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CUBATURE FORMULA FOR APPROXIMATE CALCULATION INTEGRAL OF HIGHLY OSCILLATING FUNCTION OF TREE VARIABLES (IRREGULAR CASE)

Nechuiviter O. P. – Dr. Sc., Associate Professor, Head of the Department of Information Computer Technologies and Mathematics, Ukrainian Engineering and Pedagogical Academy, Kharkiv, Ukraine.

ABSTRACT

Context. The integrals of highly oscillating functions of many variables are one of the central concepts of digital signal and image processing. The object of research is a digital processing of signals and images using new information operators.

Objective. The work aims to construct a cubature formula for the approximate calculation of the triple integral of a rapidly oscillating function of a general form.

Method. Modern methods of digital signal processing are characterized by new approaches to obtaining, processing and analyzing information. There is a need to build mathematical models in which information can be given not only by the values of the function at points, but also as a set of traces of the function on the planes and as a set of traces of the function on the lines. There are algorithms which are optimal by accuracy for calculating the integrals of highly oscillating functions of many variables (regular case), which involve different types of information in their construction. As a solution of a broader problem for the irregular case, the work presents the cubature formula for the approximate calculation of the triple integral of the highly oscillating function in a general case. The presented algorithm for approximate calculation of the integral is based on the application of operators that restore the function of three variables using a set of traces of functions on the mutually perpendicular planes. Operators use piece-wise splines as auxiliary functions. The cubature formula correlates with a formula of the Filon type. An error estimation of the approximation of the integral from the highly oscillating function by the cubature formula on the class of differential functions is obtained.

Results. The cubature formula of the approximate calculation of the triple integral from the highly oscillating function of a general form is researched.

Conclusions. The experiments confirm the obtained theoretical results on the error estimation of the approximation triple integral from the highly oscillating function in a general form by the cubature formula. The prospect of further research is to obtain an estimation of the approximation error on wider classes of functions and to prove that the proposed cubature formula is optimal by the order of accuracy.

KEYWORDS: digital signal and image processing, cubature formula, numerical integration of highly oscillating functions of many variables.

NOMENCLATURE

$H^{3,1}\left(M,\widetilde{M}\right)$ is a class of functions, which are defined in the domain and have limited derivatives;

$I^3(\omega)$ is a three-dimensional integral from highly oscillating functions in a general form;

$\Phi^3(\omega)$ is a cubature formula for calculating three-dimensional integral;

$f(x_k, y, z)$ is a trace of function on the plane x_k;

$f(x, y_j, z)$ is a trace of function on the plane y_j;

$f(x, y, z_s)$ is a trace of function on the plane z_s;

$Jf(x,y,z)$ is an operator that restores the function $f(x,y,z)$ on the traces on planes;

$Og(x,y,z)$ is an operator that restores the function $g(x,y,z)$ on the traces on planes;

$\rho\left(I^3(\omega),\Phi^3(\omega)\right)$ is an error of approximation of the integral by the cubature formula.

INTRODUCTION

Currently a rapid development of information technology is contributing to the development of mathematical modeling of phenomena and processes in such scientific areas as astronomy, radiology, computed tomography, holography and others. Methods of digital signal and image processing are widely used in solving such complex problems in these fields of science. Modern methods of digital signal and image processing are based, among other things, on new approaches to obtaining, processing and analyzing information. Today, in mathematical models, information can be given not only by the values of a function in nodes, but also as a set of traces of a function on planes or as a set of traces of a function on lines. The theory of computing integrals from highly oscillating functions of many variables (regular case) is an example when cubature formulas are chosen depending on the type of input information.

The integrals of highly oscillating functions of many variables are one of the central concepts of digital signal and image processing. Therefore, it is important to continue research in this direction, in particular, to develop cubature formulas for the approximate calculation of integrals from highly oscillating functions of many variables in a general (irregular) case.

The object of study is a digital processing of signals and images using new information operators [1].

New information operators allow restoring the functions of many variables with high accuracy. Traces of the

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function on the planes or traces of the function on the lines are used by new information operators.

The subject of study is an approximate calculation of the integrals from the rapidly oscillating functions of three variables in a general case.

The aim of the work is to construct the cubature formula for the approximate calculation of the integral of the rapidly oscillating function in a general form in the case when the sets of traces of the function on the planes will be used as information about the functions.

1 PROBLEM STATEMENT

It is necessary to construct and investigate the cubature formula of the approximate calculation of the integral of highly oscillating function in a general case

$$I^3(\omega)=\int\limits_0^1\int\limits_0^1\int\limits_0^1 f(x,y,z)\,e^{i\omega g(x,y,z)}\,dxdydz, \qquad (1)$$

when the following information is given:

$$f(x_k,y,z),\ 0\le y\le 1,\ \ 0\le z\le 1,\ x_k=k\Delta_1-\Delta_1/2,$$
$$f(x,y_j,z),\ 0\le x\le 1,\ \ 0\le z\le 1,\ y_j=j\Delta_1-\Delta_1/2,$$
$$f(x,y,z_s),\ 0\le x\le 1,\ \ 0\le y\le 1,\ z_s=s\Delta_1-\Delta_1/2,$$
$$g(\tilde{x}_p,y,z),\ 0\le y\le 1,\ \ 0\le z\le 1,\ \tilde{x}_p=p\Delta_2-\Delta_2/2,$$
$$g(x,\tilde{y}_q,z),\ 0\le x\le 1,\ \ 0\le z\le 1,\ \tilde{y}_q=q\Delta_2-\Delta_2/2,$$
$$g(x,y,\tilde{z}_r),\ 0\le x\le 1,\ \ 0\le y\le 1,\ \tilde{z}_r=r\Delta_2-\Delta_2/2,$$
$$k,j,s=\overline{1,\ell_1},\ \Delta_1=1/\ell_1,\ \ p,q,r=\overline{1,\ell_2},\ \Delta_2=1/\ell_2.$$

2 REVIEW OF THE LITERATURE

New information operators have found their effective application in various fields of science [1, 2]. In digital signal and image processing, new information operators have become an effective tool for calculating the integrals of rapidly oscillating functions [3–7]. Cubature formulas for approximate calculating of integrals of highly oscillating functions of many variables are built using various information about the function. The developed algorithms belong to Filon type methods [8, 9]. An overview of Filon type methods in the one-dimensional case can be found in [10–12], and their multidimensional analogue in [13].

Calculation of double integrals from highly oscillating functions in a regular case on the example of Fourier coefficients is considered in [3, 4]. In these works cubature formulas are built in two cases. In the first case, the information about the function was given by traces on the lines, in the second it was given by the values of the function in nodes. In the first case, it is proved that the cubature formulas are optimal by the order of accuracy on the class of differentiable functions. An algorithm for building cubature formulas with the optimal number of traces also was considered. In the second case where the information about the function was given in nodes cubature formulas proved to be effective in terms of the use of input information and the time spent on calculating the integrals.

In [7, 14, 15] the calculation of integrals from highly oscillating functions (regular case) is considered on the example of Fourier coefficients. In these works algorithms for the approximate calculation of integrals are presented in three cases. In the first case, information about function is given by set of traces of a function on planes, in the second case information about function is represented by set of traces on lines, and in the third case we deal with values of a function at nodes. By analogy with the two-dimensional case, it is proved that the cubature formulas for approximate calculating of integrals of highly oscillating functions of three variables are optimal by the order of accuracy on the class of differentiable functions. In addition, the algorithm for building cubature formulas with the optimal number of planes was considered in [15]. In the case where the information about the function was given in nodes, the cubature formulas proved to be effective in terms of the use of input information and the time spent on calculating the integral.

Frequently in mathematical modeling of technical processes there is a need to approximate the integrals of higly oscillating functions in a general form. Methods and algorithms for calculating integrals of highly oscillating functions of one variable in irregular case can be found in [16–21]. In [22, 23] methods and examples of approximate calculation of double integrals from fast-oscillating functions of general form are presented.

In the case when the information about the functions $f(x,y)$ and $g(x,y)$ was given by the sets of corresponding traces on the lines, an algorithm for the approximate calculation of the double integral from highly oscillating functions of a general form was presented in [24, 25]. This paper aims to present an algorithm for approximate calculation of the integral (1) in the case when the information about the functions $f(x,y,z)$ and $g(x,y,z)$ will be given by the corresponding sets of traces of functions on the planes. This problem has not been solved yet.

3 MATERIALS AND METHODS

This paper considers $H^{3,1}\left(M,\widetilde{M}\right)$ as a class of functions, which are defined in the domain $G=[0,1]^3$ and

$$\left|f^{(1,0,0)}(x,y,z)\right|\le M,\left|f^{(0,1,0)}(x,y,z)\right|\le M,$$
$$\left|f^{(0,0,1)}(x,y,z)\right|\le M,$$
$$\left|f(x,y,z)\right|\le \widetilde{M},\quad \left|f^{(1,1,1)}(x,y,z)\right|\le \widetilde{M}.$$

Definition 1. Under the traces of function $f(x,y,z)$ on the planes

$$x_k=k\Delta_1-\Delta_1/2,\ \ y_j=j\Delta_1-\Delta_1/2,\ \ z_s=s\Delta_1-\Delta_1/2,$$
$$k,j,s=\overline{1,\ell_1},\ \Delta_1=1/\ell_1$$

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we understand a function of two variables

$$f(x_k, y, z),\ 0 \le y \le 1,\quad 0 \le z \le 1,$$
$$f(x, y_j, z),\ 0 \le x \le 1,\quad 0 \le z \le 1,$$
$$f(x, y, z_s),\ 0 \le x \le 1,\quad 0 \le y \le 1.$$

Definition 2. Under the traces of function $g(x, y, z)$ on the planes

$$\tilde{x}_p = p\Delta_2 - \Delta_2 / 2,\ \tilde{y}_q = q\Delta_2 - \Delta_2 / 2,$$
$$\tilde{z}_r = r\Delta_2 - \Delta_2 / 2,\ p,q,r = \overline{1,\ell_2},\ \Delta_2 = 1/\ell_2$$

we understand a function of two variables

$$g(\tilde{x}_p, y, z),\ 0 \le y \le 1,\quad 0 \le z \le 1,$$
$$g(x, \tilde{y}_q, z),\ 0 \le x \le 1,\quad 0 \le z \le 1,$$
$$g(x, y, \tilde{z}_r),\ 0 \le x \le 1,\quad 0 \le y \le 1.$$

A tree-dimensional integral from highly oscillating functions of a general view is defined as (1) for $f(x, y, z),\ g(x, y, z) \in H^{3,1}\left(M, \widetilde{M}\right)$.

Let

$$X1_k = \left[x_{k-1/2}, x_{k+1/2}\right],\ Y1_j = \left[y_{j-1/2}, y_{j+1/2}\right],$$
$$Z1_s = \left[z_{s-1/2}, z_{s+1/2}\right],$$
$$h1_{1k}(x) = \begin{cases} 1, x \in X1_k, \\ 0, x \notin X1_k, \end{cases}\quad h1_{2j}(y) = \begin{cases} 1, y \in Y1_j, \\ 0, y \notin Y1_j, \end{cases}$$
$$h1_{3s}(z) = \begin{cases} 1, z \in Z1_s, \\ 0, z \notin Z1_s, \end{cases}$$
$$x_k = k\Delta_1 - \Delta_1 / 2,\qquad y_j = j\Delta_1 - \Delta_1 / 2,$$
$$z_s = s\Delta_1 - \Delta_1 / 2,\qquad k, j, s = \overline{1,\ell_1},\ \Delta_1 = 1/\ell_1,$$
$$X2_p = \left[\tilde{x}_{p-1/2}, \tilde{x}_{p+1/2}\right],\ Y2_q = \left[\tilde{y}_{q-1/2}, \tilde{y}_{q+1/2}\right],$$
$$Z2_r = \left[\tilde{z}_{r-1/2}, \tilde{z}_{r+1/2}\right].$$
$$h2_{1p}(x) = \begin{cases} 1, x \in X2_p, \\ 0, x \notin X2_p, \end{cases}\quad h2_{2q}(y) = \begin{cases} 1, y \in Y2_q, \\ 0, y \notin Y2_q, \end{cases}$$
$$h2_{3r}(z) = \begin{cases} 1, z \in Z2_r, \\ 0, z \notin Z2_r, \end{cases}$$
$$\tilde{x}_p = p\Delta_2 - \Delta_2 / 2,\qquad \tilde{y}_q = q\Delta_2 - \Delta_2 / 2,$$
$$\tilde{z}_r = r\Delta_2 - \Delta_2 / 2,\qquad p,q,r = \overline{1,\ell_2},\ \Delta_2 = 1/\ell_2.$$

Let define operator

$$Jf(x, y, z) = J_1 f(x, y, z) + J_2 f(x, y, z) + J_3 f(x, y, z) -$$
$$- J_1 J_2 f(x, y, z) - J_2 J_3 f(x, y, z) - J_1 J_3 f(x, y, z) +$$
$$+ J_1 J_2 J_3 f(x, y, z),$$

where

$$J_1 f(x, y, z) = \sum_{k=1}^{\ell_1} f(x_k, y, z) h1_{1k}(x),$$
$$J_2 f(x, y, z) = \sum_{j=1}^{\ell_1} f(x, y_j, z) h1_{2j}(y),$$
$$J_3 f(x, y, z) = \sum_{s=1}^{\ell_1} f(x, y, z_s) h1_{3s}(z).$$

The following properties are performed for the operator
$Jf(x, y, z)$:

$$Jf(x_k, y, z) = f(x_k, y, z),\quad k = \overline{1,\ell_1},$$
$$Jf(x, y_j, z) = f(x, y_j, z),\quad j = \overline{1,\ell_1},$$
$$Jf(x, y, z_s) = f(x, y, z_s),\quad s = \overline{1,\ell_1}.$$

Let define operator

$$Og(x, y, z) = O_1 g(x, y, z) + O_2 g(x, y, z) + O_3 g(x, y, z) -$$
$$- O_1 O_2 g(x, y, z) - O_2 O_3 g(x, y, z) - O_1 O_3 g(x, y, z) +$$
$$+ O_1 O_2 O_3 g(x, y, z),$$

where

$$O_1 g(x, y, z) = \sum_{p=1}^{\ell_2} g(\tilde{x}_p, y, z) h2_{1p}(x),$$
$$O_2 g(x, y, z) = \sum_{q=1}^{\ell_2} g(x, \tilde{y}_q, z) h2_{2q}(y),$$
$$O_3 g(x, y, z) = \sum_{r=1}^{\ell_2} f(x, y, \tilde{z}_r) h2_{3r}(z).$$

The following properties are performed for the operator $Of(x, y, z)$:

$$Og(\tilde{x}_p, y, z) = g(\tilde{x}_p, y, z),\quad p = \overline{1,\ell_2},$$
$$Og(x, \tilde{y}_q, z) = g(x, \tilde{y}_q, z),\quad q = \overline{1,\ell_2},$$
$$Og(x, y, \tilde{z}_r) = g(x, y, \tilde{z}_r),\quad r = \overline{1,\ell_2}.$$

The following cubature formula

$$\Phi^3(\omega) = \int_0^1 \int_0^1 \int_0^1 Jf(x, y, z) e^{i\omega Og(x,y,z)} dxdydz$$

is proposed for numerical calculation of (1).

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Theorem. Suppose that $f(x,y,z)\in H^{3,1}\left(M,\widetilde{M}\right)$, $g(x,y,z)\in H^{3,1}\left(M,\widetilde{M}\right)$. Let functions $f(x,y,z)$, $g(x,y,z)$ be defined by $N=3\ell_1+3\ell_2$ traces $f(x_k,y,z)$, $f(x,y_j,z)$, $f(x,y,z_s)$, $k,j,s=\overline{1,\ell_1}$ and $g(\tilde{x}_p,y,z)$, $g(x,\tilde{y}_q,z)$, $g(x,y,\tilde{z}_r)$, $p,q,r=\overline{1,\ell_2}$ on the systems of perpendicular planes in domain $G=[0,1]^3$. It is true that

$$\rho\left(I^3(\omega),\Phi^3(\omega)\right)\le\frac{\widetilde{M}}{64}\frac{1}{\ell_1^3}+\widetilde{M}\min\left(2;\frac{\widetilde{M}\omega}{64}\frac{1}{\ell_2^3}\right).$$

Proof. It is important to note that

$$\int_{x_k}^{x}\int_{y_j}^{y}\int_{z_s}^{z}f^{(1,1,1)}(\xi,\eta,\zeta)\,d\xi d\eta d\varsigma=$$
$$=f(x,y,z)-f(x_k,y,z)-f(x,y_j,z)-$$
$$-f(x,y,z_s)+f(x_k,y_j,z)+$$
$$+f(x_k,y,z_s)+f(x,y_j,z_s)-f(x_k,y_j,z_s).$$

Let us show the validity of equality by direct verification:

$$\int_{x_k}^{x}\int_{y_j}^{y}\int_{z_s}^{z}f^{(1,1,1)}(\xi,\eta,\zeta)\,d\xi d\eta d\varsigma=$$
$$=\int_{x_k}^{x}\int_{y_j}^{y}f^{(1,1,0)}(\xi,\eta,z)\Big|_{z_s}^{z}d\xi d\eta=$$
$$=\int_{x_k}^{x}\int_{y_j}^{y}\left(f^{(1,1,0)}(\xi,\eta,z)-f^{(1,1,0)}(\xi,\eta,z_s)\right)d\xi d\eta=$$
$$=\int_{x_k}^{x}\left(f^{(1,0,0)}(\xi,\eta,z)-f^{(1,0,0)}(\xi,\eta,z_s)\right)\Big|_{y_j}^{y}d\xi=$$
$$=\int_{x_k}^{x}\left(f^{(1,0,0)}(\xi,y,z)-f^{(1,0,0)}(\xi,y,z_s)\right)d\xi-$$
$$-\int_{x_k}^{x}\left(f^{(1,0,0)}(\xi,y_j,z)-f^{(1,0,0)}(\xi,y_j,z_s)\right)d\xi=$$
$$=f(x,y,z)-f(x_k,y,z)-f(x,y_j,z)-f(x,y,z_s)+$$
$$f(x_k,y_j,z)+f(x_k,y,z_s)+f(x,y_j,z_s)-f(x_k,y_j,z_s).$$

The integral $I^3(\omega)$ could be written in another form

$$I^3(\omega)=\int_0^1\int_0^1\int_0^1 f(x,y,z)\,e^{i\omega g(x,y,z)}\,dxdydz=$$
$$=\int_0^1\int_0^1\int_0^1 Jf(x,y,z)\,e^{i\omega Og(x,y,z)}\,dxdydz+$$
$$+\int_0^1\int_0^1\int_0^1\left[f(x,y,z)-Jf(x,y,z)\right]e^{i\omega Og(x,y,z)}\,dxdydz+$$
$$+\int_0^1\int_0^1 f(x,y,z)\left[e^{i\omega g(x,y,z)}-e^{i\omega Og(x,y,z)}\right]dxdydz.$$

Hence, it is sufficient to show that

$$\rho\left(I^3(\omega),\Phi^3(\omega)\right)=$$
$$=\left|\int_0^1\int_0^1\int_0^1 f(x,y,z)\,e^{i\omega g(x,y,z)}\,dxdydz-\right.$$
$$\left.-\int_0^1\int_0^1\int_0^1 Jf(x,y,z)\,e^{i\omega Og(x,y,z)}\,dxdydz\right|\le$$
$$\le\int_0^1\int_0^1\int_0^1\left|f(x,y,z)-Jf(x,y,z)\right|dxdydz+$$
$$+\int_0^1\int_0^1\int_0^1\left|f(x,y,z)\right|\left|e^{i\omega g(x,y,z)}-e^{i\omega Og(x,y,z)}\right|dxdydz.$$

We use the fact that

$$e^{i\omega g(x,y,z)}-e^{i\omega Og(x,y,z)}=$$
$$=\cos(\omega g(x,y,z))+i\sin(\omega g(x,y,z))-$$
$$-\cos(\omega Og(x,y,z))-i\sin(\omega Og(x,y,z))=$$
$$=-2\sin\frac{\omega g(x,y,z)+\omega O g(x,y,z)}{2}\sin\frac{\omega g(x,y,z)-\omega O g(x,y,z)}{2}+$$
$$+2i\sin\frac{\omega g(x,y,z)-O\omega g(x,y,z)}{2}\cos\frac{\omega g(x,y,z)+\omega Og(x,y,z)}{2}=$$
$$=2i\sin\frac{\omega g(x,y,z)-\omega O g(x,y,z)}{2}\times$$
$$\times\left[i\sin\frac{\omega g(x,y,z)+\omega Og(x,y,z)}{2}+\cos\frac{\omega g(x,y,z)+\omega Og(x,y,z)}{2}\right]=$$
$$=2i\sin\frac{\omega g(x,y,z)-\omega Og(x,y,z)}{2}e^{i\frac{\omega}{2}\left(g(x,y,z)+Og(x,y,z)\right)}.$$

Thus,

$$\rho\left(I^3(\omega),\Phi^3(\omega)\right)\le\int_0^1\int_0^1\int_0^1\left|f(x,y,z)-Jf(x,y,z)\right|dxdydz+$$
$$+\widetilde{M}\int_0^1\int_0^1\int_0^1\left|2i\sin\frac{\omega g(x,y,z)-\omega O g(x,y,z)}{2}e^{i\frac{\omega}{2}\left(g(x,y,z)+Og(x,y,z)\right)}\right|dxdydz\le$$

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$$\leq \sum_{k=1}^{\ell_1}\sum_{j=1}^{\ell_1}\sum_{s=1}^{\ell_1}\int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}\int_{z_{s-\frac{1}{2}}}^{z_{s+\frac{1}{2}}}\left|\int_{x_k}^{x}\int_{y_j}^{y}\int_{z_s}^{z} f^{(1,1,1)}(\xi,\eta,\varsigma)\,d\xi d\eta d\varsigma\right| dxdydz +$$

$$+2\widetilde{M}\sum_{p=1}^{\ell_2}\sum_{q=1}^{\ell_2}\sum_{r=1}^{\ell_2}\int_{\tilde{x}_{p-\frac{1}{2}}}^{\tilde{x}_{p+\frac{1}{2}}}\int_{\tilde{y}_{q-\frac{1}{2}}}^{\tilde{y}_{q+\frac{1}{2}}}\int_{\tilde{z}_{r-\frac{1}{2}}}^{\tilde{z}_{r+\frac{1}{2}}}\left|\sin\frac{\omega(g(x,y,z)-Og(x,y,z))}{2}\right| dxdydz \leq$$

$$\leq \widetilde{M}\sum_{k=1}^{\ell_1}\sum_{j=1}^{\ell_1}\sum_{s=1}^{\ell_1}\int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}|x-x_k|dx\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}|y-y_j|dy\int_{z_{s-\frac{1}{2}}}^{z_{s+\frac{1}{2}}}|z-z_s|dz +$$

$$+2\widetilde{M}\sum_{p=1}^{\ell_2}\sum_{q=1}^{\ell_2}\sum_{r=1}^{\ell_2}\int_{\tilde{x}_{p-\frac{1}{2}}}^{\tilde{x}_{p+\frac{1}{2}}}\int_{\tilde{y}_{q-\frac{1}{2}}}^{\tilde{y}_{q+\frac{1}{2}}}\int_{\tilde{z}_{r-\frac{1}{2}}}^{\tilde{z}_{r+\frac{1}{2}}}\min\left(1;\frac{\omega|g(x,y,z)-Og(x,y,z)|}{2}\right) dxdydz \leq$$

$$\leq \widetilde{M}\sum_{k=1}^{\ell_1}\sum_{j=1}^{\ell_1}\sum_{s=1}^{\ell_1}\int_{x_{k-\frac{1}{2}}}^{x_{k+\frac{1}{2}}}|x-x_k|dx\int_{y_{j-\frac{1}{2}}}^{y_{j+\frac{1}{2}}}|y-y_j|dy\int_{z_{s-\frac{1}{2}}}^{z_{s+\frac{1}{2}}}|z-z_j|dz +$$

$$+2\widetilde{M}\sum_{p=1}^{\ell_2}\sum_{q=1}^{\ell_2}\sum_{r=1}^{\ell_2}\int_{\tilde{x}_{p-\frac{1}{2}}}^{\tilde{x}_{p+\frac{1}{2}}}\int_{\tilde{y}_{q-\frac{1}{2}}}^{\tilde{y}_{q+\frac{1}{2}}}\int_{\tilde{z}_{r-\frac{1}{2}}}^{\tilde{z}_{r-\frac{1}{2}}}\min\left(1;\frac{\omega}{2}\left|\int_{\tilde{x}_p}^{x}\int_{\tilde{y}_s}^{y}\int_{\tilde{z}_r}^{z} g^{(1,1,1)}(\xi,\eta,\varsigma)d\xi d\eta d\varsigma\right|\right) dxdyd\varsigma \leq$$

$$\leq \widetilde{M}\ell_1^3\frac{\Delta_1^2}{4}\frac{\Delta_1^2}{4}\frac{\Delta_1^2}{4}+2\widetilde{M}\min\left(\sum_{p=1}^{\ell_2}\sum_{q=1}^{\ell_2}\sum_{r=1}^{\ell_2}\int_{\tilde{x}_{p-\frac{1}{2}}}^{\tilde{x}_{p+\frac{1}{2}}}\int_{\tilde{y}_{q-\frac{1}{2}}}^{\tilde{y}_{q+\frac{1}{2}}}\int_{\tilde{z}_{r-\frac{1}{2}}}^{\tilde{z}_{r+\frac{1}{2}}} dxdydz,\right.$$

$$\left.\frac{\widetilde{M}\omega}{2}\sum_{p=1}^{\ell_2}\sum_{q=1}^{\ell_2}\sum_{r=1}^{\ell_2}\int_{\tilde{x}_{p-\frac{1}{2}}}^{\tilde{x}_{p+\frac{1}{2}}}\int_{\tilde{y}_{q-\frac{1}{2}}}^{\tilde{y}_{q+\frac{1}{2}}}\int_{\tilde{z}_{r-\frac{1}{2}}}^{\tilde{z}_{r+\frac{1}{2}}}|x-\tilde{x}_p||y-\tilde{y}_q||z-\tilde{z}_r|\,dxdydz\right)=$$

$$=\frac{\widetilde{M}}{64}\Delta_1^3+2\widetilde{M}\min\left(\ell_2^3\Delta_2^3,\frac{\widetilde{M}\omega}{2}\ell_2^3\frac{\Delta_2^2}{4}\frac{\Delta_2^2}{4}\frac{\Delta_2^2}{4}\right)=$$

$$=\frac{\widetilde{M}}{64}\Delta_1^3+\widetilde{M}\min\left(2;\frac{\widetilde{M}\omega}{64}\Delta_2^3\right)=$$

$$=\frac{\widetilde{M}}{64}\frac{1}{\ell_1^3}+\widetilde{M}\min\left(2;\frac{\widetilde{M}\omega}{64}\frac{1}{\ell_2^3}\right).$$

4 EXPERIMENTS

There are two types of testing algorithms [7]:

– testing of programs in order to identify errors in their design and coding of computational algorithms for solving problems;

– testing of computational algorithms implemented by specific programs, in order to study their functionality in solving problems in this class.

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In the study of cubature formulas for the approximate calculation of integrals from highly oscillating functions of many variables, main attention is dedicated to the second type of testing. The key issues of the second type of testing include: algorithms [7]:

– definition of a set of characteristics of the computational algorithm;

– classification of solved problems and compilation of test sets;

– solving test problems, substantiation of values of characteristics and their estimations;

– verification, interpretation and confirmation of test results;

– automation of the testing procedure.

The purpose of testing is to check and identify on a set of test tasks the functionality of cubature formulas on such characteristics as E (accuracy), T (time to solve the problem on a computer), M (computer memory required).

In this paper, the subject of testing is the cubature formula $\Phi^3(\omega)$ for the approximate calculation of the integral $I^3(\omega)$ in the case when the information about the functions $f(\mathrm{x,y,z})$ and $g(\mathrm{x,y,z})$ is the corresponding traces of functions on mutually perpendicular planes. The purpose of testing is to identify the functionality of the cubature formula on such a characteristic as accuracy.

The test set of tasks includes functions of the following type:

– $f(x,y,z)=const$, $f(x,y,z)=q(x)+h(y)+t(z)$,

– $f(x,y,z)=q(x)h(y)t(z)$,

– $f(x,y,z)=q(x)h(y)t(z)+u(x)v(y)w(z)$,

– $f(x,y,z)=\sin(x+y+z)$,

– $f(x,y,z)=\cos(x+y+z)$,

– $g(x,y,z)=\mathrm{const}$, $g(x,y,z)=\phi(x)+\psi(y)+\eta(z)$,

– $g(x,y,z)=\sin(x+y+z)$,

– $g(x,y,z)=\cos(x+y+z)$.

It is due to the fact that trigometric functions belong to a wider class of functions $H^{3,r}\left(M,\widetilde{M}\right)$, $r\geq 1$, and for functions of the form $\tau(x,y,z)=\xi(x)+\zeta(y)+\theta(z)$ the operator $O\tau(x,y,z)$ exactly restores the function. The latter is easily verified by the following lemma.

Lemma. If $\tau(x,y,z)=\xi(x)+\zeta(y)+\theta(z)$, $(x,y,z)\in G$, $(x_1,y_1,z_1)\in G$, then

$$\tau(x,y,z)-\tau(x_1,y,z)-\tau(x,y_1,z)-\tau(x,y,z_1)+$$
$$+\tau(x_1,y_1,z)+\tau(x_1,y,z_1)+\tau(x,y_1,z_1)-\tau(x_1,y_1,z_1)=0.$$

The proof of the lemma is a direct test:

$$\tau(x,y,z)-\tau(x_1,y,z)-\tau(x,y_1,z)-\tau(x,y,z_1)+$$
$$+\tau(x_1,y_1,z)+\tau(x_1,y,z_1)+\tau(x,y_1,z_1)-\tau(x_1,y_1,z_1)=$$
$$=\xi(x)+\zeta(y)+\theta(z)-\xi(x_1)-\zeta(y)-\theta(z)-\xi(x)-$$

$$-\zeta(y_1)-\theta(z)-\xi(x)-\zeta(y)-\theta(z_1)+$$
$$+\xi(x_1)+\zeta(y_1)+\theta(z)+\xi(x_1)+\zeta(y)+\theta(z_1)+$$
$$+\xi(x)+\zeta(y_1)+\theta(z_1)-\xi(x\ \ )-\zeta(y_1)-\theta(z_1)=0.$$

5 RESULTS

This section provides a fragment of testing the cubature formula $\Phi^3(\omega)$ of the approximate calculation of the integral $I^3(\omega)$. Information about functions $f(x,y,z)$ and $g(x,y,z)$ is given by the corresponding traces of functions on mutually perpendicular planes. The Table 1 shows the results of calculations $I^3(\omega)$ for $f(x,y,z)=\sin(x+y+z)$ and $g(x,y,z)=\cos(x+y+z)$ for different ℓ_1, ℓ_2 and for $\omega=2\pi, \omega=5\pi, \omega=10\pi$. For each case, table 1 shows the value of the obtained approximation error

$$\varepsilon=\left|I^3(\omega)-\Phi^3(\omega)\right|,$$

as well as estimates of the approximation error E obtained in this work. According to Theorem, the following error estimation of approximation of the integral $I^3(\omega)$ by the cubature formula $\Phi^3(\omega)$ is valid:

$$E=\frac{\widetilde{M}}{64}\frac{1}{\ell_1^3}+\widetilde{M}\min\left(2;\frac{\widetilde{M}\omega}{64}\frac{1}{\ell_2^3}\right).$$

If we approximate the integral $I^3(\omega)$ for

$f(x,y,z)=\sin(x+y+z)$ $g(x,y,z)=\cos(x+y+z)$, then

$$E=\frac{1}{64\ell_1^3}+\min\left(2;\frac{\omega}{64\ell_2^3}\right).$$

All calculations were performed in the computer mathematics system Wolfram Mathematica 10.

Table 1 – Calculation $I^3(\omega)$ by cubature formula $\Phi^3(\omega)$

ℓ	ω	$\mathrm{Re}\left(\Phi^3(\omega)\right)$	$\mathrm{Im}\left(\Phi^3(\omega)\right)$	ε	E
5	2π	–0,04483571402443004	–0,005659773384241312	$1.00\cdot10^{-6}$	$9.10\cdot10^{-4}$
10	2π	–0,04483413982444644	–0,005657576013215849	$1.70\cdot10^{-6}$	$1.13\cdot10^{-4}$
5	5π	0,00716043088301338	0,002969393727490588	$2.59\cdot10^{-5}$	$2.08\cdot10^{-3}$
10	5π	0,00715783879935095	0,002951850395219029	$1.16\cdot10^{-5}$	$2.61\cdot10^{-4}$
15	5π	0.00714366140868924	0.002947434378359690	$3.58\cdot10^{-6}$	$7.73\cdot10^{-5}$
20	5π	0,00714168524396518	0,002932224529620142	$1.58\cdot10^{-5}$	$3.26\cdot10^{-5}$
25	5π	0,00715234766233286	0,002945672856298047	$5.13\cdot10^{-6}$	$1.67\cdot10^{-5}$
5	10π	–0,00180433697415137	0,000356265110351913	$7.39\cdot10^{-4}$	$4.05\cdot10^{-3}$
10	10π	–0,00140102828305083	–0,000261065518418426	$3.62\cdot10^{-6}$	$5.06\cdot10^{-4}$
15	10π	–0,00139749596727245	–0,000261837407624295	$2.41\cdot10^{-7}$	$1.05\cdot10^{-4}$
20	10π	–0,00035388799218903	–0,000260805717278864	$1.39\cdot10^{-6}$	$6.33\cdot10^{-5}$
25	10π	–0,00139663624810749	–0,00026136986950217	$8.36\cdot10^{-7}$	$3.24\cdot10^{-5}$
20	20π	–0,000353887992189033	0,00050210512582646	$5.83\cdot10^{-4}$	$1.24\cdot10^{-4}$
25	20π	–0.000246554652733453	–0.00008515953820463	$1.30\cdot10^{-5}$	$6.38\cdot10^{-5}$

6 DISCUSSION

In Table 1, it becomes obvious that with increasing ℓ, i.e. with increasing traces of functions $f(x,y,z)=\sin(x+y+z)$ and $g(x,y,z)=\cos(x+y+z)$ on the planes for each variable, the accuracy of calculations increases. In each case, for different ℓ and different ω, the accuracy of the approximate calculation of the integral $I^3(\omega)$ by the cubature formula is less $\Phi^3(\omega)$ than the theoretical estimate of the approximation error, which was obtained in Theorem. This indicates that the numerical experiment confirms the theoretical results presented in this paper.

In addition, it is important to note that in the paper all theoretical statements were made for functions $f(x,y,z)$

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and $g(x,y,z)$ which belong to the class of functions $H^{3,1}\left(M,\widetilde{M}\right)$. Obviously, functions $\sin(x+y+z)$, $\cos(x+y+z)$ are the functions that belong to a broader class of functions. This suggests that the cubature formula $\Phi^3(\omega)$ has a good approximation accuracy and should be researched on other classes of functions.

It is also worth noting that when conducting numerical experiments, it is important to always test the cubature formulas on the so-called "bad" functions of the class. The "bad" functions of the class include functions that in nodes (lines, planes) are zero, and between nodes (lines, planes) are the furthest from zero [4]. On such functions algorithms, in our case the cubature formula has the greatest error. The building of bad functions in the case when the information about the function is given by the traces of the function on the planes is a cumbersome task and requires additional research. Therefore, in this experiment we will not consider testing the cubature formula $\Phi^3(\omega)$ for a "bad" function of class. This problem will be considered in further works, when the optimality by the order of accuracy of the cubature formula $\Phi^3(\omega)$ will be proved. The study and testing of cubature formulas for the approximate calculation of integrals from highly oscillating functions of three variables (regular case) can be found in more detail in [7].

CONCLUSIONS

The problems of digital signal and image processing with the use of new information operators are considered in the paper.

The scientific novelty is that for the first time the cubature formula of approximate calculation of integrals from highly oscillating functions of three variables (irregular case) has been presented. The main difference of the proposed cubature formula is that it uses traces of functions and planes as information about function. The cubature formula has high approximation accuracy. An estimate of the approximation error on the class of differential functions is obtained.

The practical significance of this work is that the new methods of obtaining input information about the function in the form of traces of the function on the planes opens new ways in the building of mathematical models, in particular in digital signal processing.

The prospects for further research are to obtain an estimate of the approximation error on a wider class of functions and to prove that the proposed cubature formula is optimal by the order of accuracy.

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КУБАТУРНА ФОРМУЛА НАБЛИЖЕНОГО ОБЧИСЛЕННЯ ІНТЕГРАЛА ВІД ШВИДКООСЦИЛЮЮЧОЇ ФУНКЦІЇ ТРЬОХ ЗМІННИХ (ІРРЕГУЛЯРНИЙ ВИПАДОК)

Нечуйвітер О. П. – д-р фіз.-мат. наук, доцент, завідувач кафедри інформаційних комп’ютерних технологій і математики Української інженерно-педагогічної академії, Харків, Україна.

АНОТАЦІЯ

Актуальність. Інтеграли від швидкоосцилюючих функцій багатьох змінних є одним з центральних понять цифрової обробки сигналів та зображень. Об’єктом дослідження є цифрова обробки сигналів та зображень з використанням нових інформаційних операторів. Мета роботи – побудова кубатурної формули наближеного обчислення потрійного інтегралу від швидкоосцилюючої функції загального виду. Інформація про функції задається наборами слідів функцій на площинах.

Метод. Сучасні методи цифрової обробки сигналів характеризуються новими підходами до отримання, обробки та аналізу інформації. Є необхідність будувати математичні моделі, в яких інформація може задаватися не тільки значеннями функції в точках, а і як сукупність слідів функції на площинах, як набір слідів функції на лініях. Існують оптимальні за точністю алгоритми обчислення інтегралів від швидкоосцилюючих функцій багатьох змінних (регулярний випадок), які в своїй побудові передбачають різні типи задання інформації. Як розв’язання більш широкої задачі для нерегулярного випадку в роботі представлено кубатурну формулу наближеного обчислення потрійного інтегралу від швидкоосцилюючої функції у загальному виді. Представлений алгоритм наближеного обчислення інтегралу базується на використанні операторів, які відновлюють функцію трьох змінних з використанням набору слідів функцій на взаємоперпендикулярних площинах. Оператори використовують в якості допоміжних функцій кусково-сталі сплайни. Кубатурна формула відноситься до формул типу Файлона. Отримана оцінка похибки наближення інтегралу від швидкоосцилюючої функції кубатурною формулою на класі диференційовних функцій.

Результати. Досліджена кубатурна формула наближеного обчислення потрійного інтегралу від швидкоосцилюючої функції загального виду.

Висновки. Проведені експерименти підтвердили отримані теоретичні результати щодо оцінки похибки наближення потрійного інтегралу від швидкоосцилюючої функції загального виду кубатурною формулою. Перспективою подальших досліджень є отримання оцінки похибки наближення на більш широких класах функцій. А також довести, що запропонована кубатурна формула є оптимальною за порядком точності.

КЛЮЧОВІ СЛОВА: цифрова обробка сигналів та зображень, кубатурна формула, чисельне інтегрівання швидкоосцилюючих функцій багатьох змінних.

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КУБАТУРНАЯ ФОРМУЛА ПРИБЛИЖЕННОГО ВЫЧИСЛЕНИЯ ИНТЕГРАЛА ОТ БЫСТРООСЦИЛЛИРУЮЩЕЙ ФУНКЦИИ ТРЕХ ПЕРЕМЕННЫХ (ИРРЕГУЛЯРНЫЙ СЛУЧАЙ)

Нечуйвитер О. П. – д-р физ.-мат. наук, доцент, заведующий кафедрой информационых компьютерных технологий и математики Украинской инженерно-педагогической академии, Харьков, Украина.

АННОТАЦИЯ

Актуальность. Интегралы от быстроосциллирующих функций многих переменных относятся к одним из центральных понятий цифровой обработки сигналов и изображений. Объектом исследования является цифровая обработки сигналов и изображений с использованием новых информационных операторов. Цель работы – построение кубатурных формул приближенного вычисления тройного интеграла от быстроосциллирующих функции в общем виде. Информация о функциях задается наборами следов функций на плоскостях.

Метод. Современные методы цифровой обработки сигналов характеризуются новыми подходами к получению, обработке и анализу информации. Есть необходимость строить математические модели, в которых информация может задаваться не только значениями функции в точках, но и как совокупность следов функции на плоскостях, как набор следов функции на линиях. Существуют оптимальные по точности алгоритмы вычисления интегралов от быстроосциллирующих функций многих переменных (регулярный случай), которые в своей построении предусматривают различные типы задания информации. Как решение более широкой задачи для общего случая в работе представлена кубатурная формула приближенного вычисления тройного интеграла от быстроосциллирующих функции в общем виде. Представленный алгоритм приближенного вычисления интеграла базируется на использовании операторов, которые восстанавливают функцию трех переменных с использованием набора следов функций на взаимноперпендикулярных плоскостях. Операторы используют в

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качестве вспомогательных функций кусочно-постоянные сплайны. Кубатурная формула относится к формулам типа Файлона. Полученна оценка погрешности приближения интеграла от быстроосциллирующей функции кубатурной формулой на классе дифференцируемых функций.

Результаты. Исследована кубатурная формула приближенного вычисления тройного интеграла от быстроосциллирующих функций общего вида.

Выводы. Проведенные эксперименты подтвердили полученные теоретические результаты об оценке погрешности приближения тройного интеграла от быстроосциллирующих функций общего вида кубатурной формулой. Перспективой дальнейших исследований является получение оценки погрешности приближения на более широких классах функций. А также доказать, что предложенная кубатурная формула является оптимальной по порядку точности.

КЛЮЧЕВЫЕ СЛОВА: цифровая обработка сигналов и изображений, кубатурная формула, численное интегрирование быстроосциллирующих функций многих переменных.

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