (18), and taking into account the condition (17), we obtain the following result

$$\alpha D_z^{(1)}(\rho) = \frac{e_{31}}{\rho}\frac{\partial}{\partial\rho}\left[\rho\int_0^\alpha u_\rho^{(1)}(\rho, z)\,dz\right] + \quad (19)$$

$$+ e_{33}\left[u_z^{(1)}(\rho,\alpha) - u_z^{(1)}(\rho,0)\right] - \chi_{33}^{\varepsilon}\left[\Phi^{(1)}(\alpha) - \Phi^{(1)}(0)\right].$$

Let

$$u_\rho^{(1)}(\rho) = \frac{1}{\alpha}\int_0^\alpha u_\rho^{(1)}(\rho, z)\,dz, \quad (20)$$

and $u_\rho^{(1)}(\rho)$ is an averaged over the thickness of the disk radial component of the material particles displacement vector in the ring area under the electrode 1. Since $\Phi^{(1)}(\alpha) - \Phi^{(1)}(0) \equiv U_0$, then (19) takes the form

$$D_z^{(1)}(\rho) = \frac{e_{31}}{\rho}\frac{\partial}{\partial\rho}\left[\rho u_\rho^{(1)}(\rho)\right] +$$

$$+ \frac{e_{33}}{\alpha}\left[u_z^{(1)}(\rho,\alpha) - u_z^{(1)}(\rho,0)\right] - \chi_{33}^{\varepsilon}\frac{U_0}{\alpha}. \quad (21)$$

Substituting (21) into definition (7) of the amplitude value of electric charge, we can obtain

$$Q_1 = 2\pi\left\{e_{31}\left[\rho u_\rho^{(1)}(\rho)\right]\Big|_{R_1-d_1}^{R_1+d_1} + \right.$$

$$+ \frac{e_{33}}{\alpha}\int_{R_1-d_1}^{R_1+d_1}\rho\left[u_z^{(1)}(\rho,\alpha) - u_z^{(1)}(\rho,0)\right]d\rho -$$

$$\left. - \frac{\chi_{33}^{\varepsilon}}{2\alpha}\left[\left(R_1 + d_1\right)^2 - \left(R_1 - d_1\right)^2\right]U_0\right\}. \quad (22)$$

We set

$$u_z^{(1)}(z) = \frac{1}{2d_1R_1}\int_{R_1-d_1}^{R_1+d_1}\rho u_z^{(1)}(\rho, z)\,d\rho, \quad (23)$$

where $u_z^{(1)}(z)$ is an averaged over the area of the ring $\left\{R_1 - d_1 \le \rho \le R_1 + d_1;\ 0 \le \varphi \le 2\pi\right\}$ axial component of the material particles $\vec{u}(\rho, z)$ displacement vector in the ring area under the electrode 1. With the definition (23) relation (22) can be written as follows

$$Q_1 = 2\pi e_{31}\left[\left(R_1 + d_1\right)u_\rho^{(1)}\left(R_1 + d_1\right) - \right.$$

$$\left. - \left(R_1 - d_1\right)u_\rho^{(1)}\left(R_1 - d_1\right)\right] +$$

$$+ 4\pi d_1R_1\frac{e_{33}}{\alpha}\left[u_z^{(1)}(\alpha) - u_z^{(1)}(0)\right] - C_1^{\varepsilon}U_0, \quad (24)$$

where $C_1^{\varepsilon} = 4\pi d_1R_1\chi_{33}^{\varepsilon}/\alpha$ is a static electric capacity of the piezoceramic volume under the ring electrode No. 1.

Since by definition the piezoelectric transformer is a linear physical system, the averaged components of the material particles displacement vector can always be represented as follows

$$u_\rho^{(1)}(\rho) = U_0F_\rho^{(1)}(\rho), \quad u_z^{(1)}(z) = U_0F_z^{(1)}(z), \quad (25)$$

where functions $F_\rho^{(1)}(\rho)$ and $F_z^{(1)}(z)$ differ from the averaged components $u_\rho^{(1)}(\rho)$ and $u_z^{(1)}(z)$ of the material particles displacement vector only by a constant factor U_0, and have the meaning of displacements sensitivity in the ring area $\left\{R_1 - d_1 \le \rho \le R_1 + d_1;\ 0 \le \varphi \le 2\pi;\ 0 \le z \le \alpha\right\}$ to the amplitude values of electrical potential difference on the ring electrode 1. The dimension of $F_\rho^{(1)}(\rho)$ and $F_z^{(1)}(z)$ is m/V. Functions $F_\rho^{(1)}(\rho)$ and $F_z^{(1)}(z)$ numerically equal to the material particles averaged displacements of the ring area under the electrode 1 when the electric potential difference with the amplitude value of $U_0 = 1$ V is applied to this electrode.

Following suggestions (25), the expression (24) for the electric charge Q_1 calculation can be written as follows

$$Q_1 = U_0C_1^{\varepsilon}F_1(\omega, \Pi_1), \quad (26)$$

where dimensionless function $F_1(\omega, \Pi_1)$ is defined as follows

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$$F_1(\omega,\Pi_1) = \frac{e_{31}\alpha}{2\chi_{33}^{\varepsilon} d_1}\left[\left(1 + \frac{d_1}{R_1}\right)F_{\rho}^{(1)}(R_1 + d_1) - \right.$$
$$\left. -\left(1 - \frac{d_1}{R_1}\right)F_{\rho}^{(1)}(R_1 - d_1)\right] +$$
$$+\frac{e_{33}}{\chi_{33}^{\varepsilon}}\left[F_z^{(1)}(\alpha) - F_z^{(1)}(0)\right] - 1 . \quad (27)$$

Substituting (26) into the definition (6) of the electric current amplitude, and the obtained result into Ohm's law (5) for the circuit section, we can get the estimated ratio for the electrical impedance Z_1 :

$$Z_1 = -\frac{1}{i\omega C_1^{\varepsilon} F_1(\omega,\Pi_1)} . \quad (28)$$

If the dielectric under the ring electrode 1 does not have piezoelectric properties, i.e. $e_{31} = e_{33} = 0$, the function $F_1(\omega,\Pi_1) = -1$ and the expression (28) becomes as well-known formula for capacitor reactive resistance calculation with capacitance C_1^{ε}, i.e. $Z_1 = 1/(i\omega C_1^{\varepsilon})$.

Substituting (28) into the formula (4), we obtain

$$U_0 = \frac{U_1}{1 - i\omega C_1^{\varepsilon} F_1(\omega,\Pi_1) Z_i} . \quad (29)$$

It should be emphasized that the potential difference U_0 is determined by components averaged values of the material particles displacement vector of the ring area $\{R_1 - d_1 \le \rho \le R_1 + d_1 ;\ 0 \le \varphi \le 2\pi;\ 0 \le z \le \alpha\}$. This fact is of fundamental importance, since there is the possibility of equations joint solutions of a deformable piezoelectric motion.

In the case when a strong inequality $\alpha/R << 1$ takes place, i.e. when the disk can be considered as infinitely thin, the situation is considerably simplified, since the deformation ε_{zz} becomes linearly dependent on the sum of deformations $\varepsilon_{\rho\rho}$ and $\varepsilon_{\varphi\varphi}$.

From the generalized Hooke's law [20] for the elastic media with piezoelectric properties

$$\sigma_{ij} = c_{ijkl}^{E}\varepsilon_{kl} + e_{kij}\frac{\partial \Phi}{\partial x_k},$$

where σ_{ij} is a component of the resulting mechanical stresses tensor, follows that in a polarized across the thickness piezoceramic disk normal stresses $\sigma_{\rho\rho}$, $\sigma_{\varphi\varphi}$ and σ_{zz} correspond to compression and expansion deformations $\varepsilon_{\rho\rho}$, $\varepsilon_{\varphi\varphi}$ and ε_{zz} and can be defined by the following expressions:

$$\sigma_{\rho\rho} = c_{11}^{E}\varepsilon_{\rho\rho} + c_{12}^{E}(\varepsilon_{\varphi\varphi} + \varepsilon_{zz}) + e_{31}\frac{\partial \Phi}{\partial z}, \quad (30)$$

$$\sigma_{\varphi\varphi} = c_{12}^{E}\varepsilon_{\rho\rho} + c_{11}^{E}\varepsilon_{\varphi\varphi} + c_{12}^{E}\varepsilon_{zz} + e_{31}\frac{\partial \Phi}{\partial z}, \quad (31)$$

$$\sigma_{zz} = c_{12}^{E}(\varepsilon_{\rho\rho} + \varepsilon_{\varphi\varphi}) + c_{33}^{E}\varepsilon_{zz} + e_{33}\frac{\partial \Phi}{\partial z}. \quad (32)$$

In expressions (30)–(32) material constants of the same value (the elements of matrices (1) and (2)) are written by the same symbols.

On the bottom ($z = 0$) and top ($z = \alpha$) surfaces of the piezoceramic disk free from mechanical contacts with other material objects in accordance with Newton's third law the following conditions should take place:

$$\sigma_{z\rho}\big|_{z=0;\alpha} = \sigma_{zz}\big|_{z=0;\alpha} = 0 . \quad (33)$$

Since the disk is very thin, it can be argued that the quantitative characteristics of its stress-strain state does not depend on the axial coordinate values z, i.e. $\partial\sigma_{ij}/\partial z \cong 0$. It follows that the condition (33) must be satisfied at any point of the volume V of a thin piezoceramic disk. Substituting into the left side of (32) a zero, we obtain the following definition for the compression and expansion deformations in the axial direction:

$$\varepsilon_{zz} = -\frac{c_{12}^{E}}{c_{33}^{E}}(\varepsilon_{\rho\rho} + \varepsilon_{\varphi\varphi}) - \frac{e_{33}}{c_{33}^{E}}\frac{\partial \Phi}{\partial z}. \quad (34)$$

Substituting expression (34) into (30), (31) and (16), it produces the following results:

$$\sigma_{\rho\rho} = c_{11}\varepsilon_{\rho\rho} + c_{12}\varepsilon_{\varphi\varphi} + e_{31}^{*}\frac{\partial \Phi}{\partial z}, \quad (35)$$

$$\sigma_{\varphi\varphi} = c_{12}\varepsilon_{\rho\rho} + c_{11}\varepsilon_{\varphi\varphi} + e_{31}^{*}\frac{\partial \Phi}{\partial z}, \quad (36)$$

$$D_z = e_{31}^{*}(\varepsilon_{\rho\rho} + \varepsilon_{\varphi\varphi}) - \chi_{33}^{\sigma}\frac{\partial \Phi}{\partial z}, \quad (37)$$

where $c_{11} = c_{11}^{E} - (c_{12}^{E})^2/c_{33}^{E}$; $c_{12} = c_{12}^{E}(1 - c_{12}^{E}/c_{33}^{E})$; $e_{31}^{*} = e_{31} - e_{33}c_{12}^{E}/c_{33}^{E}$ are material constants for planar stress-strain state of the polarized across the thickness piezoceramic element; $\chi_{33}^{\sigma} = \chi_{33}^{\varepsilon} + e_{33}^{2}/c_{33}^{E}$ is a dielectric permittivity of the polarized across the thickness piezoceramic disk for constancy mode (equality

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to zero) of the normal mechanical stresses σ_{zz}. Equations (35) and (36) in combination with $\sigma_{z\rho} = \sigma_{zz} = 0 \forall(\rho, z) \in V$ suggest that $u_z \cong 0$ in the entire oscillating disk.

The expression (29) takes the form

$$U_0 = \frac{U_1}{1 - i\omega C_1^{\sigma} F_1^{(0)}(\omega, \Pi_1) Z_i}, \quad (38)$$

where $C_1^{\sigma} = 4\pi d_1 R_1 \chi_{33}^{\sigma} / \alpha$ is a static electrical capacitance of the ring area of infinitely thin disk under the electrode 1;

$$F_1^{(0)}(\omega, \Pi_1) = \frac{e_{31}^{*}\alpha}{2\chi_{33}^{\sigma} d_1}\left[\left(1 + \frac{d_1}{R_1}\right) F_{\rho}^{(1)}(R_1 + d_1) - \right.$$
$$\left. - \left(1 - \frac{d_1}{R_1}\right) F_{\rho}^{(1)}(R_1 - d_1)\right] - 1. \quad (39)$$

Now let us consider the processes that occur in an area of the ring electrode 2, i.e. output electrode of the piezoelectric transformer.

Obviously, in the ring area 2 $\{R_2 - d_2 \le \rho \le R_2 + d_2;\ 0 \le \varphi \le 2\pi;\ 0 \le z \le \alpha\}$ the amplitude values $u_{\rho}^{(2)}(\rho, z)$ and $u_z^{(2)}(\rho, z)$ of the harmonically time varying components of the material particles displacement vector of the oscillating piezoelectric disk can be represented as follows:

$$u_{\rho}^{(2)}(\rho, z) = U_0\, F_{\rho}^{(2)}(\rho, z),$$
$$u_z^{(2)}(\rho, z) = U_0\, F_z^{(2)}(\rho, z), \quad (40)$$

where U_0 is an electric potential difference on the exciting ring electrode 1 (Fig. 1); $F_{\rho}^{(2)}(\rho, z)$ and $F_z^{(2)}(\rho, z)$ are displacements sensitivities in the ring area 2.

The amplitude value U_2 of the voltage drop on electrical load Z_n, i.e. on the input impedance of the electronic circuit which is directly connected to the ring electrode 2, is defined as follows

$$U_2 = Z_n I_2, \quad (41)$$

where $I_2 = -i\omega Q_2$ is an amplitude of the electric current in the conductor, which connects the electrode 2 and the electrical load Z_n; Q_2 is an amplitude value of the electric charge on the ring electrode 2.

Acting in the same manner as in the determination of the electrical impedance Z_1, we can obtain the following definition of the charge Q_2:

$$Q_2 = C_2^{\varepsilon} U_0 F_2(\omega, \Pi_2) - C_2^{\varepsilon} U_2, \quad (42)$$

where $C_2^{\varepsilon} = 4\pi d_2 R_2 \chi_{33}^{\varepsilon} / \alpha$ is a static electrical capacitance of the ring area 2;

$$F_2(\omega, \Pi_2) = \frac{e_{31}\alpha}{2\chi_{33}^{\varepsilon} d_2}\left[\left(1 + \frac{d_2}{R_2}\right) F_{\rho}^{(2)}(R_2 + d_2) - \right.$$
$$\left. - \left(1 - \frac{d_2}{R_2}\right) F_{\rho}^{(2)}(R_2 - d_2)\right] + \frac{e_{33}}{\chi_{33}^{\varepsilon}}\left[F_z^{(2)}(\alpha) - F_z^{(2)}(0)\right];$$

$F_{\rho}^{(2)}(\rho)$ and $F_z^{(2)}(z)$ are averaged sensitivities.

Substituting (42) into current definition I_2, and obtained result into (41), we can come to the conclusion that

$$U_2 = f_n(\omega) U_0 F_2(\omega, \Pi_2), \quad (43)$$

where $f_n(\omega) = -i\omega C_2^{\varepsilon} Z_n \big/ \left(1 - i\omega C_2^{\varepsilon} Z_n\right)$ is a switching on function or load characteristic of the output ring electrode of the piezoelectric transformer.

In the short-circuit mode ($Z_n = 0$) function $f_n(\omega) = 0$ and $U_2 = 0$. This fact is very clear and does not require any mathematical calculations to prove its validity. In idle mode, when $Z_n \to \infty$, switching on function $f_n(\omega)$ if $\omega = 0$ is equal to zero, and at an arbitrarily small $\omega > 0$ $f_n(\omega) = 1$, i.e. in this mode switching on function is a function of Heaviside. It follows that the piezoelectric receiver of elastic vibrations is not capable to register the static pressures and deformations. This statement is not so obvious to practitioners actually cancels a large group of devices of piezoelectronics, which are presented in [22].

The rate of change of the switching on function $f_n(\omega)$ is determined by the time constant $\tau_n = C_2^{\varepsilon} Z_n$ of the circuit that connects the receiver electrode to an electrical load. The function module values $f_n(\omega)$, depending on the value of the dimensionless quantity $\Omega_n = \omega\tau_n$ are shown in Fig. 2.

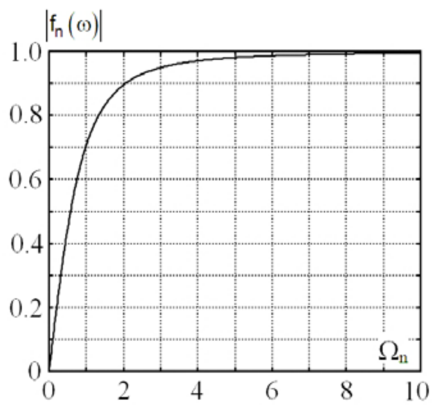

Figure 2 – Changing of switching on function module of the acoustic waves piezoelectric receiver

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After substituting (29) into (43), we can write the following definition of the transformation $K(\omega,\Pi)$ of the piezoelectric transformer

$$K(\omega,\Pi)=\frac{U_2}{U_1}=\frac{f_n(\omega)F_2(\omega,\Pi_2)}{1-i\omega C_1^{\varepsilon}F_1(\omega,\Pi_1)Z_i}. \quad (44)$$

In the case of very thin piezoceramic disk, when a strong inequality $\alpha/R<<1$ takes place an expression (44) can be written as follows

$$K^{(0)}(\omega,\Pi)=\frac{U_2}{U_1}=\frac{f_n^{(0)}(\omega)F_2^{(0)}(\omega,\Pi_2)}{1-i\omega C_1^{\varepsilon}F_1^{(0)}(\omega,\Pi_1)Z_i}. \quad (45)$$

where $f_n^{(0)}(\omega)=-i\omega C_2^{\sigma}Z_n\big/\left(1-i\omega C_2^{\sigma}Z_n\right)$;

$C_2^{\sigma}=4\pi d_2R_2\chi_{33}^{\sigma}/\alpha$;

$F_2^{(0)}(\omega,\Pi_2)=\frac{e_{31}^{*}\alpha}{2\chi_{33}^{\sigma}d_2}\left[\left(1+\frac{d_2}{R_2}\right)F_{\rho}^{(2)}(R_2+d_2)-\right.$

$\left.-\left(1-\frac{d_2}{R_2}\right)F_{\rho}^{(2)}(R_2-d_2)\right]$; $F_1^{(0)}(\omega,\Pi_1)$ is defined by (39).

Expressions (44) and (45), which have a sense of mathematical models of piezoelectric transformers operating on axially symmetric radial oscillations of piezoceramic disks, are built with a minimal number of simplifying assumptions.

To fill the definition (44) or (45) by a specific physical meaning, it is necessary to determine the components of the material particles displacement vector of the oscillating piezoceramic disk. This procedure is the subject of a separate investigation.

4 EXPERIMENTS

Let us consider a disk piezoelectric transformer (Fig. 3), primary electrical circuit of which consists of electric potential difference generator $U_1e^{i\omega t}$ (where U_1 is an amplitude value of electric potential difference) with output electrical impedance Z_g and ring electrode (position 1 in Fig. 3). The secondary electrical circuit consists of an electrode in the form of a circle (position 2) with connected electronic circuit to it with input electrical impedance Z_n, on which an electric potential difference $U_2e^{i\omega t}$ is formed. The primary and secondary circuits of piezoelectric transformer do not have a galvanic connection. The energy exchange between the primary and secondary circuits is carried out by means of axisymmetric radial vibrations of the piezoceramics material particles in the volume of thickness polarized disk (position 3 in Fig. 3).

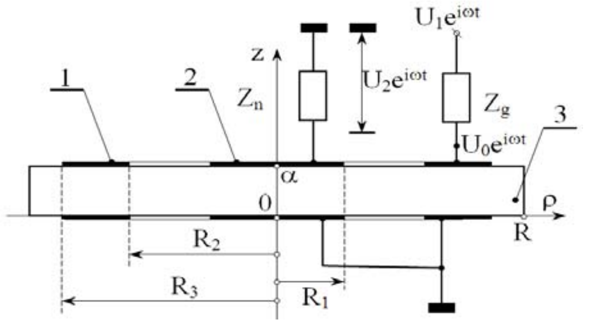

Figure 3 – Calculation scheme of disk piezoelectric transformer

It is obvious that the work of function piezoelectronic element, which is schematically shown in Fig. 3, is fully described by transformation ratio $K(\omega,\Pi)=U_2/U_1$, which is a mathematical model of the device under consideration. Scheme of construction of piezoelectric transformer's mathematical model is outlined in [23].

The elastic stresses and displacements of material particles of piezoelectric ceramics in the areas under the electrodes, and in the areas where there are no electrodes are determined in [24]. Following the method which is described in [24] we can write that

$$\sigma_{\rho\rho}^{(1)}(\rho)=c_{11}\frac{\partial u_{\rho}^{(1)}(\rho)}{\partial\rho}+c_{12}\frac{u_{\rho}^{(1)}(\rho)}{\rho}+e_{31}^{*}\frac{U_2}{\alpha}, \quad (46)$$

$$\sigma_{\rho\rho}^{(2)}(\rho)=c_{11}^{D}\frac{\partial u_{\rho}^{(2)}(\rho)}{\partial\rho}+c_{12}^{D}\frac{u_{\rho}^{(2)}(\rho)}{\rho}, \quad (47)$$

$$\sigma_{\rho\rho}^{(3)}(\rho)=c_{11}\frac{\partial u_{\rho}^{(3)}(\rho)}{\partial\rho}+c_{12}\frac{u_{\rho}^{(3)}(\rho)}{\rho}+e_{31}^{*}\frac{U_0}{\alpha}, \quad (48)$$

$$\sigma_{\rho\rho}^{(4)}(\rho)=c_{11}^{D}\frac{\partial u_{\rho}^{(4)}(\rho)}{\partial\rho}+c_{12}^{D}\frac{u_{\rho}^{(4)}(\rho)}{\rho}, \quad (49)$$

where $c_{11}=c_{11}^{E}-\left(c_{12}^{E}\right)^2\big/c_{33}^{E}$; $c_{12}=c_{12}^{E}\left(1-c_{12}^{E}/c_{33}^{E}\right)$; $c_{11}^{D}=c_{11}+\left(e_{31}^{*}\right)^2\big/\chi_{33}^{\sigma}$; $c_{12}^{D}=c_{12}+\left(e_{31}^{*}\right)^2\big/\chi_{33}^{\sigma}$ are moduli of elasticity for the mode of axially symmetric radial oscillations of the piezoceramic disk material particles in the areas under the electrodes (area No.1, where $\rho\in[0,R_1]$, and area No.3, where $\rho\in[R_2,R_3]$) and in the areas where there are no electrodes (area No.2, where $\rho\in[R_1,R_2]$, and area No.4, where $\rho\in[R_3,R]$).

The amplitude values of the radial components of the material particles displacements vectors in the areas No.1, …, No.4, are defined as follows:

$$u_{\rho}^{(1)}(\rho)=A_1J_1(\gamma\rho), \quad (50)$$

$$u_{\rho}^{(2)}(\rho)=A_2J_1(\gamma_1\rho)+A_3N_1(\gamma_1\rho), \quad (51)$$

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$$u_\rho^{(3)}(\rho) = A_4 J_1(\gamma\rho) + A_5 N_1(\gamma\rho), \quad (52)$$

$$u_\rho^{(4)}(\rho) = A_6 J_1(\gamma_1\rho) + A_7 N_1(\gamma_1\rho), \quad (53)$$

where A_1, …, A_7 are frequency-dependent constants of the radial displacements of material particles in various areas; $J_1(z)$, $N_1(z)$ ($z = \gamma\rho$; $z = \gamma_1\rho$) are Bessel and Neumann functions [25] of the first order; $\gamma = \omega\big/\sqrt{c_{11}/\rho_0}$ and $\gamma_1 = \omega\big/\sqrt{c_{11}^D/\rho_0}$ are wave numbers of the radial oscillations in the areas under the electrodes, and in the areas where there are no electrodes; ρ_0 is a piezoceramics density.

In the conditional separation boundaries the amplitudes of displacements and stresses should satisfy the conditions of dynamic and kinematic coupling, which can be written as follows:

$$\sigma_{\rho\rho}^{(1)}(R_1) - \sigma_{\rho\rho}^{(2)}(R_1) = 0, \quad (54)$$

$$u_\rho^{(1)}(R_1) - u_\rho^{(2)}(R_1) = 0, \quad (55)$$

$$\sigma_{\rho\rho}^{(2)}(R_2) - \sigma_{\rho\rho}^{(3)}(R_2) = 0, \quad (56)$$

$$u_\rho^{(2)}(R_2) - u_\rho^{(3)}(R_2) = 0, \quad (57)$$

$$\sigma_{\rho\rho}^{(3)}(R_3) - \sigma_{\rho\rho}^{(4)}(R_3) = 0, \quad (58)$$

$$u_\rho^{(3)}(R_3) - u_\rho^{(4)}(R_3) = 0. \quad (59)$$

If boundary $\rho = R$ of the piezoceramic disk is free from mechanical contacts with other material objects, then on the contour $\rho = R$ next condition should be satisfied

$$\sigma_{\rho\rho}^{(4)}(R) = 0. \quad (60)$$

Substituting expressions (46)–(53) into conditions (54)–(60), we obtain an inhomogeneous system of linear algebraic equations, which consists of seven equations, that contain seven sought constants A_1, …, A_7. It is obvious that this system of equations is solved in one way. In general terms, mentioned system of equations can be written as follows:

$$\sum_{k=1}^{7} m_{ik} A_k = P_j, \; (j, k = 1, 2, ..., 7). \quad (61)$$

The coefficients m_{jk} and right-hand parts P_j of equations (61) have the following form:

$$m_{11} = J_0(\gamma R_1) - (1-k) J_1(\gamma R_1)/(\gamma R_1); \; k = c_{12}/c_{11};$$

$$m_{12} = \xi\left[J_0(\gamma_1 R_1) - (1-k_1) J_1(\gamma_1 R_1)/(\gamma_1 R_1)\right];$$

$$\xi = \sqrt{1 + K_{31}^2}; \; K_{31}^2 = \left(e_{31}^*\right)^2 \Big/ \left(\chi_{33}^\sigma c_{11}\right); \; k_1 = c_{12}^D / c_{11}^D;$$

$$m_{13} = \xi\left[N_0(\gamma_1 R_1) - (1-k_1) N_1(\gamma_1 R_1)/(\gamma_1 R_1)\right];$$

$$m_{14} = m_{15} = m_{16} = m_{17} = 0; \; P_1 = -qU_2/\Omega;$$

$$q = e_{31}^* R/(c_{11}\alpha); \; \Omega = \gamma R; \; m_{21} = J_1(\gamma R_1);$$

$$m_{22} = J_1(\gamma_1 R_1); \; m_{31} = 0; \; m_{23} = N_1(\gamma_1 R_1);$$

$$m_{24} = m_{25} = m_{26} = m_{27} = 0; \; P_2 = 0;$$

$$m_{32} = \xi\left[J_0(\gamma_1 R_2) - (1-k_1) J_1(\gamma_1 R_2)/(\gamma_1 R_2)\right];$$

$$m_{33} = \xi\left[N_0(\gamma_1 R_2) - (1-k_1) N_1(\gamma_1 R_2)/(\gamma_1 R_2)\right.;$$

$$m_{34} = J_0(\gamma R_2) - (1-k) J_1(\gamma R_2)/(\gamma R_2);$$

$$m_{35} = N_0(\gamma R_2) - (1-k) N_1(\gamma R_2)/(\gamma R_2);$$

$$m_{36} = m_{37} = 0; \; P_3 = qU_0/\Omega; \; m_{41} = 0; \; m_{42} = J_1(\gamma_1 R_2);$$

$$m_{43} = N_1(\gamma_1 R_2); \; m_{44} = J_1(\gamma R_2); \; m_{45} = N_1(\gamma R_2);$$

$$m_{46} = m_{47} = 0; \; P_4 = 0; \; m_{51} = m_{52} = m_{53} = 0;$$

$$m_{54} = J_0(\gamma R_3) - (1-k) J_1(\gamma R_3)/(\gamma R_3);$$

$$m_{55} = N_0(\gamma R_3) - (1-k) N_1(\gamma R_3)/(\gamma R_3);$$

$$m_{56} = \xi\left[J_0(\gamma_1 R_3) - (1-k_1) J_1(\gamma_1 R_3)/(\gamma_1 R_3)\right];$$

$$m_{57} = \xi\left[N_0(\gamma_1 R_3) - (1-k_1) N_1(\gamma_1 R_3)/(\gamma_1 R_3)\right.;$$

$$P_5 = -qU_0/\Omega; \; m_{61} = m_{62} = m_{63} = 0; \; m_{64} = J_1(\gamma R_3);$$

$$m_{65} = N_1(\gamma R_2); \; m_{66} = J_1(\gamma_1 R_3); \; m_{67} = N_1(\gamma_1 R_3);$$

$$P_6 = 0; \; m_{71} = m_{72} = m_{73} = m_{74} = m_{75} = 0;$$

$$m_{76} = J_0(\gamma_1 R) - (1-k_1) J_1(\gamma_1 R)/(\gamma_1 R);$$

$$m_{77} = N_0(\gamma_1 R) - (1-k_1) N_1(\gamma_1 R)/(\gamma_1 R); \; P_7 = 0.$$

Solutions for constants A_1, A_4 and A_5, that define the radial displacements of disk material particles under the electrodes of primary and secondary electrical circuits of piezoelectric transformer are as follows:

$$A_1 = -\frac{q}{\Omega}\left(U_2 A_{11} + U_0 A_{12}\right); \; A_{11} = \frac{B_{11}}{D_0}; \; A_{12} = \frac{B_{12}}{D_0}; \quad (62)$$

$$A_4 = \frac{q}{\Omega}\left(U_2 A_{41} + U_0 A_{42}\right); A_{41} = \frac{B_{41}}{D_0}; \; A_{42} = \frac{B_{42}}{D_0}; \quad (63)$$

$$A_5 = -\frac{q}{\Omega}\left(U_2 A_{51} + U_0 A_{52}\right); \; A_{51} = \frac{B_{51}}{D_0}; \; A_{52} = \frac{B_{52}}{D_0}; \quad (64)$$

where D_0 is a determinant of the system of equations (61), and B_{11}, …, B_{52} are determinants of the following matrices:

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$$B_{11}=\begin{Vmatrix} -m_{22} & -m_{23} & 0 & 0 & 0 & 0 \\ m_{32} & m_{33} & -m_{34} & -m_{35} & 0 & 0 \\ m_{42} & m_{43} & -m_{44} & -m_{45} & 0 & 0 \\ 0 & 0 & m_{54} & m_{55} & -m_{56} & -m_{57} \\ 0 & 0 & m_{64} & m_{65} & -m_{66} & -m_{67} \\ 0 & 0 & 0 & 0 & m_{76} & m_{77} \end{Vmatrix};$$

$$B_{12}=\begin{Vmatrix} -m_{12} & -m_{13} & 0 & 0 & 0 & 0 \\ -m_{22} & -m_{23} & 0 & 0 & 0 & 0 \\ m_{32} & m_{33} & -m_{34}+m_{54} & -m_{35}+m_{55} & -m_{56} & -m_{57} \\ m_{42} & m_{43} & -m_{44} & -m_{45} & 0 & 0 \\ 0 & 0 & m_{64} & m_{65} & -m_{66} & -m_{67} \\ 0 & 0 & 0 & 0 & m_{76} & m_{77} \end{Vmatrix};$$

$$B_{41}=\begin{Vmatrix} m_{21} & -m_{22} & -m_{23} & 0 & 0 & 0 \\ 0 & m_{32} & m_{33} & -m_{35} & 0 & 0 \\ 0 & m_{42} & m_{43} & -m_{45} & 0 & 0 \\ 0 & 0 & 0 & m_{55} & -m_{56} & -m_{57} \\ 0 & 0 & 0 & m_{65} & -m_{66} & -m_{67} \\ 0 & 0 & 0 & 0 & m_{76} & m_{77} \end{Vmatrix};$$

$$B_{42}=\begin{Vmatrix} m_{11} & -m_{12} & -m_{13} & 0 & 0 & 0 \\ m_{21} & -m_{22} & -m_{23} & 0 & 0 & 0 \\ 0 & m_{32} & m_{33} & -m_{35}+m_{55} & -m_{56} & -m_{57} \\ 0 & m_{42} & m_{43} & -m_{45} & 0 & 0 \\ 0 & 0 & 0 & m_{65} & -m_{66} & -m_{67} \\ 0 & 0 & 0 & 0 & m_{76} & m_{77} \end{Vmatrix};$$

$$B_{51}=\begin{Vmatrix} m_{21} & -m_{22} & -m_{23} & 0 & 0 & 0 \\ 0 & m_{32} & m_{33} & -m_{34} & 0 & 0 \\ 0 & m_{42} & m_{43} & -m_{44} & 0 & 0 \\ 0 & 0 & 0 & m_{54} & -m_{56} & -m_{57} \\ 0 & 0 & 0 & m_{64} & -m_{66} & -m_{67} \\ 0 & 0 & 0 & 0 & m_{76} & m_{77} \end{Vmatrix};$$

$$B_{52}=\begin{Vmatrix} m_{11} & -m_{12} & -m_{13} & 0 & 0 & 0 \\ m_{21} & -m_{22} & -m_{23} & 0 & 0 & 0 \\ 0 & m_{32} & m_{33} & -m_{34}+m_{54} & -m_{56} & -m_{57} \\ 0 & m_{42} & m_{43} & -m_{44} & 0 & 0 \\ 0 & 0 & 0 & m_{64} & -m_{66} & -m_{67} \\ 0 & 0 & 0 & 0 & m_{76} & m_{77} \end{Vmatrix}.$$

Substituting definition (62) of the constant A_1 into the equation (50), and obtained result into the formula for potential calculating U_2 we can come to the conclusion that

$$U_2 = 2 f_e(\omega)\frac{e_{31}^{*}\alpha q}{\chi_{33}^{\sigma} R_1 \Omega}\left(U_2 A_{11} + U_0 A_{12}\right) J_1\left(\gamma R_1\right),$$

which implies that

$$U_2 = U_0 K_2\left(\Omega,\Pi\right); \qquad (65)$$

$$K_2\left(\Omega,\Pi\right)=\frac{2 f_e(\omega) K_{31}^2 A_{12}\left[J_1\left(\Omega R_1/R\right)/\left(\Omega R_1/R\right)\right]}{1-2 f_e(\omega) K_{31}^2 A_{11}\left[J_1\left(\Omega R_1/R\right)/\left(\Omega R_1/R\right)\right]};$$

where $K_{31}^2 = \left(e_{31}^{*}\right)^2 / \left(c_{11}\chi_{33}^{\sigma}\right)$ is a squared electromechanical coupling coefficient for the mode of

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radial oscillations of thickness polarized piezoceramic disk material particles.

Let us define the amplitude value U_0 of electric potential difference on the electrode of the piezoelectric transformer's primary electric circuit.

It is obvious that

$$U_0 = \frac{U_1 Z_3}{Z_g + Z_3}, \qquad (66)$$

where Z_3 is an electric impedance of the area No.3 under the ring electrode of the piezoelectric transformer's primary electric circuit. In accordance with Ohm's law for the electrical circuit section $Z_3 = U_0 / I_3$, where I_3 is an amplitude of the alternating current in the conductor, which connects the generator of electrical potential difference with the ring electrode. As before, we assume that $I_3 = -i\omega Q_3$, where Q_3 is an amplitude value of polarization charge under the ring electrode, which is defined as follows:

$$Q_3 = 2\pi \int_{R_2}^{R_3} \rho D_z^{(3)}(\rho)\, d\rho = \qquad (67)$$

$$= C_3^{\sigma} U_0 \left\{ \frac{2\alpha e_{31}^*}{R_3 \chi_{33}^{\sigma} \left(1 - \beta^2\right) U_0} \left[u_\rho^{(3)}(R_3) - u_\rho^{(3)}(R_2) \right] - 1 \right\},$$

where $C_3^{\sigma} = \pi \chi_{33}^{\sigma} \left(R_3^2 - R_2^2 \right) / \alpha$ is a static electrical capacitance of the ring electrode; $\beta = R_2 / R_3$ is a geometrical parameter of the ring.

Substituting (63) and (64) for the calculation of constants A_4 and A_5 into definition (52), and taking into account the expression (65), we obtain the following formula for the calculation of displacements $u_\rho^{(3)}(\rho)$:

$$u_\rho^{(3)}(\rho) = \frac{U_0 \operatorname{Re}_{31}^*}{\Omega c_{11} \alpha} \left\{ \left[K_2(\Omega, \Pi) A_{41} + A_{42} \right] J_1(\Omega\rho/R) + \right.$$
$$\left. + \left[K_2(\Omega, \Pi) A_{51} + A_{52} \right] N_1(\Omega\rho/R) \right\}. \qquad (68)$$

After calculating the values $u_\rho^{(3)}(R_2)$ and $u_\rho^{(3)}(R_3)$ according to the formula (68) it can be written that $Q_3 = C_3^{\sigma} U_0 K_3(\Omega, \Pi)$, where

$$K_3(\Omega, \Pi) = \frac{2K_{31}^2}{1 - \beta^2} \left\{ \left[K_2(\Omega, \Pi) A_{41} + A_{42} \right] J(\Omega) + \right.$$
$$\left. + \left[K_2(\Omega, \Pi) A_{51} + A_{52} \right] N(\Omega) \right\} - 1;$$
$$J(\Omega) = \left[J_1(\Omega R_3/R) - \beta J_1(\beta \Omega R_3/R) \right] / (\Omega R_3/R);$$
$$N(\Omega) = \left[N_1(\Omega R_3/R) - \beta N_1(\beta \Omega R_3/R) \right] / (\Omega R_3/R).$$

After charge determining Q_3 the electrical impedance Z_3 is determined by the expression $Z_3 = -1 / \left[i\omega C_3^{\varepsilon} K_3(\Omega, \Pi) \right]$, from which the definition of potential difference on the ring electrode follows

$$U_0 = \frac{U_1}{1 - i\omega C_0^{\sigma} Z_g K_3(\Omega, \Pi)}. \qquad (69)$$

Substituting (69) into (65) we can come to the conclusion that

$$U_2 = U_1 \frac{K_2(\Omega, \Pi)}{1 - i\omega C_0^{\sigma} Z_g K_3(\Omega, \Pi)},$$

from which the formula for the transfer ratio calculation follows

$$K(\omega, \Pi) = \frac{U_2}{U_1} = \frac{K_2(\Omega, \Pi)}{1 - i\omega C_0^{\sigma} Z_g K_3(\Omega, \Pi)}. \qquad (70)$$

Analytical structure (70) is a mathematical model of piezoelectric ring-dot transformer with ring electrode in the primary circuit.

5 RESULTS

Expression (70), which determines the transfer ratio of piezoelectric device, has a structure which is typical for electronic devices with negative feedback. It is clearly seen that the depth of feedback is directly proportional to the value of the signal source output impedance Z_g. If the value of $Z_g = 0$ the feedback disappears and transfer ratio is completely determined by a frequency dependent function $K_2(\Omega, \Pi)$.

Feedback physical content which exists in piezoelectric transformers is practically obvious. Displacements levels of piezoelectric disk material particles increases significantly at a frequency of electromechanical resonance of radial oscillations. This is accompanied by an increase of deformations and as a consequence, by an increase of levels of polarization charges on the electrodes of the primary electrical circuit. Because of this the amplitude of the electric current in the primary circuit increases, which is accompanied by an increase of voltage drop on the resistance Z_g and, accordingly, by a decrease of potential difference U_0 (see. Fig. 3).

The transfer ratio modeling of piezoelectric transformer according to (70) have been conducted, the results of which are shown in Fig. 4. As follows from the results shown in Fig. 4, the parameter change Z_g is accompanied by significant changes in the frequency characteristic of piezoceramic disk transformer.

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Fig. 5 illustrates an influence of mechanical Q_i -factor of disk material on a change of transformation ratio in a narrow band near the first electro-mechanical resonance of the radial oscillations of free (not fixed) piezoceramic disk. The numerical values of quality factor are indicated near the corresponding curves.

All calculations were performed for piezoceramic disk with radius $R = 33 \cdot 10^{-3}\ m$ and thickness $\alpha = 3 \cdot 10^{-3}\ m$, made of thickness polarized PZT type piezoceramics with following parameters: $\rho_0 = 7400\ kg/m^3$; $c_{11}^E = 112\ GPa$; $c_{12}^E = 62\ GPa$; $c_{33}^E = 100\ GPa$; $e_{33} = 20\ C/m^2$; $e_{31} = -9\ C/m^2$; $\chi_{33}^{\varepsilon} = 1800\,\chi_0$; $\chi_0 = 8{,}85 \cdot 10^{-12}\ F/m$ is a dielectric constant; $Q_i = 100$ is a quality factor of piezoceramics; $Z_n = 10\ kOhms$ is an electrical load value; $\Omega = \omega\tau_0$ is a dimensionless quantity, where $\tau_0 = R\big/\sqrt{c_{11}/\rho_0}$ is a piezoceramic disk time constant. The frequency $f = 15206\ Hz$ corresponds to the value $\Omega = 1$. The value of the electrical impedance module of the electrical signal source is shown in the figures field.

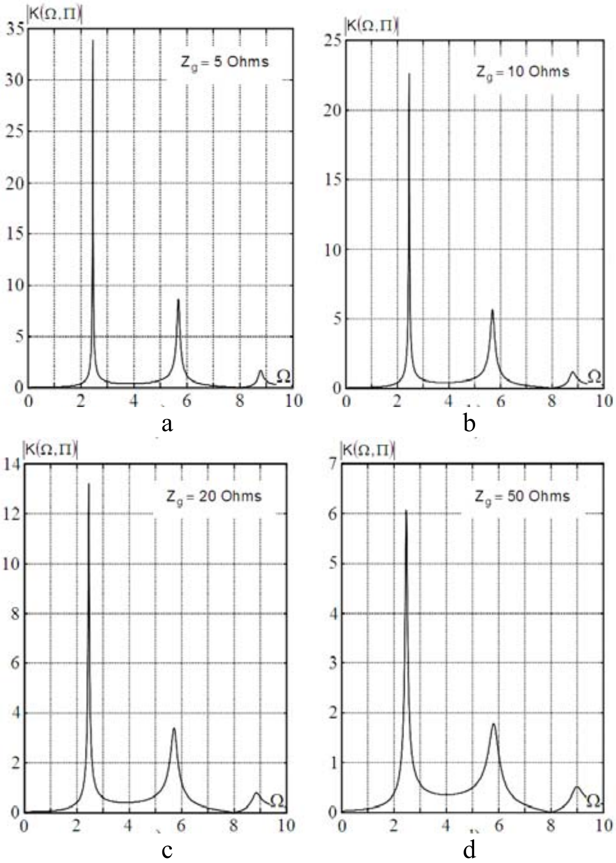

Figure 4 – Influence of the signal source output impedance Z_g on a frequency-dependent change of the transfer ratio module, when $R_1/R = 12/25$, $R_2/R = 15/25$ and $R_3/R = 0.999$:
a – Z_g = 5 Ohms; b – Z_g = 10 Ohms;
c – Z_g = 20 Ohms; d – Z_g = 50 Ohms

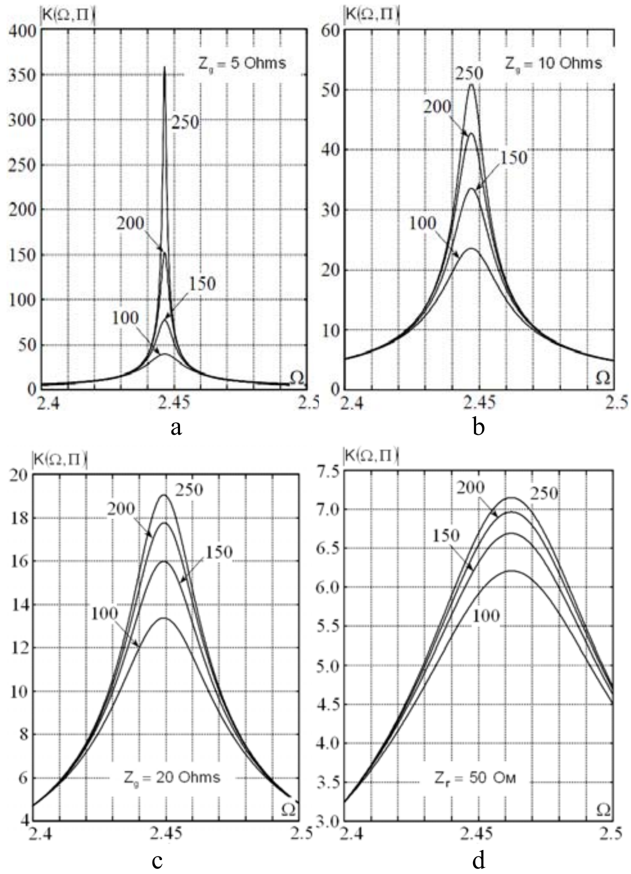

Figure 5 – Influence of the signal source output impedance Z_g on a frequency-dependent change of the transfer ratio module, when $R_1/R = 12/25$, $R_2/R = 15/25$ and $R_3/R = 0.999$:
a – Z_g = 5 Ohms; b – Z_g = 10 Ohms;
c – Z_g = 20 Ohms; d – Z_g = 50 Ohms

From the results shown in Fig. 4, 5 it can be concluded that each set of physical and mechanical piezoelectric parameters, each primary and secondary circuit electrodes configuration and fixed electrical load of piezoelectric transformer is corresponded to a fixed value of electrical signal source output impedance Z_g , with which the maximum transfer ratio is realized in a specified frequency range.

In Fig. 6 it is shown the calculated (solid line) and the experimentally obtained (dashed line) curves of the frequency dependence of the modulus of piezoceramic ring-dot disk transformer's transformation coefficient. The calculation is based on the same parameters as in the calculation of the curves $\left|K(\Omega,\Pi)\right|$ shown in Fig. 4. Naturally, the dimensions of the disk transformer in the calculation and experiment are chosen to be the same, i.e., the radius $R = 33 \cdot 10^{-3}$ m, the thickness $\alpha = 3 \cdot 10^{-3}$ m and $R_1/R = 12/25$, $R_2/R = 15/25$, $R_3/R = 0.999$. The values of the modulus of transformation coefficient of the piezoceramic disk transformer are plotted along the ordinate axis, and the frequency *f* (dimensionless value Ω) – on the abscissa axis. The frequency $f = 15206\ Hz$ corresponds to the value $\Omega = 1$.

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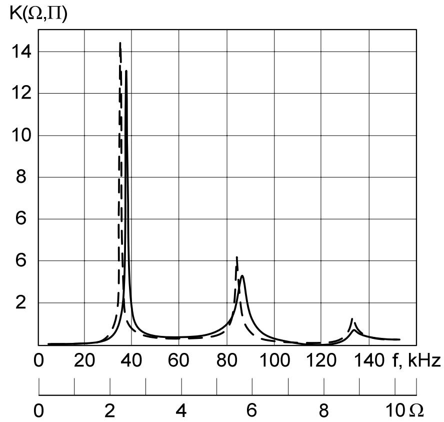

Figure 6 – Calculated (solid line) and experimentally obtained (dashed line) curves of the frequency dependence of the modulus of piezoceramic ring-dot disk transformer's transformation coefficient

6 DISCUSSION

When building the model, it was assumed that the thickness of the electrodes located on the surfaces of the disk is very small in comparison with the thickness of the disk α. In other words, the thickness of the electrodes, which, as a rule, does not exceed 15 μm, was not taken into account for constructing a mathematical model of piezoelectric transformer based on piezoceramic thin disk ($\alpha/R << 1$). It should also be noted that mathematical model (70) was built for ring-dot piezoelectric transformer (see Fig. 3) with surfaces partially covered by electrodes (area 1, $\rho \in [0, R_1]$, and area 3, where $\rho \in [R_2, R_3]$) and in the areas where there are no electrodes (area 2, where $\rho \in [R_1, R_2]$, and area 4, where $\rho \in [R_3, R]$).

As expected, the absolute values of the frequencies of resonances in calculation and experiment differ from each other. So, following the calculation, the frequencies of the first second and third electromechanical resonances are respectively equal to $f_{r1} = 37193\ Hz$, $f_{r2} = 88194\ Hz$ and $f_{r3} = 135330\ Hz$; the frequency ratio $\zeta = f_{r2}/f_{r1} = 2.371$.

The experimental values of the same quantities are, respectively, $f_{r1} = 34491\ Hz$, $f_{r2} = 83728\ Hz$, $f_{r3} = 132325\ Hz$ and $\zeta = f_{r2}/f_{r1} = 2.428$. If the experimental data are assumed to be true, the error in determining the frequency ratio is $\Delta\zeta = 2.3\%$. The obtained results are explained very simply. The numerical values of the frequencies of resonances s are determined by the dimensions and physicomechanical parameters of the material of disk element. The ratio of the resonances frequencies of the same disk is determined practically only by its dimensions. For this reason, a very satisfactory match between the theoretically and experimentally determined resonance frequency ratios is observed. The discrepancy between the absolute values of the resonance frequencies is explained by the discrepancy between the physicomechanical parameters of the piezoceramics, which were incorporated into the calculation and which are inherent in the experimentally investigated object. Comparing the curves, we can conclude that the quality factor of the material of the experimentally investigated sample is at least 1.2 times larger than included in the quality factor calculation.

Thus, it can be asserted that the character of the variation of both curves, shown in Fig. 6, in a fairly wide frequency range coincides with accuracy to details. This means that the qualitative content of the expression (70) is adequate to the processes that occur in real object. In other words, expression (70) is a mathematical model of piezoelectric ring-dot transformer with ring electrode in primary electrical circuit and sufficiently adequate to the real object and the processes occurring in it. The latter allows us to assume that the mathematical description of the stress-strain state of the disk transformer also corresponds quite well to the real state of things.

CONCLUSIONS

Physical processes in piezoelectric transformers, which operate using axially symmetric radial oscillations of the piezoceramic disk, are considered. The scheme of mathematical models constructing of the ring-dot piezoelectric transformer that is sufficiently adequate to real objects and occurring physical processes is proposed.

Main results of this work can be formulated as follows:

– mathematical model of piezoelectric transformer with ring electrode in the primary electrical circuit is constructed;

– high sensitivity of frequency characteristic of piezoelectric transformer to the values of the output impedance of the electrical signal source in the primary electrical circuit is demonstrated.

ACKNOWLEDGMENTS

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ПРИНЦИПИ ТА МЕТОДИ РОЗРАХУНКУ ПЕРЕДАТОЧНИХ ХАРАКТЕРИСТИК ДИСКОВИХ П'ЄЗОЕЛЕКТРИЧНИХ ТРАНСФОРМАТОРІВ

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АНОТАЦІЯ

Актуальність. Завдяки своїм унікальним властивостям п'єзокераміка знаходить застосування в різних областях техніки і технології. Дискові п'єзоелектричні пристрої широко використовуються в елементах інформаційних систем. Дослідження показали, що п'єзоелектричні трансформатори можуть конкурувати з традиційними електромагнітними трансформаторами як за ефективністю, так і за щільністю потужності. Кінцевою метою математичного моделювання фізичного стану коливальних п'єзокерамічних елементів є якісний і кількісний опис характеристик і параметрів існуючих в них електричних та еластичних полів.

Мета роботи – запропонувати принципи побудови математичних моделей, які в достатній мірі адекватні реальним пристроям і фізичним процесам, що відбувається в них.

Метод. Математичні моделі п'єзоелектричних трансформаторів, що працюють з використанням вісесиметричних радіальних коливань п'єзокерамичних дисків, побудовані з мінімальним числом припущень, що спрощують реальну

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ситуацію. Це дозволяє стверджувати, що запропонована схема побудови доставляє математичні моделі, які в достатній мірі адекватно відповідають реальним об'єктам і фізичним процесам, які в них існують.

Результати. Основні результати цієї роботи можна сформулювати наступним чином: побудована математична модель п'єзоелектричного трансформатора з кільцевим електродом в первинному електричному колі; показана висока чутливість частотної характеристики п'єзоелектричного трансформатора до змін значень вихідного опору джерела електричного сигналу в первинному електричному колі.

Висновки. В результаті дослідження математичної моделі реального пристрою можна визначити той набір геометричних, фізико-механічних та електричних параметрів реального об'єкта, який забезпечує реалізацію технічних показників функціонального елемента п'єзоелектроніки, обумовлених в технічному завданні. Вартість збережених ресурсів становить комерційну ціну математичної моделі. Перспективи подальших досліджень можуть полягати в побудові математичної моделі п'єзоелектричного трансформатора з секторними електродами.

КЛЮЧОВІ СЛОВА: п'єзоелектричний трансформатор, вісесиметричні коливання, фізичні процеси, математична модель.

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ПРИНЦИПЫ И МЕТОДЫ РАСЧЕТА ПЕРЕДАТОЧНЫХ ХАРАКТЕРИСТИК ДИСКОВЫХ ПЬЕЗОЭЛЕКТРИЧЕСКИХ ТРАНСФОРМАТОРОВ

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АННОТАЦИЯ

Актуальность. Благодаря своим уникальным свойствам пьезокерамика находит применение в различных областях техники и технологии. Дисковые пьезоэлектрические устройства широко используются в элементах информационных систем. Исследования показали, что пьезоэлектрические трансформаторы могут конкурировать с традиционными электромагнитными трансформаторами как по эффективности, так и по плотности мощности. Конечной целью математического моделирования физического состояния колеблющихся пьезокерамических элементов является качественное и количественное описание характеристик и параметров существующих в них электрических и упругих полей.

Цель работы – предложить принципы построения математических моделей, которые в достаточной мере адекватны реальным устройствам и происходящим в них физическим процессам.

Метод. Математические модели пьезоэлектрических трансформаторов, работающих с использованием осесимметричных радиальных колебаний пьезокерамических дисков, построены с минимальным числом упрощающих реальную ситуацию предположений. Это позволяет утверждать, что предложенная схема построения доставляет математические модели, которые в достаточной мере адекватны реальным объектам и физическим процессам, которые в них существуют.

Результаты. Основные результаты настоящей работы можно сформулировать следующим образом: построена математическая модель пьезоэлектрического трансформатора с кольцевым электродом в первичной электрической цепи; показана высокая чувствительность частотной характеристики пьезоэлектрического трансформатора к изменениям значений выходного сопротивления источника электрического сигнала в первичной электрической цепи.

Выводы. В результате исследования математической модели реального устройства можно определить тот набор геометрических, физико-механических и электрических параметров реального объекта, который обеспечивает реализацию технических показателей функционального элемента пьезоелектроники, оговоренных в техническом задании. Стоимость сохраненных ресурсов составляет коммерческую цену математической модели. Перспективы дальнейших исследований могут заключаться в построении математической модели пьезоэлектрического трансформатора с секторными электродами.

КЛЮЧЕВЫЕ СЛОВА: пьезоэлектрический трансформатор, осесимметричные колебания, физические процессы, математическая модель.

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