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THE STATES FINAL PROBABILITIES ANALYTICAL DESCRIPTION IN QUEUING SYSTEM WITH AN ENTRANCE FLOW OF REQUIREMENTS GROUPS, WITH WAITING AND LEAVING THE QUEUE

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ABSTRACT

Context. The problem of predicting the efficiency of real queuing systems in the event of a possible arrival of requirements groups and leaving of "impatient" requirements from the queue. The aim of the study was to model the operation of such systems to create opportunities to control their operation in real time.

Objective. The aim of the research is to obtain an analytical description of the state's final probabilities in a Markov queuing system with an input flow of requirements groups, with individual service of requirements, with a limited number of waiting places and with individual leaving of "impatient" requirements from the queue that is necessary to predict the values of the queuing system performance indicators.

Method. The probabilities of queuing systems states with an input flow of requirements groups with a random composition and with leaving of "impatient" requirements from the queue are described by the Kolmogorov differential equations. In a stationary state, these equations are transformed into a linearly dependent homogeneous system of algebraic equations. The structure of the equations depends on the numerical values of the input flow requirements group's parameters and the controlled service system. Therefore, an attempt to predict the efficiency of a system is faced with the need to write down and numerically solve a countable set of algebraic equations systems that is quite difficult. The key idea of the proposed method for finding an analytical description of the final probabilities for the specified queuing system was the desire to localize the influence of requirements groups in the input flow on the operation of the queuing system in multiplicative non-ordinary functions. Such functions allow obtaining the required analytical description and assessing the degree of the final probabilities transformation, in comparison with known systems, as well as assessing the predicted values of the noted queuing system efficiency indicators when choosing the parameters for controlling its operation.

Results. For the first time analytical expressions are obtained for the final probabilities of the queuing system states with an input flow of random composition requirements groups, with a limited number of waiting places, with individual service and leaving "impatient" requirements from the queue, which makes it possible to evaluate all known indicators of the system's performance.

Conclusions. The resulting description turned out to be a general case for well-known types of Markov queuing systems with non-ordinary and with the simplest input flow of requirements. The results of the numerical experiment testify in favor of the correctness of the obtained analytical expressions for the final probabilities and in favor of the possibility of their practical application in real queuing systems when solving problems of forecasting efficiency, as well as analyzing and synthesizing the parameters of real queuing systems.

KEYWORDS: Markov models, queuing systems, requirements groups, leaving the queue.

ABBREVIATIONS

QS is a queuing system.

NOMENCLATURE

A is an absolute QS capacity;

a_i is a probability of a group consisting of exactly i requirements at the input of the queuing system;

e=2,71… is a second remarkable limit;

$f_1()$ is a density distribution of the requirements flow at the input of the queuing system;

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$f_2()$ is a density distribution of service duration;

$f_3()$ is a density distribution of the requirement waiting until leaving the queue;

f_k is a non-ordinary function, which deforms the probability p_k of the queuing system k-th state when groups appear in the input flow of requirements;

$f_{n+\gamma}$ is a non-ordinary function, which deforms the probability p_{k+n} of the queuing system $(k+n)$-th state when groups appear in the input flow of requirements;

I is a flow intensity of requirements at the input of QS;

i is the number of requirements in the group;

L is a maximum number of requirements in a group;

L_{queue} is a queue length in the queuing system;

$M_L/M/n/m$ is a designation of queuing system with waiting or with leaving the queue in the Kendall-Basharin classification;

$M_L/M/n$ is a designation of the queuing system noted above but with no places to wait and it means QS with refusals;

M is a designation of an exponential distribution of the random service time of each requirement;

m is a number of places to wait;

M_L is a designation of Poisson input flow of requirements groups with random composition and with the maximum number L of requirements in a group;

$M_{b.d}$ is a mathematical expectation of the busy devices number;

$M[i]$ is a mathematical expectation of the requirements number in groups;

n is a number of identical channels (devices) in the queuing system;

P_{service} is a service probability of queuing system;

P_{refusal} is a service refusal probability;

p_k is a probability of a queuing system state in which exactly k requirements is in the system;

p_{n+k} is a probability of a queuing system state in which exactly n devices are busy by servicing and exactly γ waiting places are occupied by requirements;

S_k is a system state, at which exactly k requirements are under maintenance;

$S_{n+\gamma}$ is a system state, at which exactly n requirements are under maintenance and γ requirements are in a queue;

t is a current time;

T_{avr} is a mathematical expectation of requirement's service duration by the service device;

$T_{avr.w}$ is a mathematical expectation of time before requirement leaves the queue;

β is an inverse value to the mathematical expectation of time before requirement leaves the queue, $\beta = T_{avr.w}^{-1}$ and has the physical meaning of the intensity of requirements leaving from the queue;

γ is a current number of occupied places to wait;

λ is a parameter of requirements groups flow at the input of the queuing system and has the physical meaning of the requirements groups occurrence frequency;

λ_i is a parameter of requirements groups input partial flow that consists of exactly i requirements in the group;

μ is a performance of one service device as the inverse value to the mathematical expectation of service time, $\mu = T_{avr}^{-1}$;

v is a ratio of requirements leaving intensity β from the queue to the performance μ of the service device;

ρ is a load factor of a queuing system with a simplest flow of requirements;

ρ_i is a load factor of queuing system by a part of the input flow of requirements groups.

INTRODUCTION

In the field of transport, trade, medicine, industry, information networks, control systems and in other areas, there is often appears repeated massive demand (flow of requirements) for various services. To work out such requirements, the corresponding "service" systems are created.

The wide distribution and diversity of such systems has caused the need to develop appropriate models of queuing systems for solving problems of analysis, synthesis and control of real systems. The moments of each requirement occurrence and the duration of its working out (service) are not known in advance (are random). If all service devices are busy, requirements can wait for their turn. "Impatient" requirements may leave the queue at an unknown point in time. Therefore, most models are stochastic

In real systems, as a rule, the conditions of the central limit theorem of A. Ya. Khinchin [1] are satisfied, and an input flow of requirements, that is close to the simplest one, is automatically generated. For such conditions, there are well-known models, for example, in [2]. However, requirements can often enter the system in groups with an unknown (random) quantity in the group. In queuing systems, shock loads occur, the effectiveness of systems decreases.

To perform a forecast of the effectiveness in such system and in such conditions its possible only by numerical methods for specific numerical values of the conditions parameters. Unfortunately, the probability of "guessing" the exact values of the future set of continuous random variables (the parameters of the conditions) is strictly zero. Therefore, numerical analysis can be adequate to the real process only a posteriori, which sharply reduces its scientific significance and at the same time makes it important to search not numerical, but analytical descriptions of state probabilities and efficiency indicators of queuing systems with an input flow of groups with random composition of requirements. At present, there is an analytical description of QS models with an input flow of requirements groups and with waiting in the queue [3]. However, for the general case of real systems, in which "impatient" requirements can refuse service and leave the queue at unknown moments in time, the analytical description of the model is not known, which complicates the control of such systems and makes the topic of this article relevant.

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The object of research is a steady-state process of servicing an input flow of requirements groups in $M_L/M/n/m$ queuing system with leaving the queue.

The subject of research is the distribution law of the final state probabilities in queuing system $M_L/M/n/m$ with input flow of requirements groups and leaving the queue.

The research goal is to obtain an analytical description of final probabilities for the queuing system $M_L/M/n/m$ with input flow of requirements groups and leaving the queue which is the general case for the already known Markov models of queuing systems with an input flow of requirements groups and with the simplest input flow.

The noted final probabilities are a complete description of the systems operation and allow estimating the expected values of all known indicators of the queuing systems efficiency.

1 PROBLEM STATEMENT

The requirements groups flow with intensity I and density $f_1(t) = Ie^{-It}$ enters the queuing system. Service duration is random and has exponential distribution $f_2(t) = \mu e^{-\mu t}$. Some of the requirements in the groups that have found all service devices busy are queued. Each requirement can leave the queue without waiting for the start of service. The duration of the requirement waiting until leaving the queue is random and has exponential distribution $f_3(t) = \beta e^{-\beta t}$. By virtue of the noted distribution densities, a Markov process with continuous time and discrete states arises in the system.

This paper relies on a system of statements about the properties of a non-ordinary (general stationary) flow [1, pp. 14, 40, 41], which we present without proof.

The stationary flow of time points for the arrival of events groups without aftereffect is the simplest and is called the General Stationary Flow or non-ordinary flow.

Non-ordinary flow includes groups of i requirements ($i = 1, 2, \ldots, L$) in a group. The flow can be determined by setting the probabilities distribution law (a_i) of appearing exactly i requirements in any group of input flow. Then the flow parameter λ will be less than the flow intension ($\lambda < I$) and will include partial flows with parameters λ_i:

$$\lambda_i = \lambda a_i; \quad \lambda = \sum_{i=1}^{L} \lambda_i; \quad I = \sum_{i=1}^{L} i \cdot \lambda_i . \qquad (1)$$

A. Ya. Khinchin limit theorem [1] for random time intervals between groups of events in a non-ordinary flow is preserved and the form of the time intervals exponential distribution is preserved too, but with the parameter λ:

$$f(t) = \lambda e^{-\lambda t}, \qquad t > 0 . \qquad (2)$$

At the same time, to fulfill equality ($\lambda = I$) it is necessary and sufficient to have $a_1 = 1$. In this case, the flow of events becomes the simplest. For all other (non-ordinary) stationary flows without an aftereffect, the intensity of the flow is always greater than its parameter ($I > \lambda$).

2 REVIEW OF THE LITERATURE

The model developed to describe one system or process often become relevant in other areas. Thus, in 1909, A. K. Erlang [4] developed a model for calculating the part of calls that can be served at a telephone station.

The work process at the telephone station included the receipt and service of applications from subscribers to switch communication channels with other subscribers. After the end of the call, the channel was released and could be used to service the next request. The application that arrived at the telephone station at the time when all channels were busy received a denial of service. The moments of applications receipt and the end of their service were random.

The Erlang-developed model of the requests mass service system at the telephone station turned out to be a universal tool for describing the processes of service in different systems and in different fields of human activity. Each of these areas and systems has its own peculiarities, which led to the development of more complex models and to the appearance of an independent scientific direction – the queuing theory.

Currently, queuing system models are being actively used for analysis, for predicting efficiency and for optimizing decisions made in various areas. These include the following areas: telecommunication networks [5, 6, 7, 8, 9, 10], socio-economic systems [11, 12], production systems [13, 14, 15, 16] and logistic systems [17, 18, 19], computing systems [20, 21], traffic management systems [22, 23, 24, 25] and others.

An interesting direction in the theory of queuing systems is the construction of models with an infinite number of devices, since it is these models that make it possible to describe complex technical systems for which the number of devices can be relatively large. For example, L. Brown, N. Gans, A. Mandelbaum, and A. Sakov [5] use such systems to simulate a call center in which agents provide telephone services almost no refusals. In such a company, customer service should start immediately. Therefore, the number of working operators should be large enough and should be monitored using the appropriate model.

Infinitely linear systems are also used as an approximation for multiline systems in cases where the probability of denial to service is negligible [26, 27, 28, 29, 30, 31].

At the initial stage, most studies of the queuing theory were performed under the assumption that the incoming flow of requests is the simplest [32, 33].

However, the development of computer and mobile systems has led to the need to create new mathematical models of requirements flows at the system input, which are not Poisson or non-ordinary flows. This was the reason for the increased interest in the study of systems with more complex incoming flows. Systems with non-Poisson flows were studied by such authors as G. P.

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Klimov [34], G. Sh. Tsitsiashvili [35], P. P. Bocharov, A. V. Pechinkin [36], A. N. Moiseev and A. A. Nazarov, [37], S. P. Moiseeva [38], E. A. Doorn and A. A. Jages [39], V. F. Matveev, V. G. Ushakov [40] and others.

So, in the book of Matveev V. F. and Ushakov V. G. [40] was obtained generating function of the requirements number in the system for which the incoming flow is a superposition of independent flows with the same number of requirements in packs. For non-Poisson input flows in a system with an unlimited number of service channels E. A. Doorn and A. A. Jagers [39] obtained estimates of the variance for the number of busy servers.

Another important direction in the development of the queuing theory is the study of the systems operation in the conditions of the incoming flow, which includes groups of requirements with previously unknown composition. Thus, groups of motorcade cars can arrive at a gas station, visitors can arrive at a roadside restaurant in groups at the time of vehicles arrival, and the customers flow to the hotel includes both single customers and groups of several people, families for example. Such a flow is called non-ordinary.

A description of queuing system models with non-ordinary input flow can be found in works of A. A. Shakhbazov [41], Jung-Shyr Wu and Jyh-Yeong Wang [42], N. O. Kutselay and S. V. Safonov [43], O. Yu. Bogoyavlenskaya [44], V. B. Monsik, A. A. Skrynnikov, and A. U. Fedotov, in works of A. V. Pechinkin [45] and A. G. Tatashev, M. Akhilgova, S. A. Shchebunyaev.

In the general case, the probabilities of states in queuing systems $M_L/M/n/m$ with a non-ordinary input flow of requirements are described by Kolmogorov differential equations.

In the stationary state of the queuing system, these equations are transformed into a linearly dependent system of algebraic equations. The final probabilities of the queuing system states can be found by numerically solving the system of algebraic equations using the methods well known in linear algebra [2] – complete exclusion, inverse matrix, Kramer determinants. It should be noted that in this case the determinant of the algebraic equations system is always zero. Therefore, it is impossible to apply the Kramer determinant method directly.

One of the variants of the noted system algebraic equations numerical solution is the well-known matrix geometric method of Ramaswami [46]. This method is characterized as a method for the analysis of quasi-birth-death processes, continuous-time Markov chain whose transition rate matrix has a repetitive block structure. In this method, the final probabilities of the queuing system states are found using numerical calculations of the elements of the Neut's rate matrix [46].

Analytical description of models is sometimes possible to find for some performance indicators, as a rule, for single-channel systems (N. O. Kutselay and S. V. Safonov [43], O. Yu. Bogoyavlenskaya [44) with a specific composition of requirements in input flow groups (V. B. Monsik, A. A. Skrynnikov, A. U. Fedotov and A. V. Pechinkin [45]).

The search for regularities that could provide an analytical description of the final probabilities in the general case of a queuing system with a non-ordinary input flow of requirements were engaged in A. A. Shakhbazov [41], Jung-Shyr Wu and Jyh-Yeong Wang [42]. In all the studies noted, it was concluded that the final probabilities sought could ultimately be found only by numerical methods for a specific flow structure.

But the goal of queuing systems describing, as a rule, is the development of control tools for their operation based on predicting their efficiency when the parameters of the system and/or the parameters of input flow requirements change.

In this case, the number of different variants of systems of algebraic equations, requiring a numerical solution, can be estimated at 10^5–10^6 and more, which makes it difficult to write down so many different systems of equations itself, and also raises doubts about the possibility of their timely numerical solution and the choice of a values rational set of control parameters for service system.

The solution to the control problem can be in the search for an analytical description of the QS models for the most general conditions under which both single requirements and their groups can appear in the input flow.

The most complete analytical description of such QS was obtained in [3] for three types of Markov multichannel queuing systems: with refusals, with a limited and with an unlimited number of waiting places under the conditions of an input flow with a random composition of requirements groups and individual service of each requirement in the group.

At the same time, the models of queuing systems with input flow of requirements groups and with leaving requirements from queue, which are the most close to real service systems, did not receive their description in this work.

As a result, the relevance of the problem of analytical description for the state's final probabilities and performance indicators of queuing system with a non-ordinary input flow of requirements and with the leaving of single requirements from the queue becomes obvious, which also makes the topic of the article – relevant.

3 MATERIALS AND METHODS

In order to demonstrate the logic of obtaining an analytical description of the final probabilities, let us consider a relatively easily visible example for the $M_2/M/n/m$ system (Fig. 1) with waiting, with individual service for each requirement and with individual requirements leaving the queue.

At the entrance of the $M_2/M/3/4$ queuing system with waiting and leaving of individual requirements from the queue, a non-ordinary flow is coming. It consists of two ($L = 2$) partial flows with parameters $\lambda_1 = \lambda a_1$ and $\lambda_2 = \lambda a_2$.

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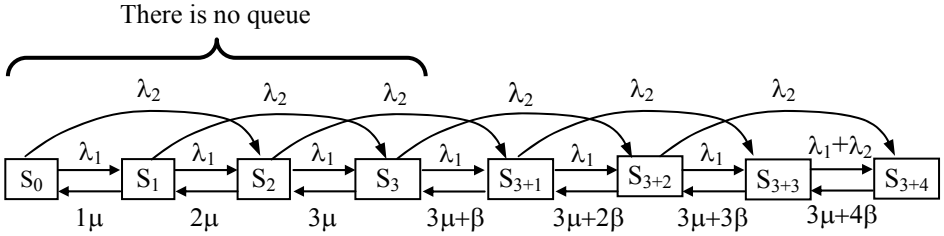

Figure 1 – The model Graph of queuing system $M_2/M/3/4$ with waiting and leaving of individual requirements from the queue

Let's call a group of requirements as a request and define expressions for the coefficients (ρ_i, $i = 0; 1$) of the system load by a part of the entrance flow of requests and for the coefficient ν as follows:

$$\rho_0 = \frac{\lambda_1 + \lambda_2}{\mu}; \quad \rho_1 = \frac{\lambda_2}{\mu}; \quad \nu = \frac{\beta}{\mu}. \tag{3}$$

The model graph (Fig. 1) is described by the system of linearly independent Kolmogorov differential equations for state probabilities p_k, $0 \le k \le 3$ and $p_{3+\gamma}$, $1 \le \gamma \le 4$, where $(p_{k<0} = 0)$:

$$\left.\begin{aligned} &p'_k(t) = -(\lambda + k\mu)p_k(t) + \lambda_1 p_{k-1}(t) + \\ &+ \lambda_2 p_{k-2}(t) + (k+1)\mu p_{k+1}(t), \ k = 0, ..., 2; \\ &p'_{3+\gamma}(t) = -(\lambda + 3\mu + \gamma\beta)p_{3+\gamma}(t) + \\ &+ \lambda_1 p_{3+\gamma-1}(t) + \lambda_2 p_{3+\gamma-2}(t) + \\ &+ [3\mu + (\gamma+1)\beta]p_{3+\gamma+1}(t); \qquad \gamma = 0, ..., 4; \end{aligned}\right\} \tag{4}$$

For the stationary mode of the queuing system operation, equations (4) will take the form:

$$\left.\begin{aligned} &(k+1)\mu p_{k+1} = (\lambda + k\mu)p_k - \lambda_1 p_{k-1} - \\ &- \lambda_2 p_{k-2}, \qquad k = 0, ..., 2; \\ &[3\mu + (\gamma+1)\beta]p_{n+\gamma+1} = (\lambda + 3\mu + \\ &+ \gamma\beta)p_{3+\gamma} - \lambda_1 p_{3+\gamma-1} - \lambda_2 p_{3+\gamma-2}; \ \gamma = 0, ..., 4; \end{aligned}\right\} \tag{5}$$

Then let's perform a sequential summation of the left and right sides of the equations in the system (5) for the steady state conditions:

$$\left.\begin{aligned} &\sum_{i=0}^{k}(i+1)\mu p_{i+1} = \sum_{i=0}^{k}[(\lambda + i\mu)p_i - \\ &- \lambda_1 p_{i-1} - \lambda_2 p_{i-2}], \qquad k = 0, ..., 2; \\ &\sum_{i=0}^{2}(i+1)\mu p_{i+1} + \sum_{i=1}^{3+\gamma}[3\mu + (\gamma+1)\beta]p_{n+\gamma+1} = \\ &= \sum_{i=0}^{2}[(\lambda + i\mu)p_i - \lambda_1 p_{i-1} - \lambda_2 p_{i-2}] + \\ &\sum_{i=0}^{\gamma}[(\lambda + 3\mu + i\beta)p_{3+i} - \\ &- \lambda_1 p_{3+i-1} - \lambda_2 p_{3+i-2}]; \qquad \gamma = 0, ..., 4; \end{aligned}\right\} \tag{6}$$

From formulas (6) we can get:

$$k\mu p_k = \lambda p_{k-1} + \lambda_2 p_{k-2}, \quad k = 1, ..., 3; \tag{7}$$

$$\begin{aligned}[3\mu + \gamma\beta]p_{3+\gamma} &= \lambda p_{3+\gamma-1} + \\ &+ \lambda_2 p_{3+\gamma-2}, \gamma = 1, ..., 4.\end{aligned} \tag{8}$$

Using the notations (3) for the conditions of the steady state, from formulas (7) and (8) we can find:

$$k p_k = \rho_0 p_{k-1} + \rho_1 p_{k-2}, \ k = 1, ..., 3; \tag{9}$$

$$\begin{aligned}(3 + \gamma\nu)p_{3+\gamma} &= \rho_0 p_{(3+\gamma)-1} + \\ &+ \rho_1 p_{(3+\gamma)-2}, \qquad \gamma = 1, ..., 4.\end{aligned} \tag{10}$$

For further reasoning, we recall the well-known Erlang formulas [4] for the queuing system M/M/n with refusals:

$$p_k = \frac{\rho^k}{k!}p_0, \ k = 1, \dots, n; \qquad \rho = \frac{I}{\mu}. \tag{11}$$

And then let's choose the form of analytical expressions for the final probabilities of the queuing system states, taking into account the need to localize the non-ordinary properties of the input request flow in a separate multicomponent function f_k and $f_{n+\gamma}$:

$$p_k = \frac{\rho_0^k}{k!}p_0 f_k, \quad k = 0, ..., n; \tag{12}$$

$$p_{n+\gamma} = \frac{\rho_0^n}{n!}\frac{\rho_0^\gamma}{\prod_{i=1}^{\gamma}(n + i\nu)}p_0 f_{n+\gamma}, \gamma = 1, ..., m. \tag{13}$$

Substituting (12) into (9) and (13) into (10), we find:

$$f_k = f_{k-1} + f_{k-2}\frac{\rho_1}{\rho_0^2}(k-1); k = 1, ..., n; n = 3; \tag{14}$$

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$$f_{n+\gamma} = f_{n+\gamma-1} + f_{n+\gamma-2}\frac{\rho_1}{\rho_0^2}\times \\ \times[n+(\gamma-1)v]; \quad n=3;\gamma=1,...,4. \tag{15}$$

For the convenience of writing the general case of the non-ordinary functions expressions, we note that the sum of elements whose lower boundary is greater than the upper one, do not contain any element, therefore it is equal to zero. Elements with a negative index in this case have no physical meaning therefore they are also equal to zero.

Now let's consider the general case of a $M_L/M/n/m$ queuing system with an input flow of requirements groups and with the leaving of individual requirements from the queue.

In this case, the input of the queuing system receives a flow of requirements groups, which consists of L partial flows with parameters $\lambda_i = \lambda a_i$, $i = 1,\ldots, L$. Then expressions (3) take the form:

$$\rho_i = \frac{1}{\mu}\sum_{j=1+i}^{L}\lambda_j, \ i=0,...,L. \tag{16}$$

For the general case of QS $M_L/M/n/m$ with single "impatient" requirements leaving the queue, expressions (14) and (15) take the form:

$$f_k = f_{k-1} + \sum_{i=2}^{k}\left[f_{k-i}\frac{\rho_{i-1}}{\rho_0^i}\times \right. \\ \left. \times\prod_{j=1}^{i-1}(k-j)\right]; \ k=2,\ldots,n; \tag{17}$$

$$f_{n+1} = \sum_{k=0}^{n}\frac{\rho_{n-k}\cdot n!}{k!\,\rho_0^{n+1-k}}f_k; \tag{18}$$

$$f_{n+\gamma} = \prod_{i=1}^{\gamma-1}(n+iv)\sum_{k=0}^{n}\left[\frac{\rho_{n+\gamma-k-1}\cdot n!}{\rho_0^{n+\gamma-k}\cdot k!}f_k + \right. \\ \left. + \sum_{j=1}^{\gamma-1}\frac{\rho_{\gamma-j-1}}{\rho_0^{\gamma-j}\cdot\prod_{i=1}^{j}(n+iv)}f_{n+j}\right], \ \gamma=2,...,m. \tag{19}$$

One can make sure that for the considered example (Fig. 1) with the conditions $\rho_i = 0$, $i>1$; $n=3$, $m=4$ expressions (17), (18) and (19) are transformed into expressions (14) and (15), respectively.

To determine the value of the first non-ordinary function (f_0), we substitute the value $k=0$ in formula (12) and then we can get:

$$p_0 = p_0 \cdot f_0. \tag{20}$$

From equation (20) follows the equality $f_0 = 1$. The value of the non-ordinary function (f_1) can be found from formula (14) or from (17). If $k=1$, then we get:

$$f_1 = f_{1-1} + 0 = f_0. \tag{21}$$

Thus, the numerical values of the first two non-ordinary functions become known:

$$f_0 = f_1 = 1. \tag{22}$$

To find the zero state probability p_0 of the QS, we use the condition of normalizing the probabilities. Substituting there formulas (12) and (13), we obtain:

$$\sum_{k=0}^{n+m}p_k = 1; \rightarrow \sum_{k=0}^{n}\frac{\rho_0^k}{k!}p_0 f_k + \\ + \frac{\rho_0^n}{n!}p_0\sum_{\gamma=1}^{m}\frac{\rho_0^\gamma}{\prod_{i=1}^{\gamma}(n+iv]}f_{n+\gamma} = 1. \tag{23}$$

Then we take out the common factor p_0 outside the brackets and find its value:

$$p_0 = \left(\sum_{k=0}^{n}\frac{\rho_0^k}{k!}f_k + \frac{\rho_0^n}{n!}\sum_{\gamma=1}^{m}\frac{\rho_0^\gamma}{\prod_{i=1}^{\gamma}(n+iv)}f_{n+\gamma}\right)^{-1}. \tag{24}$$

To verify the correctness of the solution obtained, we'll find the value of non-ordinary functions f_k for the case of non-ordinary input flow of requirements degeneration into the simplest flow $a_1 = 1$; $a_i = 0$, $i>1$. In this case, the parameters of the partial flow of requests for service immediately two or more requirements are equal to zero ($\lambda_i = \lambda a_i = 0, i>1$). Then from formulas (1) and (16) it follows $\rho_0 = \rho; \rho_i = 0; i>0$.

If we substitute the obtained values ρ_i into formulas (17) and (19), taking into account the equality $f_0 = f_1 = 1$, we can see that the second term in formula (17) vanishes and the first part of non-ordinary functions becomes equal to one:

$$f_k = f_{k-1} = 1; \quad k=1,...,n. \tag{25}$$

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In formula (19), the first term in the square bracket becomes equal to zero. The rest of the terms are nonzero only for the condition $j = \gamma - 1$, under which $\rho_{\gamma-j-1} = \rho_0 \neq 0$:

$$f_{n+\gamma} = \prod_{i=1}^{\gamma-1}(n+iv)\left[0 + \frac{\rho_0}{\rho_0 \cdot \prod_{i=1}^{\gamma-1}(n+iv)} f_{n+\gamma-1}\right]. \quad (26)$$

From (26) follows the equality:

$$f_{n+\gamma} = f_{n+\gamma-1} = 1, \ \gamma = 1, ..., m. \quad (27)$$

As a result, expressions (13) and (24) for the final probabilities of the $M_L/M/n/m$ queuing system with the leaving of requirements from the queue are transformed into well-known formulas for the same system $M/M/n/m$ but with the simplest input flow of requirements:

$$\left.\begin{aligned} p_k &= \frac{\rho^k}{k!} p_0, \qquad k = 0, ..., n; \\ p_{n+\gamma} &= \frac{\rho^n}{n!}\frac{\rho^\gamma}{n^\gamma} p_0, \ \gamma = 1, ..., m; \ \rho = \frac{I}{\mu}; \end{aligned}\right\} \quad (28)$$

$$p_0 = \left(\sum_{k=0}^{n}\frac{\rho^k}{k!} + \frac{\rho^n}{n!}\sum_{\gamma=1}^{m}\frac{\rho^\gamma}{n^\gamma}\right)^{-1}. \quad (29)$$

This result testifies in favor of the research correctness and the expressions obtained for the non-ordinary functions (17), (19) and final probabilities (12), (13) and (24) for the $M_L/M/n/m$ system with a non-ordinary input flow, with the waiting and leaving of individual "impatient" requirements from the queue.

The obtained description (12), (13), (16)–(19) of a queuing system is a general case for well-known Markov queuing systems with a non-ordinary input flow of requests and without leaving the queue $M_L/M/n/m$ and for a queuing system with refusals $M_L/M/n$ [3], as well as for the QS with the simplest input flow of requirements and leaving the queue $M/M/n/m$, for the QS without leaving the queue $M/M/n/m$ and for the QS $M/M/n$ with refusals.

To verify this statement, let us consider a variant of the description (12), (13), (16)–(19) transition into the description of a queuing system with waiting $M_L/M/n/m$ and without leaving the queue.

In the queuing system $M_L/M/n/m$ with waiting, the leaving of requirements from the queue is not provided.

Therefore, in formulas (3), (13), (19), and (24) of the queuing system $M_L/M/n/m$ model with waiting and with the leaving of "impatient" requirements from the queue, the value β and the coefficient v are equal to zero:

$$\beta = 0; \ v = 0. \quad (30)$$

The formulas for the load factors of the system as a part of the input flow of requirements (16), for the non-ordinary functions f_k (17) and for the probabilities p_k (12) remain unchanged. The analytical description of the final probabilities for the states in the $M_L/M/n/m$ QS with the presence of requirements in the queue takes the form:

$$p_{n+\gamma} = \frac{\rho_0^n}{n!}\frac{\rho_0^\gamma}{n^\gamma} p_0 f_{n+\gamma}, \ \gamma = 1, ..., m; \quad (31)$$

$$f_{n+\gamma} = n^{\gamma-1}\left[\sum_{k=0}^{n}\frac{\rho_{n+\gamma-k-1}\cdot n!}{\rho_0^{n+\gamma-k}\cdot k!} f_k + \sum_{j=1}^{\gamma-1}\frac{\rho_{\gamma-j-1}}{\rho_0^{\gamma-j}\cdot n^j} f_{n+j}\right]; \quad (32)$$

$$p_0 = \left(\sum_{k=0}^{n}\frac{\rho_0^k}{k!} f_k + \frac{\rho_0^n}{n!}\sum_{\gamma=1}^{m}\left[\left(\frac{\rho_0}{n}\right)^\gamma f_{n+\gamma}\right]\right)^{-1}. \quad (33)$$

The analytical description for the final probabilities of the $M_L/M/n$ queuing system with refusals, with individual servicing of each requirement and with the flow of requirements groups at the system input includes formulas (12), (16), (17), and (22), as well as the formula (24), which in this case will have the following form:

$$p_0 = \left(\sum_{k=0}^{n}\frac{\rho_0^k}{k!} f_k\right)^{-1}. \quad (34)$$

Subsequent transformations of expressions (31) – (34) for the types of queuing systems mentioned above are given in [3].

4 EXPERIMENTS

To evaluate the performance of a queuing system with the input flow of requirements groups and with the waiting and leaving of individual "impatient" requirements from the queue, sometimes the input flow is replaced with the simplest flow and is used the $M/M/n/m$ model.

To check the admissibility of such a replacement, we will use the known example [3], in which we change the number of service devices and waiting places, and also take into account the possibility of "impatient" requirements leaving the queue. The graph of the marked model is presented in Fig. 2.

The parameters of the considered QS model $M_L/M/n/m$ are presented in Table 1 (items 1–9) and turn out to be equal: I=2 [requirements/minute]; $n = 4$; $m = 3$; $\beta = 0{,}25$; $L = 8$; $a_i = 1/L$, $i = 1, ..., L$; $\mu = 1$ [minute^{-1}]; $\rho_i = 0$, for $i > 7$.

In such a system, the total performance of service devices is bigger than the intensity of the input requirements flow. Then, in the case of deterministic input flow, all requirements must be served.

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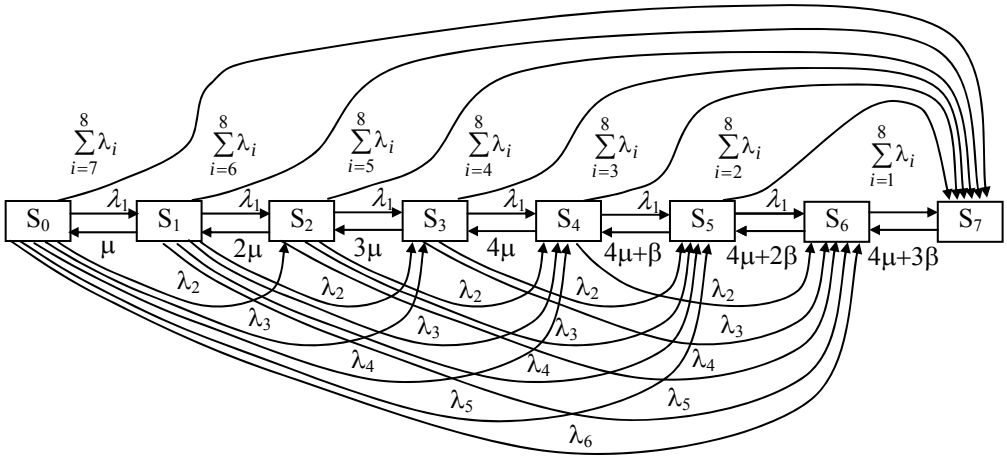

Figure 2 – The model graph of queuing system M_8 /M/4/3 with the waiting and leaving of individual "impatient" requirements from the queue

Table 1 – Evaluation of the influence of requirements groups in the input flow on the state's probabilities in the queuing systems with leaving of requirements from the queue (see Fig. 3)

Model M_L /M/n/m (names and values of model parameters)												Model M/M/n/m		
#	Name	Value	#	Name	Value	#	Name	Value	#	Name	Value	#	Name	Value
1	*n*	4	11	M[*i*]	4.5	21	$\mathbf{f_0}$	1	31	p_2	0.119	41	p_0	0.133
2	*m*	3	12	ρ=*I*/μ	2	22	$\mathbf{f_1}$	1	32	p_3	0.086	42	p_1	0.266
3	μ	3	13	ρ_0	0.444	23	$\mathbf{f_2}$	2.969	33	p_4	0.064	43	p_2	0.266
4	*I*	2	14	ρ_1	0.389	24	$\mathbf{f_3}$	14.5	34	p_5	0.057	44	p_3	0.177
5	*L*	8	15	ρ_2	0.333	25	$\mathbf{f_4}$	97.53	35	p_6	0.049	45	p_4	0.089
6	λ	0.444	16	ρ_3	0.278	26	$\mathbf{f_5}$	825.4	36	p_7	0.040	46	p_5	0.042
7	a_i	0.125	17	ρ_4	0.222	27	$\mathbf{f_6}$	7168	37	$M_{b.d}$	0.933	47	p_6	0.019
8	λ_i	0.194	18	ρ_5	0.167	28	$\mathbf{f_7}$	63004	38	A	0.933	48	p_7	0.008
9	β	0.25	19	ρ_6	0.111	29	P_0	0.405	39	*Lqueue*	0.275	49	*Lqueue*	0.102
10	*v*	0.25	20	ρ_7	0.056	30	P_1	0.180	40	$\boldsymbol{P}_{\text{serv.gr}}$	**0.466**	50	$\boldsymbol{P}_{\text{serv.Er}}$	**0.843**

The law of states' probability distribution in a queuing system allows finding the calculation formulas for the following characteristics: the mathematical expectation of the busy devices number; the mathematical expectation of the requirements number in groups; absolute system capacity; service probability; queue length in the queuing system and for service refusal probability:

$$\left.\begin{aligned} M_{b.d} &= \sum_{k=1}^{n} k \cdot p_k ;\ M[i] = \sum_{i=1}^{L} i \cdot a_i ; \\ \lambda &= \tfrac{I}{M[i]};\quad \lambda_i = \lambda \cdot a_i,\ i = 1, ..., L; \end{aligned}\right\} \qquad (35)$$

$$\left.\begin{aligned} A &= \mu \cdot M_{b.d} ;\ P_{service} = \frac{A}{I}; \\ P_{refusal} &= 1 - P_{service} ;\ L_{queue} = \sum_{\gamma=1}^{m} \gamma \cdot p_{n+\gamma} . \end{aligned}\right\} \qquad (36)$$

5 RESULTS

For the considered version of the queuing system model, the non-ordinary functions (17)-(19) will take the specific form for states without queue (37) and for states with a queue (38)–(40):

$$\left.\begin{aligned} f_0 &= f_1 = 1;\ f_2 = f_1 + f_0 \frac{\rho_1}{\rho_0^2}(2-1); \\ f_3 &= f_2 + f_1 \frac{\rho_1}{\rho_0^2} 2 + f_0 \frac{\rho_2}{\rho_0^3} 2 \cdot 1; \\ f_4 &= f_3 + f_2 \frac{\rho_1}{\rho_0^2} 3 + f_1 \frac{\rho_2}{\rho_0^3} 3 \cdot 2 + f_0 \frac{\rho_3}{\rho_0^4} 3 \cdot 2 \cdot 1; \end{aligned}\right\}. \qquad (37)$$

$$f_5 = 24\frac{\rho_4}{\rho_0^5} f_0 + 24\frac{\rho_3}{\rho_0^4} f_1 + 12\frac{\rho_2}{\rho_0^3} f_2 + 4\frac{\rho_1}{\rho_0^2} f_3 + f_4 ; \qquad (38)$$

$$\begin{aligned} f_6 = (4+1v)\Bigg[& 24\frac{\rho_5}{\rho_0^6} f_0 + 24\frac{\rho_4}{\rho_0^5} f_1 + \\ & + 12\frac{\rho_3}{\rho_0^4} f_2 + 4\frac{\rho_2}{\rho_0^3} f_3 + \frac{\rho_1}{\rho_0^2} f_4 \Bigg] + f_5 ; \end{aligned} \qquad (39)$$

$$\begin{aligned} f_7 = (4+1v)(4+2v)\Bigg[& 24\frac{\rho_6}{\rho_0^7} f_0 + 24\frac{\rho_5}{\rho_0^6} f_1 + 12\frac{\rho_4}{\rho_0^5} f_2 + \\ & + 4\frac{\rho_3}{\rho_0^4} f_3 + \frac{\rho_2}{\rho_0^3} f_4 \Bigg] + (4+2v)\frac{\rho_1}{\rho_0^2} f_5 + f_6 . \end{aligned} \qquad (40)$$

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The indicators (35), (36) and probabilities of the model states (Fig. 2) in the steady state are described by the formulas (1), (12), (3), (13), (16), (17), (18), (19), (21) and (24).

Let us use the noted formulas and estimate the probabilities (p_k, $p_{n+\gamma}$) of the system states and the probability of servicing requirements in queuing systems with leaving the queue of individual requirements, with the same number of servicing devices and waiting places, with the same intensity of the incoming flow of requirements but no requirement groups.

In the first case, a flow of requirements groups arrives at the input of the system, and in the second case, the input flow of requirements is the simplest and includes only single requirements. The calculation results are presented in table 1 and in the Fig. 3.

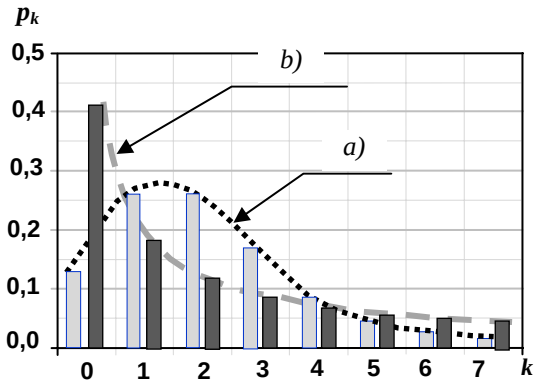

Figure 3 – Final probabilities p_k of the states in the same queuing systems with leaving individual requirements from the queue and with the same intensity:
a) QS without any groups of requirements in the input flow (model M/M/4/3);
b) QS with groups of requirements in the composition of the real input flow

Quantitative estimates (see Table 1, Fig. 3) lead to the following conclusion.

6 DISCUSSION

In a number of cases, the main indicator of the queuing system efficiency is the probability of servicing requirements, on which other indicators depend. Therefore, we will consider the numerical values of this indicator for the compared queuing systems.

The appearance of groups in the input flow of requirements changes the probability distribution of the considered systems states (Table 1 items 41–48, items 29–39, and Fig. 3) and leads to a decrease in the probability of service (Table 1 item 40) by about 44% compared to the probability service in the model with the simplest input flow of requirements (Table 1, item 50).

A decrease in the probability of service, as the main indicator of efficiency, can be significant for the results of the system's operation and requires a quantitative forecast for timely action in managing of the system operation.

The influence of the requirements groups composition on the change in the final probabilities is concentrated in the multiplicative non-ordinary functions (17), (19) (Table 1 items 21–28), which each time reflect the magnitude of the transformation of the system specific states probabilities with a group input flow in comparison with the same system, but with the simplest input flow of requirements, and may have an order value 10^4–10^5.

The considered features determine the need to take into account the composition of the groups in the input flow of requirements when conducting assessments and when managing the corresponding queuing systems.

CONCLUSIONS

In the course of the research, the analytical expressions for the final probabilities of states in the M_L/M/n/m queuing system with an input flow of requirements groups, with individual service of requirements and with leaving of "impatient" requirements from the queue were obtained for the first time.

The influence of the requirements groups composition on the change in the final probabilities of queuing system is concentrated in the multiplicative non-ordinary functions (17), (19), which each time reflect the magnitude of the transformation of the system specific states probabilities and can have a value of the order of 10^4–10^5.

An analytical description of the final probabilities and performance indicators allows the use of calculation automation tools, for example Microsoft Excel, to obtain the results of estimates almost instantly and to select the values of the parameters for controlling the operation of the service system in real time.

The scientific novelty of the results obtained lies in the creation of possibilities for predicting the effectiveness of QS M_L/M/n/m with leaving of "impatient" requirements from the queue and of known types of Markov queuing systems with an input flow of requirements groups, with individual service for each requirement and with a random number of requirements in groups.

The obtained description (12), (13), (16)–(19) of a queuing system is a general case for other well-known types of Markov queuing systems with a non-ordinary input flow of requests and without leaving the queue M_L/M/n/m and for a queuing system with refusals M_L/M/n [3], as well as for the QS with the simplest input flow of requirements and leaving the queue M/M/n/m, for the QS without leaving the queue M/M/n/m and for the QS with refusals M/M/n.

At the same time, the well-known Markov service models for the simplest flow of requirements turned out to be a special case of the considered models with an input flow of requirements groups. In the new formulas, all the features of requirements groups servicing are localized in recurrent expressions for the non-ordinary functions, which makes it easier to perform calculations in real time.

The practical significance of the results obtained lies in creating conditions for the directed solution of problems of analysis, synthesis and control of Markov queuing systems in the general case of a requirements

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groups input flow with a random number of requirements in groups. The formulas obtained for calculating the values of the non-ordinary functions are recurrent and convenient for practical calculations. The numerical values of these functions clearly show the deformation of the final states probabilities in queuing systems with an input flow of requirements groups compared to known queuing systems with the simplest input flow of requirements.

Prospects for further research may include the construction of queuing systems models with incomplete availability of servicing device. Each of the systems under consideration is an actual model of real systems in economics, medicine, modern communication systems and in other areas.

REFERENCES

1. Khinchin A. Ya. Pod red. B. V. Gnedenko Raboty po matematicheskoy teorii massovogo obsluzhivaniya. Moscow, Fizmatgiz, 1963, 236 p.
2. Venttsel' Ye. S. Issledovaniye operatsiy. Moscow, Sovetskoye radio, 1972, 552 p.
3. Gorodnov V. P. The analytical description of final probabilities for states of queuing systems with input flow of groups of requirements, *Radio Electronics, Computer Science, Control*, 2019, No. 4 (51), pp. 25–37 DOI https://doi.org/10.15588/1607-3274-2019-4-3/.
4. Erlang A.K. The Theory of Probabilities and Telephone Conversations, *Nyt Tidsskrift for Mathematic Ser. B 20*, 1909.
5. Brown L., Gans N., Mandelbaum A. et.al Statistical Analysis of a Telephone Call Center, *Queueing-Science Perspective Journal of the American Statistical Association*, 2005, Vol. 100, Issue 469, pp. 36–50. DOI: https://doi.org/10.1198/016214504000001808/.
6. Gaydamaka Yu. V., Zaripova E. R., Samuilov K. E. Modeli obsluzhivaniya vyzovov v seti sotovoy podvizhnoy svyazi. Moscow, RUDN, 2008, 72 p.
7. Lakatos L., Szeidl L., Telek M. Introduction to queueing systems with telecommunication applications, books.google.com, 2012.
8. Lozhkovsky A. G. Teoriya massovogo obsluzhivaniya v telekommunikatsiyakh: uchebnik. Odessa, ONAS im. A. S. Popova, 112 p.
9. Tarasov V. N., Bakhareva N. F., Akhmetshina E. G. Modeli teletrafika na osnove sovremennoy teorii massovogo obsluzhivaniya, *Infokommunikatsionnyye tekhnologii*, 2018, Vol. 16, № 1, pp. 68–74.
10. Tsitsiashvili G. Sh., Osipova M. A., Samuilov K. E. et al. Primeneniye mnogokanal'nykh sistem massovogo obsluzhivaniya s otkazami k konstruirovaniyu telekommunikatsionnykh setey, *Dal'nevostochnyy matematicheskiy Zhurnal*, 2018, Vol. 18:1, pp. 123–126.
11. Ebadi M., Ahmadi-Javid A. Socio-economic design of control charts for monitoring service processes: a case study of a restaurant system, *Journal Quality Technology & Quantitative Management,* 2018, Published online. DOI: https://doi.org/10.1080/16843703.2018.1519880/.
12. Liu Zhongyia, Liu Jingchenb, Zhai Xinb et al. Police staffing and workload assignment in law enforcement using multi-server queueing models, *European Journal of Operational Research,* 2019, Vol. 276, Issue 2, pp. 614–625. DOI: https://doi.org/10.1016/j.ejor.2019.01.004
13. Albey E., Bilge U., Uzsoy R. Multi-dimensional clearing functions for aggregate capacity modeling in multi-stage production systems, *International Journal of Production Research,* 2017, Vol. 55, Issue 14, pp. 4164–4179. DOI: https://doi.org/10.1080/00207543.2016.1257169/.
14. Korolkova L. I., Pereverzev P. P. Optimizatsiya protsessov predpriyatiya na osnove novoy metodiki rascheta kharakteristik mnogofaznoy sistemy massovogo obsluzhivaniya s nepreryvnoy zagruzkoy bez promezhutochnykh nakopiteley, *Sovremennyye problemy nauki i obrazovaniya*, 2012, № 3.
15. Papadopoulos H. T., Heavey C. Queueing theory in manufacturing systems analysis and design: A classification of models for production and transfer lines, *European Journal of Operational Research*, 1996, Vol. 92, Issue 1, pp. 1–27. DOI: https://doi.org/10.1016/0377-2217(95)00378-9
16. Zavanella L., Zanoni S., Ferretti I. et al. Energy demand in production systems: A Queuing Theory perspective, *International Journal of Production Economics,* 2015, Vol. 170, Part B, pp. 393–400. DOI: https://doi.org/10.1016/j.ijpe.2015.06.019/.
17. Istomina A. A., Badenikov V. Y., Istomin A. L. Optimal'noye upravleniye tovarnymi zapasami na osnove teorii massovogo obsluzhivaniya, *FGBOU VO «Angarskiy gosudarstvennyy tekhnicheskiy universitet»,* 2016, № 10, pp. 148–152,
18. Plotkin B. K., Delukin L. A. Ekonomiko-matematicheskiye metody i modeli v kommercheskoy deyatel'nosti i logistike: Uchebnik. SPb, Izd-vo, 2015, 345 p.
19. Popov A. V., Obrezanova E. R., Sinebryukhova E. Yu. Veroyatnostnoye modelirovaniye logisticheskoy sistemy gruzoperevozok. Radíoyelektronní í komp'yuterní sistemi, *Radioelektronni i komp'yuterni systemy*, 2012, № 1 (53), pp. 144–151.
20. Balsamo S., De Nitto V Personè, Inverardi P. A review on queueing network models with finite capacity queues for software architecture performance prediction, *Performance Evaluation*, 2003, Vol. 51, Issue. 2, pp. 269–288. Access mode: DOI: https://doi.org/10.1016/S0166-5316(02)00099-8/.
21. Kleinrok L. Vychislitel'nyye sistemy s ocheredyami. Moscow, Mir, 1979, 600 p.
22. Afanas'yeva L. G., Bulinskaya E. V. Matematicheskiye modeli transportnykh sistem, osnovannyye na teorii ocheredey, *Trudy MFTI*, 2010, Vol. 2, Issue 4, pp. 6–10.
23. Assad A. A. Models for rail transportation, *Transportation Research Part A: General,* 1980, Vol. 14, Issue 3, pp. 205–220. Access mode: DOI: https://doi.org/10.1016/0191-2607(80)90017-5/.
24. Kazakov A., Lempert A. A., Zharkov M. L. Modelirovaniye transportno-peresadochnykh uzlov na osnove sistem massovogo obsluzhivaniya – mnogofaznykh i c bmap-potokom, *Vestnik ural'skogo gosudarstvennogo universiteta putey soobshcheniya*, 2016, № 4 (14), pp. 4–14. DOI: https://doi.org/10.20291/2079-0392-2016-4-4-14/.
25. Rachinskaya M. A., Fedotkin M. A. Postroyeniye i issledovaniye veroyatnostnoy modeli tsiklicheskogo upravleniya potokami maloy intensivnosti, *Vestnik Nizhegorodskogo universiteta im. N. I. Lobachevskogo*, 2014, № 4 (1), pp. 370–376.
26. Grachev V. V. Moiseev A. N., Nazarov A. A. et al. Mnogofaznaya model' massovogo obsluzhivaniya sistemy raspredelennoy obrabotki dannykh, Doklady TUSURa, 2012, № 2 (26), Part 2, pp. 248–251.

DOI 10.15588/1607-3274-2020-4-4

27. Mandelbaum A., Pats G. State-dependent queues: approximations and applications, *Stochastic Networks, IMA Volumes in Mathematics, Springer*, 1995, pp. 239–282.
28. Mandelbaum A., Zeltyn S. The impact of customers patience on delay and abandonment: some empirically driven experiments with the M/M/n + G queue, *Operations Research*, 2004, Vol. 26, pp. 377–411.
29. Pankratova E. V. Issledovaniye matematicheskikh modeley neodnorodnykh beskonechnolineynykh SMO, *TSU,* 2016, pp. 1–19.
30. Puhalskii A. A., Reed J. E. On many-server queues in heavy traffic, *Annals of Applied Probability*, 2008, Vol. 20, pp. 129–195.
31. Reed J. E. The G/GI/N queue in the Halfin-Whitt regime I: infinite-server queue system equations, *The Stern School, NYU*, 2007, pp. 1– 59. DOI: https://doi.org/10.1214/09-AAP609/.
32. D'Auria B. Stochastic decomposition of the M/G/∞ queue in a random environment, *Operations Research Letters*, 2007, Vol. 35, pp. 805–812.
33. Saaty T. L. Elements of queuing theory: with applications. New York, Dover Pubns, 1983, 423 p.
34. Klimov G. P. Teoriya massovogo obsluzhivaniya. Moscow, MGU, 2011, 307 p.
35. Tsitsiashvili G. Sh. Invariantnyye svoystva sistem massovogo obsluzhivaniya s neskol'kimi potokami, *Dal'nevostochnyy matematicheskiy zhurnal,* 2018, Vol. 18:2, pp. 267–270.
36. Bocharov P. P., Pechinkin A. V. Teoriya massovogo obsluzhivaniya. Moscow, Izd-vo RUDN, 1995, 520 p.
37. Moiseev A. N., Nazarov A. A. Beskonechnolineynyye sistemy i seti massovogo obsluzhivaniya. Tomsk, Izd-vo NTL, 2015, 240 p.
38. Moiseeva S. P. Razrabotka metodov issledovaniya matematicheskikh modeley nemarkovskikh sistem obsluzhivaniya s neogranichennym chislom priborov i nepuassonovskimi vkhodyashchimi potokami: dis. doktora fiz.-mat. Nauk. Tomsk, NI TGU, 2014, 260 p.
39. Doorn E. A., Jagers A. A. Note on the GI/GI/∞ system with identical service and interarrival-time distributions, *Journal of queueing systems*, 2004, Vol. 47, pp. 45–52.
40. Matveev V. F., Ushakov V. G. Sistemy massovogo obsluzhivaniya. Moscow, Izd-vo MGU, 1984, 242 p.
41. Shakhbazov A. A. Ob odnoy zadache obsluzhivaniya neordinarnogo potoka trebovaniy, *Dokl. AN SSSR*, 1962, Vol. 145:2, pp. 289–292.
42. Jung-Shyr Wu, Jyh-Yeong Wang Refining the diffusion approximation for M/G/m queuing systems with group arrivals, *International Journal of Systems Science*, 1992, Vol. 23, Issue 1, pp. 127–133. DOI: https://doi.org/10.1080/00207729208949194/.
43. Kutselay N. O., Safonov S. V. Obsluzhivaniye neordinarnogo potoka trebovaniy, *Molodoy uchenyy,* 2018, № 23, pp. 1–2.
44. Bogoyavlenskaya O.Yu. Statsionarnoye raspredeleniye dliny ocheredi v sisteme s neordinarnym potokom i distsiplinoy razdeleniya protsessora, *Trudy Petrozavodskogo gosudarstvennogo universiteta, seriya «Matematika»*, 1996, Vol. 3, pp. 3–10.
45. Pechinkin A. V. Inversionnyy poryadok obsluzhivaniya s veroyatnostnym prioritetom v sisteme obsluzhivaniya s neordinarnym potokom, *Matematicheskiye issledovaniya. Ser. Veroyatnost' i prilozheniya*, 1989, Vol. 109, pp. 83–94.
46. Ramaswami V. A. duality theorem for the matrix paradigms in queueing theory, *Communications in Statistics. Stochastic Models*, 1990, pp. 151–161. DOI: https://doi:10.1080/15326349908807141/.

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АНАЛІТИЧНИЙ ОПИС ФІНАЛЬНИХ ЙМОВІРНОСТЕЙ СТАНІВ В СИСТЕМІ МАСОВОГО ОБСЛУГОВУВАННЯ З ВХІДНИМ ПОТОКОМ ГРУП ВИМОГ, З ОЧІКУВАННЯМ І ВІДХОДОМ З ЧЕРГИ

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АНОТАЦІЯ

Актуальність Завдання прогнозування ефективності реальних систем масового обслуговування в разі можливого надходження груп вимог і відходу «нетерплячих» заявок з черги. Метою дослідження було моделювання роботи таких систем для створення можливостей контролю їх роботи в режимі реального часу.

Метод. Ймовірності станів систем масового обслуговування з вхідним потоком груп вимог з випадковим складом і з відходом «нетерплячих» вимог з черги описуються диференціальними рівняннями Колмогорова. У стаціонарному стані ці рівняння перетворюються в лінійно залежну однорідну систему алгебраїчних рівнянь. Структура рівнянь залежить від числових значень параметрів груп вимог вхідного потоку і керованої системи обслуговування. Тому спроба прогнозувати ефективність системи стикається з необхідністю написати і чисельно вирішити рахункову безліч систем алгебраїчних рівнянь, що досить складно. Ключовою ідеєю запропонованого методу пошуку аналітичного опису фінальних ймовірностей для згаданої системи масового обслуговування було прагнення локалізувати вплив груп вимог у вхідному потоці на роботу системи масового обслуговування в мультиплікативних функціях неординарності. Такі функції дозволяють отримати необхідний аналітичний опис і оцінити ступінь трансформації фінальних ймовірностей в порівнянні з відомими системами, а також оцінити прогнозні значення відомих показників ефективності системи масового обслуговування при виборі параметрів управління її роботою.

Результати. Вперше отримано аналітичні вирази для фінальних ймовірностей станів системи масового обслуговування з вхідним потоком груп вимог випадкового складу, з обмеженою кількістю місць очікування, з індивідуальним обслуговуванням і відходом «нетерплячих» вимог з черги, що дає можливість оцінити всі відомі показники роботи системи

Висновки. Отриманий опис виявився загальним випадком для відомих типів марковских систем масового обслуговування з неординарним і найпростішим вхідним потоком вимог. Результати чисельного експерименту свідчать на користь коректності отриманих аналітичних виразів для фінальних ймовірностей і на користь можливості їх практичного

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застосування в реальних системах масового обслуговування при вирішенні завдань прогнозування ефективності, а також аналізу і синтезу параметрів. реальних систем масового обслуговування.

КЛЮЧОВІ СЛОВА: марковские моделі, системи масового обслуговування, групи вимог, відхід з черги.

УДК 519.872

АНАЛИТИЧЕСКОЕ ОПИСАНИЕ ФИНАЛЬНЫХ ВЕРОЯТНОСТЕЙ СОСТОЯНИЙ В СИСТЕМЕ МАССОВОГО ОБСЛУЖИВАНИЯ С ВХОДНЫМ ПОТОКОМ ГРУПП ТРЕБОВАНИЙ, С ОЖИДАНИЕМ И УХОДОМ ИЗ ОЧЕРЕДИ

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АННОТАЦИЯ

Актуальность. Задача прогнозирования эффективности реальных систем массового обслуживания в случае возможного поступления групп требований и ухода «нетерпеливых» заявок из очереди. Целью исследования было моделирование работы таких систем для создания возможностей контроля их работы в режиме реального времени.

Метод. Вероятности состояний систем массового обслуживания с входным потоком групп требований со случайным составом и с уходом «нетерпеливых» требований из очереди описываются дифференциальными уравнениями Колмогорова. В стационарном состоянии эти уравнения преобразуются в линейно зависимую однородную систему алгебраических уравнений. Структура уравнений зависит от числовых значений параметров групп требований входного потока и управляемой системы обслуживания. Поэтому попытка прогнозировать эффективность системы сталкивается с необходимостью написать и численно решить счетное множество систем алгебраических уравнений, что достаточно сложно. Ключевой идеей предложенного метода поиска аналитического описания финальных вероятностей для упомянутой системы массового обслуживания было стремление локализовать влияние групп требований во входном потоке на работу системы массового обслуживания в мультипликативных функциях неординарности. Такие функции позволяют получить необходимое аналитическое описание и оценить степень трансформации финальных вероятностей по сравнению с известными системами, а также оценить прогнозные значения известных показателей эффективности системы массового обслуживания при выборе параметров управления ее работой.

Результаты. Впервые получены аналитические выражения для финальных вероятностей состояний системы массового обслуживания с входным потоком групп требований случайного состава, с ограниченным количеством мест ожидания, с индивидуальным обслуживанием и уходом «нетерпеливых» требований из очереди, что дает возможность оценить все известные показатели работы системы.

Выводы. Полученное описание оказалось общим случаем для известных типов марковских систем массового обслуживания с неординарным и простейшим входным потоком требований. Результаты численного эксперимента свидетельствуют в пользу корректности полученных аналитических выражений для финальных вероятностей и в пользу возможности их практического применения в реальных системах массового обслуживания при решении задач прогнозирования эффективности, а также анализа и синтеза параметров. реальных систем массового обслуживания.

КЛЮЧЕВЫЕ СЛОВА: марковские модели, системы массового обслуживания, группы требований, уход из очереди.

ЛІТЕРАТУРА / ЛИТЕРАТУРА

1. Хинчин А. Я. Работы по математической теории массового обслуживания. Под ред. Б. В. Гнеденко / А. Я. Хинчин. – М. : Физматгиз, 1963. – 236 с.
2. Вентцель Е. С. Исследование операций / Е. С. Вентцель. – М. : Советское радио, 1972. – 552 с.
3. Gorodnov V. P. The analytical description of final probabilities for states of queuing systems with input flow of groups of requirements/ V. Gorodnov // Radio Electronics, Computer Science, Control. – 2019. – № 4 (51). – P. 25–37. DOI https://doi.org/10.15588/1607-3274-2019-4-3/.
4. Erlang A.K. The Theory of Probabilities and Telephone Conversations / A. K. Erlang // Nyt Tidsskrift for Mathematic Ser. B 20. – 1909.
5. Brown L. Statistical Analysis of a Telephone Call Center / [L. Brown, N. Gans, A. Mandelbaum et al.] //Queueing-Science Perspective Journal of the American Statistical Association.– 2005.– Vol. 100, Issue 469. – P. 36–50. DOI: https://doi.org/10.1198/016214504000001808/.
6. Гайдамака Ю.В. Модели обслуживания вызовов в сети сотовой подвижной связи: учебно-методическое пособие. / Ю. В. Гайдамака, Э. Р. Зарипова, К. Е. Самуйлов. – М. : РУДН, 2008. – 72 р.
7. Lakatos L. Introduction to queueing systems with telecommunication applications. / L. Lakatos, L. Szeidl, M. Telek // books.google.com. – 2012.
8. Ложковский А. Г. Теория массового обслуживания в телекоммуникациях : учебник / А. Г. Ложковский. – Одесса : ОНАС им. А. С. Попова – 112 с.
9. Tarasov V. N. Модели телетрафика на основе современной теории массового обслуживания. / В. Н. Тарасов, Н. Ф. Бахарева, Э. Г. Ахметшина // Инфокоммуникационные технологии. – 2018. – Т. 16, № 1. – С. 68–74.
10. Цициашвили Г. Ш. Применение многоканальных систем массового обслуживания с отказами к конструированию телекоммуникационных сетей / Г. Ш. Цициашвили, М. А. Осипова, К. Е. Самуйлов и др. // Дальневост. матем. журн. – 2018. – Т. 18, № 1. – С. 123–126.
11. Ebadi M. Socio-economic design of control charts for monitoring service processes: a case study of a restaurant system / M. Ebadi, A. Ahmadi-Javid // Journal Quality Technology & Quantitative Management. – 2018. DOI: https://doi.org/10.1080/16843703.2018.1519880/.
12. Liu Zhongyia. Police staffing and workload assignment in law enforcement using multi-server queueing models / Zhongyia Liu, Jingchenb Liu, Xinb Zhai et al. // European Journal of Operational Research. – 2019. – Vol. 276, Issue 2. – P. 614–625. DOI: https://doi.org/10.1016/j.ejor.2019.01.004
13. Albey E. Multi-dimensional clearing functions for aggregate capacity modeling in multi-stage production systems / E. Albey, U. Bilge, R. Uzsoy // International Journal of Production Research. – 2017. – Vol. 55, Issue 14. – P. 4164–4179. DOI: https://doi.org/10.1080/00207543.2016.1257169/.

DOI 10.15588/1607-3274-2020-4-4

14. Королькова Л. И. Оптимизация процессов предприятия на основе новой методики расчета характеристик многофазной системы массового обслуживания с непрерывной загрузкой без промежуточных накопителей [Текст] / Л. И. Королькова, П. П. Переверзев // Современные проблемы науки и образования. – 2012. – № 3.
15. Papadopoulos H. T. Queueing theory in manufacturing systems analysis and design: A classification of models for production and transfer lines / H. T. Papadopoulos, C. Heavey // European Journal of Operational Research. – 1996. – Vol. 92, Issue 1. – P. 1–27. DOI: https://doi.org/10.1016/0377-2217(95)00378-9
16. Zavanella L. Energy demand in production systems: A Queuing Theory perspective. / L. Zavanella, S. Zanoni, I. Ferretti, et al. // International Journal of Production Economics. – 2015. – Vol. 170, Part B – P. 393–400. DOI: https://doi.org/10.1016/j.ijpe.2015.06.019/.
17. Истомина А. А. Оптимальное управление товарными запасами на основе теории массового обслуживания / А. А. Истомина, В. Я. Баденков, А. Л. Истомин // Вестник ангарского государственного технического университета. – 2016. – № 10. – С. 148–152.
18. Плоткин Б. К. Экономико-математические методы и модели в коммерческой деятельности и логистике : учебник / Б. К. Плоткин, К. Л. А. Делюкин. – СПб. : Изд-во, 2015. – 345 с.
19. Попов А. В. Вероятностное моделирования логистической системы грузоперевозок / А. В. Попов, Е. Р. Обрезанова, Е. Ю. Синебрюхова // Радіоелектронні і комп'ютерні системи. – 2012. – № 1 (53). – С. 144–151.
20. Balsamo S. A review on queueing network models with finite capacity queues for software architecture performance prediction. / S. Balsamo, De Nitto V Personè, P. Inverardi // Performance Evaluation. – 2003. – Vol. 51, Issue. 2. – P. 269–288. DOI: https://doi.org/10.1016/S0166-5316(02)00099-8/.
21. Клейнрок Л. Вычислительные системы с очередями / Л. Клейнрок. – М. : Мир, 1979. – 600 с.
22. Афанасьева Л. Г. Математические модели транспортных систем основанные на теории очередей / Л. Г. Афанасьева, Е. В. Булинская // Труды МФТИ. – 2010. – Т. 2, № 4. – С. 6–10.
23. Assad A. A. Models for rail transportation / A. A. Assad // Transportation Research Part A: General. – 1980. – Vol. 14, Issue 3. – P. 205–220. DOI: https://doi.org/10.1016/0191-2607(80)90017-5/.
24. Казаков Л. А. Моделирование транспортно-пересадочных узлов на основе систем массового обслуживания – многофазных и с BMAP-потоком / А. Л. Казаков, А. А. Лемперт, М. Л. Жарков // Вестник Уральского государственного университета путей сообщения. – 2016. – № 4 (14). – С. 4–14. DOI: https://doi.org/10.20291/2079-0392-2016-4-4-14/.
25. Рачинская М. А. Построение и исследование вероятностной модели циклического управления потоками малой интенсивности. / М. А. Рачинская, М. А. Федоткин // Вестник Нижегородского университета им. Н. И. Лобачевского. – 2014. – № 4 (1). – С. 370–376.
26. Многофазная модель массового обслуживания системы распределенной обработки данных. / В. В. Грачев, А. Н. Моисеев, А. А. Назаров и др. // Журнал «Доклады ТУСУР». – 2012. – № 2, Часть 2 (26). – С. 248–251.
27. Mandelbaum A. State-dependent queues: approximations and applications / A. Mandelbaum, G. Pats // Stochastic Networks, IMA Volumes in Mathematics, Springer. – 1995. – P. 239–282.
28. Mandelbaum A. The impact of customers patience on delay and abandonment: some empirically driven experiments with the M/M/n + G queue / A. Mandelbaum, S. Zeltyn // Operations Research. – 2004. – Vol. 26. –P. 377–411.
29. Панкратова Е. В. Исследование математических моделей неоднородных бесконечнолинейных СМО / Е. В. Панкратова // ТГУ. – 2016. – С. 1–19.
30. Puhalskii A. A. On many-server queues in heavy traffic / A. A. Puhalskii, J. E. Reed // Annals of Applied Probability. – 2008. – Vol. 20. – P. 129–195.
31. Reed J. E. The G/GI/N queue in the Halfin-Whitt regime I: infinite-server queue system equations / J. E. Reed // The Stern School, NYU. – 2007. – P. 1–59. DOI: https://doi.org/10.1214/09-AAP609/.
32. D'Auria B. Stochastic decomposition of the M/G/∞ queue in a random environment. / B. D'Auria // Operations Research Letters. – 2007. – Vol. 35. – P. 805–812.
33. Saaty T. L. Elements of queuing theory: with applications / T. L. Saaty. – New York : Dover Pubns, 1983. – 423 p.
34. Климов Г. П. Теория массового обслуживания / Г. П. Климов. – М. : МГУ, 2011. – 307 р.
35. Цициашвили Г. Ш. Инвариантные свойства систем массового обслуживания с несколькими потоками / Г. Ш. Цициашвили // Дальневосточный математический журнал. – 2018. – Т. 18:2. – С. 267–270.
36. Бочаров П. П. Теория массового обслуживания. / П. П. Бочаров, А. В. Печинкин – М. : Изд-во РУДН, 1995. – 520 с.
37. Моисеев А. Н. Бесконечно линейные системы и сети массового обслуживания / А. Н. Моисеев, А. А. Назаров. – Томск : Изд-во НТЛ, 2015. – 240 с.
38. Моисеева С. П. Разработка методов исследования математических моделей немарковских систем обслуживания с неограниченным числом приборов и непуассоновскими входящими потоками: дис. доктора физ.-мат. наук / С. П. Моисеева. – Томск : ТГИ, 2014. – 260 с.
39. Doorn E. A. Note on the GI/GI/∞ system with identical service and interarrival-time distributions. / E. A. Doorn, A. A. Jagers // Journal of queueing systems. – 2004. – Vol. 47. – P. 45–52.
40. Матвеев В. Ф. Системы массового обслуживания. / В. Ф. Матвеев, В. Г. Ушаков – М : Изд-во МГУ, 1984. – 242 с.
41. Шахбазов А. А. Об одной задаче обслуживания неординарного потока требований. / А. А. Шахбазов // Докл. АН СССР. – 1962. – Т. 145:2. – С. 289–292.
42. Jung-Shyr Wu, Jyh-Yeong Wang Refining the diffusion approximation for M/G/m queuing systems with group arrivals. / Wu Jung-Shyr, Wang Jyh-Yeong // International Journal of Systems Science. – Vol. 23. – 1992. – Issue 1. – P. 127–133. DOI: https://doi.org/10.1080/00207729208949194/.
43. Куцелай Н. О. Обслуживание неординарного потока требований. / Н. О. Куцелай, С. В. Сафонов // Молодой ученый. – 2018. – № 23. – С. 1–2.
44. Богоявленская О. Ю. Стационарное распределение длины очереди в системе с неординарным потоком и дисциплиной разделения процессора / О. Ю. Богоявленская // Труды Петрозаводского государственного университета, серия «Математика». – 1996. – Вып. 3. – С. 3–10.
45. Печинкин А. В. Инверсионный порядок обслуживания с вероятностным приоритетом в системе обслуживания с неординарным потоком / А. В. Печинкин // Математические исследования. Сер. Вероятность и приложения. – 1989. – Т. 109. – С. 83–94.
46. Ramaswami V. A duality theorem for the matrix paradigms in queueing theory / V. Ramaswami // Communications in Statistics. Stochastic Models, 1990. – P. 151–161. DOI: https://doi :10.1080/15326349908807141/

DOI 10.15588/1607-3274-2020-4-4