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UNIVERSAL METHOD FOR COMPUTATIONAL MODELING OF THRESHOLD PHENOMENON IN THE NONSTEADY BIOLOGICAL PROCESSES

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ABSTRACT

Context. In modern conditions occur abrupt changes in ecosystems. The species composition of Caspian Sea is changing rapidly. The dynamics of populations acquires an extreme character with the development of rapid invasions. The mathematical description of scale transformations requires new modeling methods. Complicated population regimes of changes have features of the threshold phenomenon in process of its development.

Objective. We set the goal of computational modeling of practically important scenarios – groups of situations that relate to extreme and transitional dynamics of ecosystems, like outbreaks at the onset of dangerous invasions. We are developing a method that, on the basis of the survival model of generations, will conduct a description of sudden transitions to rapid but limited outbreak of numbers or, on contrary, a collapse of stocks like Atlantic cod in 1992 or Peruan anchovy *Engraulis ringens* in 1985. The purpose of our modeling is to improve the accuracy of forecasts of the population size when experts are estimates a rational strategy for the exploitation of biological resources.

Method. Situations of abrupt but short-term changes in population processes cannot be calculated by traditional mathematical models and expressed in terms of asymptotic dynamics – closed limit trajectory sets. The basis of the idea of the method proposed by us is the formalization of nonlinear efficiency of reproduction, which changes in a threshold manner only in strictly defined environmental conditions. We use continuous-discrete time in the model for early ontognosis of the cod fish and insect pests. The method with triggers allows us to take into account in simulation experiments logic and motivation of making decisions by experts, people who manage the strategy of exploiting biological resources. Models assess variability for development of situations

Results. We have implemented new method of bounded trigger functionals into hybrid system of the equations, that acting in selected specific states of biosystems. Analysis of new model scenarios with modifications of functionals in the basic hybrid system for extreme situations in fish and insect pests is carried out.

Conclusions. We consider the method to be universal, since selection of the functional can be adapted to a wide class of models using differential equations on a fixed interval.

KEYWORDS: mathematical biology, modeling of threshold effects, hybrid models, trigger functional method, population outbreaks of insect, collapse of peruan anchovy.

ABBREVIATIONS

ODU is an allowable catch quota.

NOMENCLATURE

$N(t)$ is a current size of one generation of a population;

$w(t)$ is indicator for size development of individuals;

α, β are juvenile mortality rates;

μ is a food supply parameter for generation;

M is a maximum food abundance;

r_μ is a reproduction rate of food organisms;

δ is a correction factor for the rate of development;

λ is a fertility of individuals average over the population;

S is an amount of Atlantic cod parent fish stock;

R is a size of the final replenishment;

R_τ, L_τ are a time gaps of the hybrid model time set of continuous intervals;

η, τ are parameters of time delay in equations;

$t \in [0, T]$ is time interval of juvenile ontogenesis of the northern cod or an insect pest;

t_m is an intra-frame time threshold;

$q \in [0,1)$ is fraction of fishing removal rate from a fish stock;

μ, ϑ are metabolism indicators in the Bertalanffy model;

c_2 is a resource exhaustion parameter;

σ is a scaling indicator for trigger functional Ψ;

l is a scaling indicator for trigger functional Ξ;

Ψ is a trigger functional of the threshold effect of aggregated group of the Atlantic cod fish;

$Q[n]$ is a variable control function;

Λ is a disconnected set of intervals;

S_Ψ is a Schwarzian derivative;

K is a carried capacity of ecological area;

Ξ is a trigger functional of depletion effect of an insect forest pests.

INTRODUCTION

Various biological objects can exhibit identical qualitative dynamics [1]. Such dynamics can be complex. It refers to the transitional regimes of the existence of biosystems. In many ways, this problem is similar to the analysis of electrical circuits with inductive elements, but only when suddenly one element of the circuit breaks down and begins to behave unpredictably. In the same way, at first glance, biosystems also fall into the modes of functioning prohibited by the principles of their regulation. A good example is the body's confrontation with an infection that is fatal. The immune system is too complicated and most importantly it is individual to build its mathematical models, but then we will look in detail at

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some qualitatively similar examples. It is interesting to compare and summarize mathematically the forms of manifestation of self-regulation in the dynamics of processes for such different studied biological objects as sea fish (Atlantic northern cod or sturgeon in Volga River) and insects, which affect forests in Australia and Canada. In a previous paper [2], we built a hybrid model for a special situation of degradation of sturgeon in the Caspian Sea. We received a change of stationary and two different chaotic modes of behavior. According to statistics [3], it turned out that the reproduction of three close species of sturgeon is expressed by completely different functional dependencies on the graphs. It is difficult to explain in ecology, but different species have a common population dynamics, like cod and anchovy populations. On contrary, they have no similarity of changes in numbers to the stellate and russian sturgeon fish in the Caspian Sea, which being similar in evolution.

The object of study is the methodology for constructing computational model scenarios of the process on separate time frames. In the logical functions of the simulation experiments of the hybrid model we will include the properties of nonsteady biological processes, which include the action of rapid threshold phenomena and the non-regular, aperiodic changes in population characteristics. The method of constructing the variable structure of the model will allow us to consider complex processes with specific examples of collapse and pest outbreaks.

The subject of study is the algorithmic implementation for computational modeling, which allow calculating a moment of occurrence of the most important phenomena of collapse of fish stocks and determine the duration of pest invasion in the model of insect outbreak scenarios.

The purpose of the work is to improve the accuracy of predicting the state of fish populations for calculating the rational distribution of the fishing quota during the exploitation of biological resources. Flexible regulation of the fishing quota based on the comparison of model scenarios will avoid degradation of valuable fish populations. We will show that all three biology nonlinear effects can be computationally simulated in one way – including the trigger functional of limited scope. It is important that types of such trigger functional and its area of its effect naturally differ in real the examples in biology simulation problems.

1 PROBLEM STATEMENT

The overall goal of our ongoing work is to expand the methodological arsenal of mathematical biology. The focus of our current research is a comparative modeling of the common properties of the specific development of the special nature of environmental processes. Here we include the transition to outbreaks of numbers that have threshold effects. For a variety of situations of ecological confrontation between species that have reliably statistically determined threshold effects, an effective solution is to propose a general method for modifying the population model of the generation of successive generations. By the term “threshold effect” we mean a sharp change in a efficiency of reproduction or a rapid increase in loss, which is disproportionate to the change in the total population size. Here we give a simple example, if density of organisms in the experiment reaches a critical one, then a massive death will occur due to a lack of oxygen. Otherwise, insufficient numbers manifest themselves, as in social insects it leads to poor food availability and low survival of offspring, in Caspian sturgeon fishes to loss of eggs at spawning grounds in the Volga River.

As a result of our improvement of methods, the theory of optimal bioresources management will be able to simulate a scenario capable of detailing the internal mechanics of feedback in transient environmental processes. For example, the scenario as a result of a slight excess of the level of impact is a rapid and unexpected for biologists degradation of numerous previously and valuable stocks of fish. A classic example is the immediate collapse Canadian cod fishery during 1992. The solution of an important problem will be to show that various actual scenarios are common from the point of view of nonlinear dynamics and can be obtained by modifying systems of differential equations using the method of trigger functionals. The method is applicable to analysis of situations in the collapse of bioresources, rapid and unexpected degradation of fish stocks, which is not replaced by restoration contrary to forecasts and calculations of specialists.

For the problem of describing situations, we will use computational scenarios with functional iterations as $x_{n+1} = \varphi(x_n) + Q[n]$, where $Q[n]$ expresses a variable control function. φ must have clear population properties. Mathematically, we consider the following properties of such iterative model for nonlinear effects to be an actual:

$$1.\ \exists x_0 < x_1, x^* > x_1 : \lim\nolimits_{n\to\infty} \varphi(x_0) = x^*.$$

$$2.\ \forall x_0 > x_1 : \lim\nolimits_{n\to\infty} \varphi(x_0) = x^*.$$

$$3.\ \exists y_0 < x_1 : \lim\nolimits_{n\to\infty} \varphi(y_0) \in \Lambda, \inf \Lambda < x^*.$$

$$4.\ \forall x_0 < x_2 : \lim\nolimits_{n\to\infty} \varphi(x_0) = 0.$$

$$5.\ \varphi(x_{\max}) > x_1, \varphi(x_{\min}) > x_2, x_{\max} < x_{\min}.$$

This means that the function φ must have at least two extremum points. Set Λ arise as disconnected set of intervals. It is important that the function will have multistability, ability to attract the trajectory of initial point x_0 to different attractors. An important change in the scenarios will be the value that the function takes at the points of its extrema. Equilibrium states in the iteration model can appear and disappear. We do not fix the number of equilibrium positions, its value ranges from 2 to 4 points.

2 REVIEW OF THE LITERATURE

Modeling of ecological processes based on the formalism of hybrid computing structures is noted as promising, but there is not very much work in this area. A number of studies were carried out with predicatively redefinable parameters in models based on automata with transitions

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developed for a number of tasks to account for direct anthropogenic interference [4]. Interesting works were carried out on the analysis of changes in Lake Chao in China with eutrophication, with a sharp increase in the runoff of nutrients [5]. Specialists in the problems of hydrophysics modeled changes in the composition of the fish population of Lake Sevan with a sharp decrease in level of the lake [6]. Ecodynamic models must assess variability for the development of a situation in a changing environment. Biological justification for application of the numerical solution of a continuous model on time interval in a form of an evolution operator in discrete iterations is based on the Ricker theory of replenishment formation in fish stocks from, which he developed to analyze the management of the pink salmon fishery in British Columbia in [7]. The theory has several modern variants and modifications [8] with different functions [9]. Their difference is in the occurrence of nonlinear effects in the dynamics of iterations. These bifurcations are obtained for different models of the opposite interpretation. The parameters in the ecodynamics models are interpreted in completely different ways, but their increase causes the same rearrangements of the phase portrait and bifurcations of attractors. The "Feigenbaum cascade" is an example of the versatility of behavior, the phenomenon, against their will, many authors observe when calculating models of the dynamics of bioresources [10]. The computational structure of replenishment models was previously successfully applied only to stocks in a stable state, not in extreme conditions. We need to expand the model for a situation of drastic change, and consider examples of contradictions for two functions. For the famous Ricker function $\psi(x) = axe^{-bx}$ we have properties of older derivatives:

$$\begin{aligned}
&\psi'(x) = a\mathbf{e}^{-bx}(1-bx),\\
&\psi''(x)=ab\mathbf{e}^{-bx}(bx-2),\\
&\psi'''(x)=ab^2\mathbf{e}^{-bx}(3-bx)\ ,\\
&\psi^{(n)}(x)=(-1)^n ab^{n-1}\mathbf{e}^{-bx}(bx-n),\\
&S_\psi=b^2\frac{-b^2x^2+4bx-6}{2(1-bx)^2},\ \text{thus}\ S_\psi<0\ \forall\ x\in\Re.
\end{aligned}$$

Where Schwarzian derivative S_Ψ always less than zero. For the alternative Shepherd function $f(x) = ax\big/1+(x/K)^b$ by analogous procedures, we obtain the following relations of the bifurcation parameters of the derivative at the stationary points of all iterations $\forall n : f(x^*) = f^n(x^*)$:

$$f(x^*) = x^* = K\sqrt[b]{a-1},$$

$$\frac{df(x)}{dx} = \frac{(K^b + x^b)aK^b - ab(Kx)^b}{(K^b + x^b)^2}, \frac{df(x^*)}{dx} = \frac{a - ba + b}{a}.$$

For b <1 there are no critical points, for b=2 the reproductive function of the stock and recruitment has a critical point x=K, but K is the carried capacity of ecological area. Thus, the parameters a and b have the opposite interpretation and the behavior of the models cannot be reconciled. We need to develop a fluctuation model for fish stocks without a cycle period doubling cascade. Compared to previously known models, our new development will be able to generate brief chaos modes. Such regimes of chaos are very limited in time and can be detected on the basis of observational data on state of the population. Some modern researchers, when constructing models of aquatic biosystems, deliberately construct discrete equations without the possibility of any chaotic regime. The appearance of the chaos regime is unpredictable and reduces the possibilities for interpreting the simulation results. Many phenomena in modeling are not explainable.

3 MATERIALS AND METHODS

To build a computational model of populations with clearly defined stages in the development of individuals, we use the structure of time with a discrete and continuous component of time. We decided to build a method for analyzing nonlinear processes on the basis of a mathematical description of the survival of generations of populations in early ontogenesis. The approach can be improved in several ways. To the changing conditions and factors in ecological and physiological development of species, our computational models are logical to form, in an algorithmic form, logical-event structures and to consider them as scenarios. The first idea of our method is that the new model of a population process is formed on the basis of a dynamically redefined system. In ecology problems about fish and insects, the key biological aspect allows us to consider the consequences of changes in their life cycle as a factor of nonlinear dynamics. Hybrid time is formalized as a multiset of tuples. This time introduces an event component to the management of the change in a continuous process as a composition of sets:

$$\bigcup_n \{R\_\tau_n, [t_{n-1}, t_n], L\_\tau_n\}.$$

This time with right and left time gaps form with a continuous and discrete component is constructed from a set of ordered frames of non-fixed length. The second idea of the method is to establish events with a set of predicates of the first order, with which changes in the system or control action are associated.

The method of organizing continuous-event computing structures was originally proposed by us to explain the problem of low efficiency of artificial reproduction of sturgeon in the Caspian Sea. Such a low effect of the release of fry to the Caspian Sea could not be explained by ichthyologists. To build a model of phased ontogenesis, we propose a predicatively redefinable computational structure with three successive modes of change in the generation state. We write the new formula of general model with generalized three stages of development of ontogenesis in the interval (without trigger functional), but with inclusion of delay:

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$$\frac{dN}{dt} = \begin{cases} -(\alpha_1 w(t)N(t)+\beta)N(t), \ t<\tau \\ -\alpha_2 N(t)/w(\tau)-\beta N(t), \ t>\tau, w(t)<\bar{w} \\ -\alpha_3 w(t)N(t)N(t-\eta), \ t_w<t<T. \end{cases} \tag{1}$$

The system (1) is written by a differential equation with a set of possible forms for the right part and additionally with set of predicates for changing the mode of calculations. In hybrid model (1) we differently interpreted juvenile mortality rates of fish or insects. The parameter value $\bar{w}$ need as the threshold level of average dimensional development for a generation. The form for the implementation of the model (1) in the computing environment is the hybrid automaton. We construct such an automaton with transitions on the basis of a graph with oriented arcs – ways of transforming the right-hand side. The rebuildings of the right-hand side occur either by timing the time (we will call such transitions «timed arcs») or predicative transitions (by calculating internal model variables) that will be associated with the conditions of other equations in the system. In our method, the changes will be between state change modes. It differs from transitions between the states themselves, as was usual in discrete-event models. The basic hybrid structure is solved (1) numerically in conjunction with an auxiliary indicator of the dimensional development of a generation. The second equation for example of simulation of young sturgeon artificial cultivation is written as follows (2) with the correction indicator δ under the square root:

$$\frac{dw}{dt} = \frac{\mu}{\sqrt[3]{(N(t)N(\tau)+\delta)^2}} . \tag{2}$$

The model (2) fixedly reflects the availability of food supply, but we can set this indicator by the third equation:

$$\frac{d\mu}{dt} = r_\mu \mu \left(1-\frac{\mu}{M}\right) - \varepsilon N(t) . \tag{3}$$

The model (3) with maximum food abundance and fixed reproduction rate of food organisms was tested to assess the efficiency of rearing sturgeon fry in artificial conditions and showed a slowdown in the growth rate of individuals with an increase of initial generation $N(0)$. The method, which allows us to expand the base model, made the idea meaningful for the extreme dynamics of many invasive species with high fecundity of females. For example, for sudden insect outbreaks with an incomplete cycle of transformations, such spices as aphids and psyllids. These insects have no natural defense against enemies. The state and activity of many harmful insects of phytophages are regulated by their enemies-parasites. Parasites wasps usually attack one of the development stages of a pest, most often eggs. The reaction of parasites due to presence of clusters of available victims, namely the value of $N(0)$. The reaction to the initial number of insect eggs will cause this modification for the specific survival of insects:

$$\frac{dN}{dt} = -(\alpha_1 w(t)N(0)+\beta)N(t) : t<\tau . \tag{1a}$$

It is important for insect species to calculate numerically the fluctuations of juvenile survival $N(0) \to N(T) \equiv \varphi(N(0))$. We will obtain a functional dependence on the initial (eggs, spawn fish caviar) group of individuals $N(0) = \lambda S$ in model variation (1a). The reaction of their enemies is due to the crowding of the victims most accessible to attack. Next, we consider examples of modifying the model of equations (1), (2) and new equation (3) by our method of trigger functionals for cases of special situations in dynamics in fish and insects. Next, we will review and show a series of computer experiments for hybrid models with trigger effects.

4 EXPERIMENTS

The method successfully applied to calculate the reproduction of sturgeon in the Caspian Sea, but not only, for other fish populations even more relevant. In case of small numbers, the role of unfavorable environmental factors in reproduction of populations increases many times. The spawning efficiency of *Acipenser stellatus* stellate sturgeon according to data [11], in terms of the number of juveniles that roll from the river to the Caspian Sea, decreases sharply if the number of spawning fish stocks (due to systematic overfishing) becomes less than at the threshold. *Acipenser stellatus* now is dying out.

A disproportionate reduction in reserves replenishment cannot be foreseen by correlation methods. The main novelty of our approach is introduction into the hybrid, predicatively redefinable dynamic system of trigger functionals with only a limited area of impact, which we take from conditions of the ecological problem being solved by our new models.

In our equation (1) of decay for the number of generations, there are two death rates, directly dependent on population density α and independent rate β. The value α takes into account the quickest exhaustion of resources, necessary for development as number of larvae increases.

It turned out to be important for assessing the stocks of migratory fish, that with a low density of fish producers entering the river, it makes sense for us to take into account loss of their reproduction at the stage $t=0$. The effect of loss of reproductive activity of fish is implemented in model with trigger functional $\Psi[S]$ with a limited range of values it takes in this way:

$$\begin{aligned} &\Psi(S) = 1+\exp(-\sigma S^2), \\ &\lim\nolimits_{S\to\infty} \Psi(S) = 1, \Psi(0) = 2, \\ &\frac{dN}{dt} = -\alpha w(t)N^2(t) - \Psi[S]\beta N(t). \end{aligned} \tag{4}$$

In this addition function the new parameter reflects the severity of the threshold effect, which acts because of the reduced probability of spawning fish pairs in the river. The numerical solution of the system (1–3) supplemented

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by the functional in (4) we use as an operator of the evolution of functional iterations. We investigate the phase porters for iterations $R_{n+1} = \varphi(R_n) - qR_n$, where $q \in [0,1)$ the proportion of the harvest is taken from the fish stock. Now population size is R value.

We obtained a trajectory with two possible asymptotic states, but the regions of attraction of these alternative stable states are separated by a fractal repeller. The formation of a chaotic repeller with a change in the position of the extremes of dependence $\varphi(\lambda S)$, generates a different type of transitional chaotic motion than in the well-known Feigenbaum Cantor-like attractor [12].

In our computational experiment scenario of the collapse of commercial stocks develops from two phases. 1) A stable population gets only after an increase in commercial withdrawal in the aperiodic mode of fluctuations already with a smaller number, but with the illusion of recovery. 2) For the situation, if that moratorium was not introduced in a timely manner, in 10–12 model seasons the second phase of degradation named “collapse” will occur decisively (as in computational experiment Fig. 1).

The resulting computational scenario summarizes situation of collapse of the large long-lived fish fishery. For the mathematical implementation of such a collapse scenario of two phases, we propose to organize two nonlinear effects in the dynamics of iterations. The first is the inverse tangent bifurcation with a reduction of attracting equilibrium state. The second is the boundary crisis of the interval attractor, which is formed after reduction of a stable point.

We used examples of the instantaneous collapse of Canadian cod stocks near Labrador in 1992 and extended depletion of 2000 in sturgeon fishes of the Caspian Sea. The collapse of Atlantic cod was caused by predictions about the too-good state of these bioresources, the main thing is an overestimation of the reserves of cod, which were calculated by cohort models. Before the critical threshold, efficiency of reproduction of the population according to our model is quite high, it introduces deceptive expectations. A moratorium on catching cod was introduced with expectation of virtual “reserve” generations entering reproductive age. Theoretically, there should be 4–5 generations not covered by the fishery. But their abundant generations, to all surprise, were not in the networks. Otherwise, the cod fishing would recover after 7 years. Restoration of the Canada’s Northern cod subpopulation of the Atlantic cod occurs gradually due to influx of individuals from the southern ocean regions [13].

Methods for assessing stocks of sturgeon and cod significantly overestimated their actual number of fish. The official catch of cod stopped too late. For the Caspian Sea sturgeons phase of pseudo-stabilization of the stock has been stretched for 15 years, from 1977–1992, but the phase of a sharp fall in the catch is similar to the collapse of cod in Canada. The fall in stocks and collapse of short-cycle fish also proceeds in two stages in a similar way. In Fig. 2 from [14] we have seen real dynamics of the catch of anchovy fish off the coast of Peru, which collapsed in 1982. The phases before collapse are similar to the previously mentioned examples. Obviously the difference – anchovy recovery is more efficient than Canada’s cod multi-age stock. Population outbreaks are typical for small, numerous rapidly maturing fish species. Cod matures for a long time, sturgeon fish for a very long time.

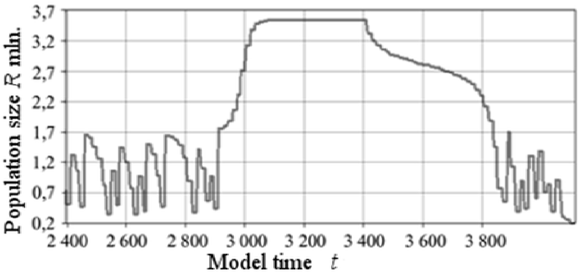

Figure 1 – Model scenario of dynamics of fish stocks collapse with two stages of degradation process

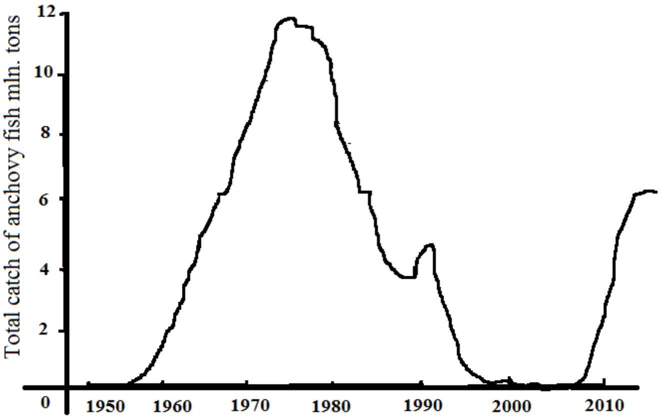

Figure 2 – Dynamics of collapse and recovery of the population anchovy *Engraulis ringens* fish in Peru

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To recover from small groups they need to go into short-cycle forms. The cod of Canada has not recovered in a quarter of a century since 1992, the cod will require another quarter of a century to form the former reserves [15]. The abundance of sturgeon fish will never recover in the Caspian Sea. Artificial reproduction of fish did not show the efficiency expected from this technology, and the natural ways of spawning fish were destroyed by dams. Artificial reproduction of stellate sturgeon was a failure action in its effectiveness.

The commonality of the phases of the dynamics of such different stocks under extreme pressure pressure looks like a real biological nonsense. After all, the spawning stock of large predatory fish of cod consists of a whole series of generations. Anchovy is more likely to undergo rapid collapse as in Fig. 2 by the dynamics of total catch. Anchovy type recovery scenario will require explicit introduction of probability into the model. The recovery of the anchovy population in Peru after a long time of collapse was like a rapid outbreak of breeding. The dynamics of a small group outbreak scenario requires an addition to the computational structure.

We need to include the probabilistic model component exactly in the form of a trigger functional Ψ, conducting numerical experiments with a random new variable among the parameters of the functional. For the practice of fishing, this means that when the population goes into sporadic states. An observer has no mechanisms for calculating stock size. There are no reliable ways to predict the optimal seizure rate, based on the statistical processing of controversial data on fish capture.

Pest outbreaks are a global problem for mathematical biology. Outbreaks of different insects differ in the phases of launch, the transition to the so-called eruptive dynamics, exit at its peak and the stage of completion of a dangerous phenomenon. Insect outbreaks are a pronounced transitional and short mode.

One of the most common scenarios for outbreak transition is a threshold one. We represent this threshold mathematically as an unstable equilibrium in iterations of a complex functional dependence. It would seem that in this task may be difficult? The root of the problem is that the threshold must be overcome spontaneously, which means without external influence. This must be a rare occurrence. A pest outbreak cannot occur even every year, otherwise there will be no forest at all. The chaotic repeller due to the locally disconnected boundary of the regions of attraction of alternative attractors remarkably solves this mathematical problem of modeling.

In the model proposed above with a functional (4), there is one solution, when a spontaneous overcoming of unstable equilibrium starts a pest outbreak with access to a stable equilibrium of a large population. However, this is how we solve only the first part of the problem of modeling an insect outbreak. Such phenomena will soon end and just quickly.

The ecosystem will not sustain such a state for a long time. The forest pest population will destroy the resources it needs. We need to describe the spontaneous termination of an insect outbreak. For this purpose, we otherwise modify the model (1) with our universal method of functionals in such form:

$$\begin{aligned}&\Xi(N(\tau))=1+\frac{\mathbf{e}^{c_1N(\tau)}}{l+c_2\mathbf{e}^{c_1N(\tau)}},\ \lim_{N(\tau)\to\infty}\Xi(N(\tau))=1+\frac{1}{c_2},\\&\frac{dN}{dt}-\alpha_2N(t)\Xi(N(\tau))/w(\tau)-\beta N(t),\ t>\tau,\end{aligned} \tag{5}$$

We estimate $c_2>1$ to characterize the swiftness of resource exhaustion. In (5) we vary the level of numbers at which the effect will noticeably manifest. We will implement the trigger functionality for the development stage number II. We describe a spontaneous, unpredictable transition from transitional chaotic dynamics to a stable equilibrium in a computational experiment. The results of the study of the model scenario constitute the phase of eruptive development of outbreak in fig. 3. Equilibrium with large numbers may seem stable for a long time. However, in the new model scenario, any equilibrium will disappear due to the functional Ξ.

In the model scenario within (5) we obtained an outbreak of insects is a rare single event. We have obtained a computational structure, which is correctly called a dynamic system with a redetermined evolution operator.

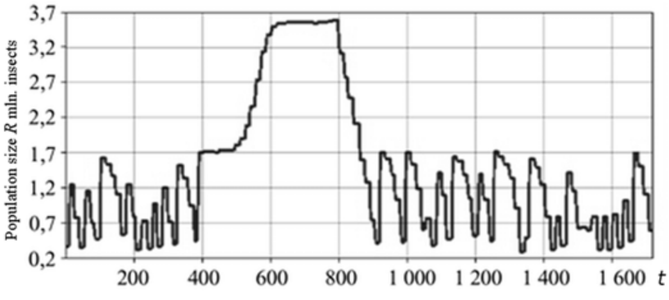

Figure 3 – Modeling the passage of phases of a single outbreak of insect pest population

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6 RESULTS

The main result lies in our method of constructing of hybrid models for threshold phenomenon in biosystems. We abandoned the method of a single parametric function for iterations. For dynamics of the model with repeated activity of the pest, we observe a sensitive dependence on the initial conditions. It is characteristic of all chaotic modes and expressed here in the spontaneous overcoming of threshold for outbreak. Similarly, as in our model, psyllid species show peak of activity. These are harmful small insects of Psyllidae family with three stages of development (an incomplete cycle of transformations) in the eucalyptus forests of Australia [15]. Abundance of psyllids pests is controlled by parasitic wasps, which themselves are an object of attack for other parasites. The model uses only the necessary types of bifurcations to complete this outbreak process.

Thus, insect outbreaks remain unpredictable for fundamental mathematical reasons due to changing regimes of chaos and stability. The sensitivity of dynamics to initial conditions is important for first stage of the process.

7 DISCUSSION

The obtained variant of dynamics of insects is not the only possible scenario for completion of an extreme population process. A repeated outbreak for a pest is a real ecological option. This scenario can also be modeled using our method. Now replace variable $w(\tau)$ in the equation of stage number II: Make the pest outbreak frequent and irregular. Modification will quickly return to the model the presence of a stationary state with a high number. So we will describe an early repeated outbreak of a pest after transient chaotic oscillations of unpredictable duration. The outbreak in such insects will end itself as a short burst in a month. Forest management and regulation cannot help in this pest control scenario.

The scenario of a collapse according to our model is a natural development of the situation under expert control. The degradation process consists of two or three phases. The number of phases before collapse depends on the average fecundity of the species. Fertility is a more important factor than the length of the life cycle in the ontogeny of a given biological species. Stock collapse is developing rapidly, collapse cannot be stopped. However, we have shown two phenomena that we hear as signs and harbingers of a future collapse. Fishing moratoriums for one season should be introduced at the first clear signs of stock reduction. Regulation using a constant quota does not help with prevent of rick of collapse, a variable quota likewise does not guarantee optimal control. Only a limit on fuel for vessels ships can protect against the rapid overfishing of biological resources. The sturgeon fish of the Caspian Sea can no longer be saved, but the Atlantic cod will restore its abundance near the Canadian cost. We have analyzed and compared various data on collapse situations of bioresources such as anchovy, stellate sturgeon, cod, crab and halibut in the Pacific Ocean [16], and we were able to find common dynamic characteristics of such events. The population always falls into an unstable regime of fluctuations in abundance after large catches. Forecasts from experts in this case, for reasons not yet clear, are always given very favorable for fishing.

CONCLUSIONS

We have shown on the current environmental problems that hybrid structures are difficult to use, but flexible and variable computational modeling tools. With a help of point introduction of trigger functionals, we obtained a circumscribing for occurrence of various ecological processes with abrupt and threshold changes. The method can be applied to analysis of transient phenomena and extreme states. The analysis of such different situations as rapid collapse of commercial stocks of large and even some small fish, an outbreak of insect pests, intermittent growth rates in ontogenesis will be relevant. Using predicates, we will be able to embed the logic of expert decisions into the hybrid model. The method has many directions for its application. For example, it is interesting to replace the well-known Bertalanffy equation with a hybrid structure, but this will be discussed in the next article. The stochastic component remains an important factor in modeling at the minimum population size [16].

The scientific novelty of obtained results is that the method of trigger functional can act purposefully, causing only the necessary changes in the phase portrait and bifurcation of attractors. Using the predominant method, we can change the boundaries of the regions of attachment of attractors. The new hybrid automata with transitions and initialization functions are able to realize more clearly and justified than in the works [17–19]. The approach will allow interaction of several hybrid automata for simulation anabolism and catabolism of different organisms.

The practical significance of obtained results is that we have reached a computational description of the stages of such a complex process as the collapse of bio-resources using the example of anchovy fish in Peru. We compared this collapse with the previously studied problem of the rapid extinction of sturgeon fishes in the Caspian Sea.

Prospects for further research are to study the process of extreme pulsating outbreak of leaf-worm butterfly of a dangerous pest in the forests of Canada.

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УНІВЕРСАЛЬНИЙ МЕТОД ОБЧИСЛЮВАЛЬНОГО МОДЕЛЮВАННЯ ПОРОГОВИХ ЕФЕКТІВ В НЕСТАЦІОНАРНИХ ЕКОЛОГІЧНИХ ПРОЦЕССАХ

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АНОТАЦІЯ

Актуальність. В сучасних умовах відбуваються різкі зміни в екосистемах. Стрімко змінюється видовий склад біоценозів. Динаміка популяцій набуває екстремальний характер при розвитку стрімких інвазій. Для математичного опису масштабних трансформацій потрібні нові методи моделювання. Складні популяційні режими змін мають ряд особливостей порогового розвитку. Нами розвивається методика гібридних обчислень для опису нелінійних ефектів в екодинаміці.

Мета роботи. Ми ставимо завдання обчислювального моделювання практично важливих сценаріїв – групи ситуацій, які відносяться до екстремальної та перехідною динаміці екосистем, як спалахів при початку небезпечних інвазій. Розвиваємо спосіб, який буде на основі моделі виживання поколінь проводити опис раптових переходів до стрімкої, але обмеженою спалаху чисельності. Аналогічно, метод можна застосовувати до аналізу ситуацій колапсу біоресурсів – швидкої і несподіваної деградації запасів риб, що не змінюється відновленням всупереч прогнозам і розрахунками фахівців.

Метод. Ситуації різких, але нетривалих зміни в популяційних процесах неможливо розрахувати традиційними математичними моделями і висловити термінами асимптотической динаміки – замкнутими граничними множинами траєкторії. В основі ідеї пропонованого нами методу формалізація нелінійної ефективності відтворення, яка змінюється пороговим чином тільки в суворо визначених умовах середовища, для чого використано безперервно-дискретний час.

Результати. У гібридній системі рівнянь реалізований метод тригерних функціоналів, що діють в виділених особливих станах біосистем. Проводиться аналіз нових модельних сценаріїв з модифікаціями функціоналів в базовій гібридній системі для екстремальних ситуацій у риб і комах-шкідників.

Висновки. Метод ми вважаємо універсальним, так підбір функціонала може бути адаптований до широкого класу моделей, що використовують диференціальні рівняння на фіксованому інтервалі.

КЛЮЧОВІ СЛОВА: математична біологія, моделювання порогових ефектів, гібридні моделі, метод тригерних функціоналів, спалахи чисельності популяцій, колапс перуанського анчоуса.

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УНИВЕРСАЛЬНЫЙ МЕТОД ВЫЧИСЛИТЕЛЬНОГО МОДЕЛИРОВАНИЯ ПОРОГОВЫХ ЭФФЕКТОВ В НЕСТАЦИОНАРНЫХ ЭКОЛОГИЧЕСКИХ ПРОЦЕССАХ

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АННОТАЦИЯ

Актуальность. В современных условиях происходят резкие изменения в экосистемах. Стремительно меняется видовой состав биоценозов. Динамика популяций приобретает экстремальный характер при развитии стремительных инвазий. Для математического описания масштабных трансформаций требуются новые методы моделирования. Сложные популяционные режимы изменений имеют ряд особенностей порогового развития.

Цель работы. Мы ставим задачу вычислительного моделирования практически важных сценариев – группы ситуаций, которые относятся к экстремальной и переходной динамике экосистем, как вспышек при начале опасных инвазий. Развиваем способ, который будет на основе модели выживаемости поколений проводить описание внезапных переходов к стремительной, но ограниченной вспышки численности. Аналогично, метод применим к анализу ситуаций коллапса биоресурсов – быстрой и неожиданной деградации запасов рыб, которая не сменяется восстановлением вопреки прогнозам и расчетам специалистов. Нами развивается методика гибридных вычислений для описания нелинейных эффектов в экодинамике.

Метод. Ситуации резких, но непродолжительных изменениях в популяционных процессах невозможно рассчитать традиционными математическими моделями и выразить терминами асимптотической динамики – замкнутыми предельными множествами траектории. В основе идеи предлагаемого нами метода формализация нелинейной эффективности воспроизводства, которая меняется пороговым образом только в строго определенных условиях среды, для чего использовано непрерывно-дискретное время.

Результаты. В гибридной системе уравнений реализован метод триггерных функционалов, действующих в выделенных особых состояниях биосистем. Проводится анализ новых модельных сценариев с модификациями функционалов в базовой гибридной системе для экстремальных ситуаций у рыб и насекомых-вредителей.

Выводы. Метод мы считаем универсальным, так подбор функционала может быть адаптирован к широкому классу моделей, использующих дифференциальные уравнения на фиксированном интервале.

КЛЮЧЕВЫЕ СЛОВА: математическая биология, моделирование пороговых эффектов, гибридные модели, метод триггерных функционалов, вспышки численности популяций, коллапс перуанского анчоуса.

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