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APPROXIMATE SOLUTIONS FOR THE KOLMOGOROV-WIENER FILTER WEIGHT FUNCTION FOR CONTINUOUS FRACTIONAL GAUSSIAN NOISE

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ABSTRACT

Context. We consider the Kolmogorov-Wiener filter for forecasting of telecommunication traffic in the framework of a continuous fractional Gaussian noise model.

Objective. The aim of the work is to obtain the filter weight function as an approximate solution of the corresponding Wiener-Hopf integral equation. Also the aim of the work is to show the convergence of the proposed method of solution of the corresponding equation.

Method. The Wiener-Hopf integral equation for the filter weight function is a Fredholm integral equation of the first kind. We use the truncated polynomial expansion method in order to obtain an approximate solution of the corresponding equation. A set of Chebyshev polynomials of the first kind is used.

Results. We obtained approximate solutions for the Kolmogorov-Wiener filter weight function for forecasting of continuous fractional Gaussian noise. The solutions are obtained in the approximations of different number of polynomials; the results are obtained up to the nineteen-polynomial approximation. It is shown that the proposed method is convergent for the problem under consideration, i.e. the accuracy of the coincidence of the left-hand and right-hand sides of the integral equation increases with the number of polynomials. Such convergence takes place due to the fact that the correlation function of continuous fractional Gaussian noise, which is the kernel of the corresponding integral equation, is a positively-defined function.

Conclusions. The Kolmogorov-Wiener filter weight function for forecasting of continuous fractional Gaussian noise is obtained as an approximate solution of the corresponding Fredholm integral equation of the first kind. The proposed truncated polynomial expansion method is convergent for the problem under consideration. As is known, one of the simplest telecommunication traffic models is the model of continuous fractional Gaussian noise, so the results of the paper may be useful for telecommunication traffic forecast.

KEYWORDS: Kolmogorov-Wiener filter weight function, continuous fractional Gaussian noise, Chebyshev polynomials of the first kind, telecommunication traffic forecast, method convergence.

NOMENCLATURE

T is the time interval along which the input data are observed;

k is the time interval for which the forecast should be made;

$h(t)$ is the Kolmogorov-Wiener filter weight function;

H is the Hurst exponent;

$S_n(t)$ are the Chebyshev polynomials of the first kind which are orthogonal on the time interval $t \in (0,T)$;

$R(t)$ correlation function of fractional Gaussian noise.

INTRODUCTION

The problem of telecommunication traffic forecast is a topical problem of telecommunications. Traffic in telecommunication systems with data burst transfer is a self-similar process [1]. It should be stressed that self-similar processes take place in a huge variety of different systems (see, for example, [2]), and their forecast is investigated not only for telecommunication traffic, but also for other systems (see, for example, [3, 4]).

One of the simplest models of self-similar telecommunication traffic is the model where the traffic is considered to be a fractional Gaussian noise [1]. In [5] it is stressed that it is reasonable to consider traffic as a continuous random process because of a large amount of data. So, in a simple model the traffic can be treated as a continuous fractional Gaussian noise.

Fractional Gaussian noise is a stationary random process, so the Kolmogorov-Wiener filter may be used in order to investigate the traffic forecast. This paper is devoted to the obtaining of the weight function of the corresponding filter. In this paper we consider only the case where the Hurst exponent $H > 0.5$.

The weight function under consideration obeys a Fredholm integral equation of the first kind [6]. We propose to use the truncated polynomial expansion method in order to obtain an approximate solution of the corresponding integral equation. The correlation function of fractional Gaussian noise, which is the kernel of the considered integral equation, is a positively-defined function [7], so the proposed method should be convergent.

The obtained results may be important for forecasting of telecommunication self-similar traffic.

The object of study is the Kolmogorov-Wiener filter for continuous fractional Gaussian noise.

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The subject of study is the weight function of the corresponding filter.

The aim of the work is to obtain the corresponding weight function as an approximate solution of the Fredholm integral equation of the first kind. Also the aim of the work is to show the convergence of the truncated polynomial expansion method for the problem under consideration.

1 PROBLEM STATEMENT

As is known [6], the Kolmogorov-Wiener weight function obeys the Wiener-Hopf integral equation

$$\int_0^T d\tau h(\tau) R(t-\tau) = R(t+k). \quad (1)$$

The correlation function of continuous fractional Gaussian noise for $H > 0.5$ is as follows [7]

$$R(t) = 2H(2H-1)\sigma^2 |t|^{2H-2}, \quad (2)$$

where σ is the process variance. After substitution of (2) into (1) one can obtain

$$\int_0^T d\tau h(\tau) |t-\tau|^{2H-2} = (t+k)^{2H-2}, \; t \in (0,T), \quad (3)$$

where the fact that $k > 0$, and obviously $t+k > 0$, is used. The problem is to obtain the function $h(\tau)$ as an approximate solution of the integral equation (3).

2 REVIEW OF THE LITERATURE

As is known, traffic in telecommunication systems with data burst transfer is a self-similar process, and one of the simplest self-similar traffic models is the model of fractional Gaussian noise [1]. Fractional Gaussian noise is a stationary random process [7], so its forecast may be obtained on the basis of the Kolmogorov-Wiener filter. According to [5], the traffic is considered as a continuous random process, which is reasonable in case of a large amount of data.

In the case of a continuous process the Kolmogorov-Wiener weight function obeys a Fredholm integral equation of the first kind [6]. An exact analytical solution of such an equation meets difficulties, so we use the truncated polynomial expansion method in order to obtain an approximate solution of the corresponding equation.

In our previous papers [8–10] we investigated the corresponding method for the case of fractal processes with a power-law structure function. In [8] we used polynomials which are orthogonal on the time interval $t \in (0,T)$ without weight, and in [9, 10] we used the Chebyshev polynomials of the first and second kind, respectively. It was shown that the behavior of the method convergence is identical for all the polynomial sets investigated in [8–10], and the method is not necessarily convergent for fractal processes with a power-law structure function. The reason is as follows. The method convergence is guaranteed [11] if the kernel of the corresponding integral equation is a positively-defined function. As can be seen from eq. (1), the kernel of the integral equation under consideration coincides with the correlation function of the random process under consideration. The correlation function of a fractal random process with a power-law structure function is not a positively-defined function, so for that case the method convergence is not guaranteed.

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But, as is known [7], in the case where $H > 0.5$ the correlation function of fractional Gaussian noise, in contrast to the correlation function of a process with a power-law structure function, is a positively-defined function. So the truncated polynomial expansion method should be convergent for the problem under consideration. In this paper a set of Chebyshev polynomials of the first kind is used.

3 MATERIALS AND METHODS

In the framework of the truncated polynomial expansion method [8–10] the function $h(\tau)$, which is the solution of the integral equation (3), is sought as a truncated orthogonal polynomial series

$$h(\tau) = \sum_{m=0}^{l-1} g_m^{[l]} S_m(\tau), \quad (4)$$

where l is the number of polynomials and $g_m^{[l]}$ are coefficients multiplying the polynomials. The function $h(\tau)$ in the form (4) is the solution in the l-polynomial approximation.

The polynomials $S_m(\tau)$ should be orthogonal on $\tau \in (0,T)$. In this paper this set is constructed on the basis of the Chebyshev polynomials of the first kind. As is known [12], the explicit expressions for the Chebyshev polynomials of the first kind are

$$T_m(x) = \sum_{j=0}^{[m/2]} C_m^{2j} (x^2-1)^j x^{m-2j}, \quad (5)$$

where $[y]$ is the integer part of y and

$$C_m^{2j} = \frac{m!}{(2j)!(m-2j)!}. \quad (6)$$

As is shown in [9], the following orthogonality relation is valid:

$$\int_0^T T_n\left(\frac{2y}{T}-1\right) T_m\left(\frac{2y}{T}-1\right) w(y) dy = \frac{T}{2} A_n \delta_{mn}, \quad (7)$$

where

$$w(y)=\left(1-\left(\frac{2y}{T}-1\right)^2\right)^{-1/2},\ A_n=\begin{cases}\pi, n=0;\\ \pi/2, n\neq 0,\end{cases} \tag{8}$$

and

$$\delta_{mn}=\begin{cases}1, m=n;\\ 0, m\neq n,\end{cases} \tag{9}$$

is the Kronecker delta. So, the polynomials

$$S_n(t)=T_n\left(\frac{2t}{T}-1\right) \tag{10}$$

are orthogonal on $t\in(0,T)$ with the weight $w(t)$, and they may be used in the expansion (4).

By substituting (4) into (3) one can obtain

$$\sum_{m=0}^{l-1} g_m^{[l]}\int_0^T d\tau S_m(\tau)\left|t-\tau\right|^{2H-2}=(t+k)^{2H-2}, \tag{11}$$

and by multiplying both sides of (11) by $S_n(t)$ and integrating over t one can obtain the following set of linear algebraic equations

$$\sum_{m=0}^{l-1} g_m^{[l]}G_{mn}=B_n,\ n=\overline{0,l-1}, \tag{12}$$

where G_{mn} are the so-called integral brackets

$$G_{mn}=\int_0^T\int_0^T dt d\tau S_n(t)S_m(\tau)\left|t-\tau\right|^{2H-2} \tag{13}$$

and the coefficients B_n are calculated as

$$B_n=\int_0^T d\tau S_n(t)(t+k)^{2H-2}\ , \tag{14}$$

the functions $S_n(t)$ are taken from (10).

Let us discuss the properties of the integral brackets (13). First of all, if we interchange the integration variables in (13), we obtain

$$\begin{gathered}G_{mn}=\int_0^T\int_0^T dt d\tau S_n(t)S_m(\tau)\left|t-\tau\right|^{2H-2}=\{t\leftrightarrow\tau\}=\\ =\int_0^T\int_0^T d\tau dS_n(\tau)S_m(t)\left|t-\tau\right|^{2H-2}=G_{nm}\ ,\end{gathered} \tag{15}$$

so the integral brackets obey the property

$$G_{mn}=G_{nm}\ . \tag{16}$$

Let us make the following change of the variables:

$$\begin{gathered}G_{mn}=\left\{x=\frac{2\tau}{T}-1, y=\frac{2t}{T}-1\right\}=\\ =\frac{T^2}{4}\int_{-1}^{1}\int_{-1}^{1} dxdy\left(S_n\left(\frac{T(x+1)}{2}\right)\times\right.\\ \left.\times S_m\left(\frac{T(y+1)}{2}\right)\left|\frac{Tx}{2}-\frac{Ty}{2}\right|^{2H-2}\right).\end{gathered} \tag{17}$$

On the basis of (13) and (17) one can see that

$$\begin{gathered}G_{mn}=\frac{T^2}{4}\int_{-1}^{1}\int_{-1}^{1} dxdyT_n(x)T_m(y)\left|\frac{Tx}{2}-\frac{Ty}{2}\right|^{2H-2}=\\ =\{x\to -x, y\to -y\}=\\ =\frac{T^2}{4}\int_{-1}^{1}\int_{-1}^{1} dxdyT_n(-x)T_m(-y)\left|\frac{Tx}{2}-\frac{Ty}{2}\right|^{2H-2}.\end{gathered} \tag{18}$$

As can be seen from (5), the following property is valid:

$$T_n(x)=\begin{cases}T_n(-x), n\,\vdots\,2;\\ -T_n(-x), n\not\vdots\,2,\end{cases} \tag{19}$$

which leads to the fact that

$$\begin{gathered}T_n(x)T_m(y)=-T_n(-x)T_m(-y)\ ,\\ \text{if } m,n \text{ are of different parity.}\end{gathered} \tag{20}$$

So on the basis of (20) and (18) it can be seen that

$$\begin{gathered}G_{mn}=-G_{mn}\Rightarrow G_{mn}=0\ ,\\ \text{if } m,n \text{ are of different parity.}\end{gathered} \tag{21}$$

So the integral brackets obey the properties (16) and (21). This fact significantly reduces the computing time, because by a straightforward calculation one should calculate G_{mn} only for $m\geq n$ where m, n are of the same parity.

The algorithm of the weight function calculation is as follows. First of all, one should calculate the integral brackets G_{mn} and the coefficients B_n (see (13) and (14)). Then the set of linear algebraic equations (12) for the coefficients $g_m^{[l]}$ should be solved. The approximate solution of the integral equation (3) in the l-polynomial approximation is given by expression (4). In this paper the corresponding numerical results are obtained with the help of the Wolfram Mathematica package.

4 EXPERIMENTS

Of course, an interesting question is whether the proposed method is convergent. The kernel of the integral equation (3) is a positively-defined function, so the trun-

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cated polynomial expansion method should be convergent for the problem under consideration.

In order to check the convergence of the method we numerically compare the left-hand and the right-hand sides of the integral equation (3) for different numbers of polynomials. The integral on the left-hand side of eq. (3) is numerically calculated with the help of the Wolfram Mathematica as

$$\int_0^T d\tau h(\tau)|t-\tau|^{2H-2} = \int_0^t d\tau h(\tau)(t-\tau)^{2H-2} + \\ +\int_t^T d\tau h(\tau)(\tau-t)^{2H-2}. \qquad (22)$$

The numerical investigation is made for the set of parameters

$$T = 100, \ k = 3, \ H = 0.8. \qquad (23)$$

In what follows the comparison of the left-hand and the right-hand sides of the integral equation (3) is illustrated by graphs where the dotted line illustrates the graph for the left-hand side and the solid line illustrates the right-hand side.

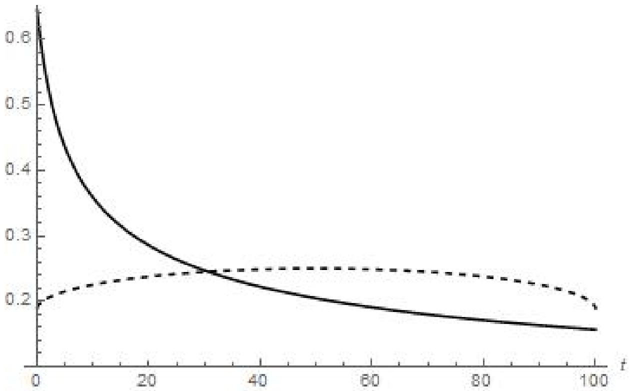

Figure 1 – Comparison of the left-hand and right-hand sides of eq. (3) for parameters (23) for the one-polynomial approximation

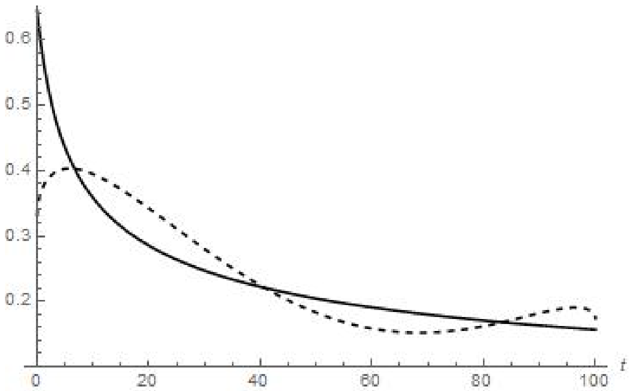

Figure 2 – Comparison of the left-hand and right-hand sides of eq. (3) for parameters (23) for the three-polynomial approximation

As can be seen from Fig. 1 and Fig. 2, the approximations of a small number of polynomials are not accurate.

However, as can be seen from Fig. 1 – Fig. 7, the accuracy of coincidence of the left-hand and the right-hand sides of the integral equation (3) increases with the number of polynomials.

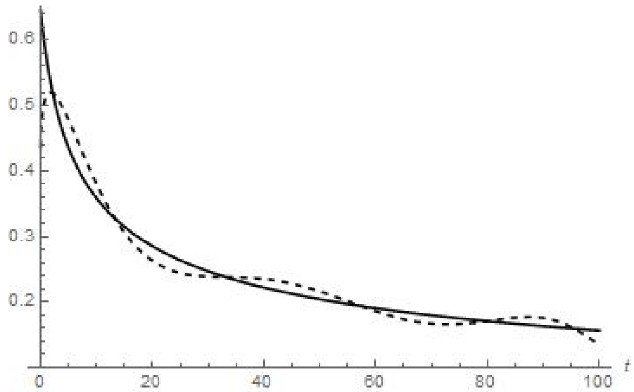

Figure 3 – Comparison of the left-hand and right-hand sides of eq. (3) for parameters (23) for the six-polynomial approximation

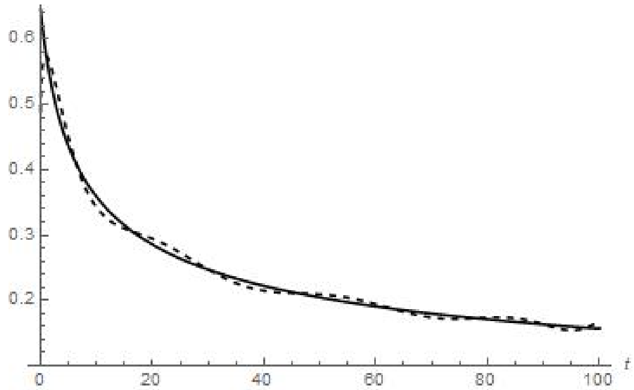

Figure 4 – Comparison of the left-hand and right-hand sides of eq. (3) for parameters (23) for the nine-polynomial approximation

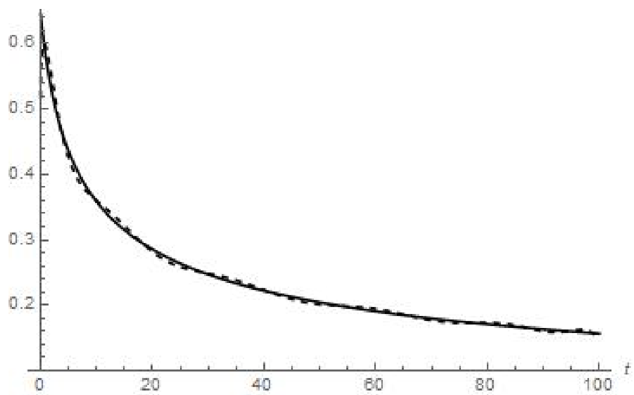

Figure 5 – Comparison of the left-hand and right-hand sides of eq. (3) for parameters (23) for the twelve-polynomial approximation

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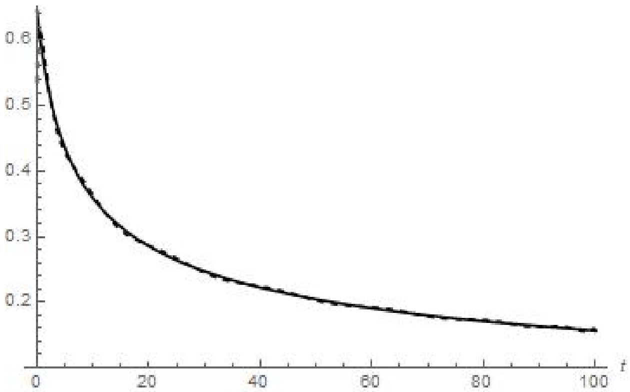

Figure 6 – Comparison of the left-hand and right-hand sides of eq. (3) for parameters (24) for the fifteen-polynomial approximation

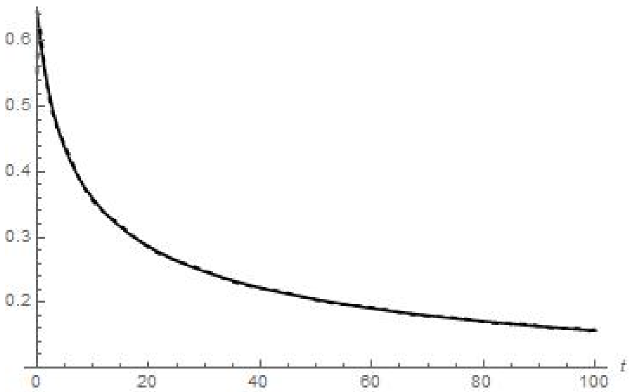

Figure 7 – Comparison of the left-hand and right-hand sides of eq. (3) for parameters (24) for the nineteen-polynomial approximation

So one can conclude that the proposed method of truncated polynomial expansion is convergent for the problem under consideration.

5 RESULTS

The Kolmogorov-Wiener filter weight function for forecasting of fractional Gaussian noise is investigated as an approximate solution of the corresponding integral equation (3). The truncated polynomial expansion method based on the Chebyshev polynomials of the first kind is used.

The results are investigated up to the nineteen-polynomial approximation. The approximations of higher-than-nineteen polynomials are not investigated because the Wolfram Mathematica package is not able to calculate them adequately. It is shown that the method is convergent, i.e. the accuracy of coincidence of the left-hand and the right-hand sides of (3) increases with the number of polynomials. However, it should be stressed that the approximations of small numbers of polynomials are not accurate, and one should use the approximation of a rather large number of polynomials.

6 DISCUSSION

This paper is devoted to the investigation of the Kolmogorov-Wiener filter weight function for forecasting of continuous fractional Gaussian noise. As is known [1], one of the simplest models of telecommunication traffic in systems with data burst transfer is the model of fractional Gaussian noise. So the results of the paper may be useful for telecommunication traffic forecasting.

The integral equation for the corresponding weight function is the Wiener-Hopf integral equation (1) which can be expressed in the form (3). In fact, we deal with the Fredholm integral equation of the first kind. The weight function is obtained with the help of the truncated polynomial expansion method. In this paper the method is based on the Chebyshev polynomials of the first kind which are orthogonal on the time interval $t \in (0, T)$.

The numerical investigation is made for the parameters (23). The approximations up to the 19-polynomial one are investigated. The approximations of small numbers of polynomials are not accurate, but it is shown that the approximation accuracy increases with the number of polynomials, the approximations of rather large numbers of polynomials are rather accurate. So the method convergence is illustrated for the parameters (23). The kernel of the integral equation (3) is a positively-defined function, so, according to [11], in the framework of the problem under consideration the truncated polynomial expansion method should be convergent not only for the parameters (23), but also for other numerical values of the parameters.

CONCLUSIONS

The Kolmogorov-Wiener weight function for forecasting of fractional Gaussian noise is investigated as an approximate solution of the corresponding Wiener-Hopf integral equation. The truncated polynomial expansion method is used, the Chebyshev polynmials of the first kind orthogonal on $t \in (0, T)$ are chosen.

The scientific novelty of the paper is the fact that in contrast to the previously investigated model [8–10], the method convergence is shown for the model of fractional Gaussian noise.

The practical significance is that the obtained results may be applied to the telecommunication traffic forecast in systems with data burst transfer.

Prospects for further research are to investigate an exact analytical solution of the corresponding integral equation.

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НАБЛИЖЕНІ РОЗВ'ЯЗКИ ДЛЯ ВАГОВОЇ ФУНКЦІЇ ФІЛЬТРА КОЛМОГОРОВА-ВІНЕРА ДЛЯ НЕПЕРЕРВНОГО ФРАКТАЛЬНОГО ГАУССОВОГО ШУМУ

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АНОТАЦІЯ

Актуальність. Розглянуто фільтр Колмогорова-Вінера для прогнозування телекомунікаційного трафіку в рамках моделі неперервного фрактального гауссового шуму.

Мета роботи. Метою роботи є отримати вагову функцію фільтра як наближений розв'язок відповідного інтегрального рівняння Вінера-Хопфа. Метою роботи також є показати збіжність запропонованого методу розв'язання даного рівняння.

Метод. Інтегральне рівняння Вінера-Хопфа на вагову функцію фільтра є інтегральним рівнянням Фредгольма першого роду. Ми використовуємо метод обірваного розвинення за ортогональними поліномами з метою отримати наближений розв'язок відповідного рівняння. Використано поліноми Чебишева першого роду.

Результати. Нами отримано наближені розв'язки для вагової функції фільтра Колмогорова-Вінера для прогнозування неперервного фрактального гауссового шуму. Розв'язки отримано у наближеннях різної кількості поліномів, результати отримано до наближення дев'ятнадцяти поліномів включно. Показано, що для задачі, що розглядається, запропонований метод є збіжним, тобто точність співпадіння лівої та правої частин інтегрального рівняння зростає зі зростом кількості поліномів. Така збіжність має місце, бо кореляційна функція фрактального гауссового шуму, яка є ядром відповідного інтегрального рівняння, є позитивно визначеною функцією.

Висновки. Вагова функція фільтра Колмогорова-Вінера для прогнозування неперервного фрактального гауссового шуму отримана як наближений розв'язок відповідного інтегрального рівняння Фредгольма першого роду. Запропонований метод обірваного розвинення за ортогональними поліномами є збіжним для задачі, що розглядається. Як відомо, однією з найпростіших моделей телекомунікаційного трафіку є модель неперервного фрактальногогауссового шуму, тож результати статті можуть бути корисними для прогнозування телекомунікаційного трафіку.

КЛЮЧОВІ СЛОВА: вагова функція фільтра Колмогорова-Вінера, нерерервний фрактальний гаусів шум, поліноми Чебишева першого роду, прогнозування трафіку, збіжність методу.

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ПРИБЛИЖЕННЫЕ РЕШЕНИЯ ДЛЯ ВЕСОВОЙ ФУНКЦИИ ФИЛЬТРА КОЛМОГОРОВА-ВИНЕРА ДЛЯ НЕПРЕРЫВНОГО ФРАКТАЛЬНОГО ГАУССОВА ШУМА

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АННОТАЦИЯ

Актуальность. Рассмотрен фильтр Колмогорова-Винера для прогнозирования телекоммуникационного траффика в рамках модели непрерывного фрактального гауссова шума.

Цель работы. Целью работы является получить весовую функцию фильтра как приближенное решение соответствующего интегрального уравнения Винера-Хопфа. Также целью работы является показать сходимость предложенного метода решения соответствующего уравнения.

Метод. Интегральное уравнение Винера-Хопфа на весовую функцию фильтра является интегральным уравнением Фредгольма первого рода. Мы используем метод оборванного разложения по ортогональным полиномам, чтобы получить приближенное решение соответствующего уравнения. Использованы полиномы Чебышева первого рода.

Результаты. Получены приближенные решения для весовой функции фильтра Колмогорова-Винера для прогнозирования непрерывного фрактального гауссова шума. Решения получены в приближениях различного числа полиномов; результаты получены вплоть до приближения девятнадцати полиномов. Показано, что для рассматриваемой задачи предложенный метод является сходящимся, т.е. точность совпадения левой и правой частей интегрального уравнения растет с увеличением числа полиномов. Такая сходимость имеет место, потому что корреляционная функция фрактального гауссова шума, которая есть ядром соответствующего интегрального уравнения, является положительно определенной функцией.

Выводы. Весовая функция фильтра Колмогорова-Винера для прогнозирования непрерывного фрактального гауссова шума получена как приближенное решение соответствующего интегрального уравнения Фредгольма первого рода. Предложенный метод оборванного разложения по ортогональным полиномам является сходящимся для рассматриваемой задачи. Как известно, одной из простейших моделей телекоммуникационного траффика является модель непрерывного фрактального гауссова шума, так что результаты статьи могут быть полезны для прогнозирования телекоммуникационного траффика.

КЛЮЧЕВЫЕ СЛОВА: весовая функция фильтра Колмогорова-Винера, непрерывный фрактальный гауссов шум, полиномы Чебышева первого рода, прогнозирование траффика, сходимость метода.

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