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TWO PAIRS OF DUAL QUEUEING SYSTEMS WITH CONVENTIONAL AND SHIFTED DISTRIBUTION LAWS

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ABSTRACT

Context. The relevance of studies of G/G/1 systems is associated with the fact that they are in demand for modeling data transmission systems for various purposes, as well as with the fact that for them there is no final solution in the general case. We consider the problem of deriving a solution for the average delay of requests in a queue in a closed form for ordinary systems with Erlang and exponential input distributions and for the same systems with distributions shifted to the right.

Objective. Obtaining a solution for the main characteristic of the system – the average delay of requests in a queue for two pairs of queuing systems with ordinary and shifted Erlang and exponential input distributions, as well as comparing the results for systems with normalized Erlang distributions.

Methods. To solve the problem posed, the method of spectral solution of the Lindley integral equation was used, which allows one to obtain a solution for the average delay for the systems under consideration in a closed form. For the practical application of the results obtained, the method of moments of the theory of probability was used.

Results. Spectral solutions of the Lindley integral equation for two pairs of systems are obtained, with the help of which calculation formulas are derived for the average delay of requests in the queue in a closed form. Comparison of the results obtained with the data for systems with normalized Erlang distributions confirms their identity.

Conclusions. The introduction of the time shift parameter into the distribution laws of the input flow and service time for the systems under consideration transforms them into systems with a delay with a shorter waiting time. This is because the time shift operation reduces the value of the variation coefficients of the intervals between the arrivals of claims and their service time, and as is known from the queuing theory, the average delay of requests is related to these variation coefficients by a quadratic dependence. If a system with Erlang and exponential input distributions works only for one fixed pair of values of the coefficients of variation of the intervals between arrivals and their service time, then the same system with shifted distributions allows operating with interval values of the coefficients of variations, which expands the scope of these systems. The situation is similar with shifted exponential distributions. In addition, the shifted exponential distribution contains two parameters and allows one to approximate arbitrary distribution laws using the first two moments. This approach makes it possible to calculate the average latency and higher-order moments for the specified systems in mathematical packets for a wide range of changes in traffic parameters. The method of spectral solution of the Lindley integral equation for the systems under consideration has made it possible to obtain a solution in closed form, and these obtained solutions are published for the first time.

KEYWORDS: Erlang and exponential distribution laws, Lindley integral equation, spectral expansion solution method, Laplace transform.

ABBREVIATIONS

LIE is a Lindley integral equation;
QS is a queuing system;
PDF is a probability distribution function.

NOMENCLATURE

$a(t)$ is a density function of the distribution of time between arrivals;

$A^*(s)$ is a Laplace transform of the function $a(t)$;

$b(t)$ is a density function of the distribution of service time;

$B^*(s)$ is a Laplace transform of the function $b(t)$;

c_λ the coefficient of variation of time between arrivals;

c_μ the coefficient of variation of service time;

E_2 is a ordinary Erlang distribution of the second order;

E_2^- is a shifted Erlang distribution of the second order;

G is a arbitrary distribution law;

M is a exponential distribution law;

M^- is a shifted exponential distribution law;

$\bar{W}$ is a average waiting time in the queue;

$W^*(s)$ is a Laplace transform of waiting time density function;

λ is a Erlang (exponential) distribution parameter for input flow;

μ is a Erlang (exponential) distribution parameter for of service time;

ρ is a system load factor;

$\bar{\tau}_\lambda$ is a average time between arrivals;

$\overline{\tau_\lambda^2}$ is a second initial moment of time between arrivals;

$\bar{\tau}_\mu$ is a average service time;

$\overline{\tau_\mu^2}$ is a second initial moment of service time;

$\Phi_+(s)$ is a Laplace transform of the PDF of waiting time;

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$\psi_+(s)$ is a first component of spectral decomposition;

$\psi_-(s)$ is a second component of spectral decomposition.

INTRODUCTION

This article is devoted to the analysis of two pairs of QSs, including the Erlang distribution law as a special case of a more general gamma distribution law and an exponential distribution. The task is to derive solutions for the average delay of requests in the queue, which is the main characteristic for any QS. This characteristic, for example, is used to estimate packet delays in packet-switched networks when they are modeled using QS. The considered QSs, according to the three-position symbolism introduced by Kendall for their classification, we denote by E_2/M/1 and M/E_2/1. Here the distribution law for E_2 differs from the previously considered normalized Erlang distribution.

We also investigated the above systems with time-shifted input distributions in order to obtain a solution for the average delay. In queuing theory, studies of G/G/1 systems are especially relevant because there is no solution in the final form for the general case and one has to carry out research for special cases of distribution laws. In the study of G/G/1 systems, an important role is played by the method of spectral solution of the Lindley integral equation [1]. The paper proposes new models of queuing with shifted second-order Erlang distributions, as a special case of the Gamma distribution law.

In the previous works of the authors [2–7], it was noted that the shift of the distribution laws in the QS by the value $t_0 > 0$ leads to a decrease in the average delay of requests in the queue due to a decrease in the coefficients of variation of the time intervals of arrivals c_λ and servicing c_μ. It is known that the average delay is related to these coefficients of variation by a quadratic dependence [1].

The object of study is the queueing systems type G/G/1.

The subject of study is the average queue delay in conventional systems E_2/M/1 and M/E_2/1 and in the same systems, but with shifted input distributions.

The purpose of the work is to obtain a solution in a closed form for the main characteristic of the system – the average delay in the queue for the above QS.

1 PROBLEM STATEMENT

The paper poses the problem of finding a solution for the delay of requests in the queue in conventional QS systems E_2/M/1 and M/E_2/1 and in the same QS with shifted input distributions. Here E_2 means the second-order Erlang distribution as a special case of a more general Gamma distribution law and has the form $f_\lambda(t) = \lambda^2 te^{-\lambda t}$ for describing the distribution density of the arrival intervals, in contrast to the normalized distribution with the density function $f_\lambda(t) = 4\lambda^2 te^{-2\lambda t}$. Moreover, these density functions differ in numerical characteristics.

In a brief presentation of the method of spectral expansion of the LIE solution, we will adhere to the approach and symbols of the author of the classics of the queuing theory [1]. At the heart of the LIE solution by the spectral expansion method is to find for expression $F_\lambda^*(-s)F_\mu^*(s)-1$ a representation in the form of a product of two factors, which would give a rational function of *s*. Consequently, to find the distribution law of the delay of requests in the queue, the following spectral expansion $F_\lambda^*(-s)F_\mu^*(s)-1=\psi_+(s)/\psi_-(s)$ is necessary. Here $\psi_+(s)$ and $\psi_-(s)$ are some rational functions of *s* that can be factorized. Functions and must satisfy special conditions according to [1].

To solve this problem, it is first necessary to construct spectral solutions of the form $F_\lambda^*(-s)F_\mu^*(s)-1=\psi_+(s)/\psi_-(s)$ for these systems, considering special conditions in each case.

2 REVIEW OF THE LITERATURE

The method of spectral decomposition of the solution of the Lindley integral equation used in this work is presented in detail for the first time in the classics of the queuing theory [1]. This method was used by the authors in [2–7] and in many other works in the study of QS with shifted distributions. The spectral solution method is widely used not only in queuing theory, but also in mathematics, physics, electromagnetism and other fields [8–12]. In both foreign and Russian-language literature, the authors have not found research results in this subject area.

The closest to this area are works [15, 16], where the questions of accessing Internet web resources as queues with time lag, described by Wiener-Hopf processes, are investigated.

The problems of approximating distribution laws using several initial moments of time intervals are covered in [11–14], and the results of new research in the queuing theory [18–27].

3 MATERIALS AND METHODS

As you know, the two-parameter gamma distribution is given by the density function of the form

$$f(t)=\begin{cases}\dfrac{\beta^{-\alpha}t^{\alpha-1}e^{-t/\beta}}{\Gamma(\alpha)}, & t\ge 0,\\ 0, & t<0,\end{cases}$$

where Γ(α) is a gamma function equal $\Gamma(z)=\int\limits_0^\infty t^{z-1}e^{-t}dt$ for any real number $z>0$, $\alpha>0$, $\beta>0$. In the case of integers, this distribution turns into an Erlang distribution of order α. For example, when replacing $\lambda = 1/\beta, k=\alpha$, we get the usual Erlang distribution of order *k*:

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$$f_\lambda(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!}.$$

For a second-order distribution, the density function has the form $f_\lambda(t) = \lambda^2 t e^{-\lambda t}$. This distribution differs from the previously considered normalized Erlang distribution, where $f_\lambda(t) = 4\lambda^2 t e^{-2\lambda t}$. The Erlang distribution was normalized in order to make the mathematical expectation independent of the order of the distribution k, therefore, the numerical characteristics of the two forms of writing the distribution will change.

The main differences between the normal (gamma-derived) and normalized Erlang distributions E_2 are shown below. Differences in numerical characteristics:

– for the usual Erlang distribution

$$\overline{\tau}_\lambda = 2/\lambda,\ \overline{\tau_\lambda^2} = 6/\lambda^2,\ c_\lambda^2 = 1/2,$$

– for normalized distribution

$$\overline{\tau}_\lambda = 1/\lambda,\ \overline{\tau_\lambda^2} = 3/(2\lambda^2),\ c_\lambda^2 = 1/2.$$

Differences in the distribution parameter obtained by the method of moments:

– for the usual Erlang distribution $\lambda = 2/\overline{\tau}_\lambda$,

– for normalized distribution $\lambda = 1/\overline{\tau}_\lambda$.

Thus, the indicated distribution laws differ in both parameter and numerical characteristics, except for the coefficient of variation. As we will see below, systems formed by ordinary and normalized Erlang distributions will have different spectral expansions. Due to such a difference between the distributions, we are interested in the fact whether this difference will affect the final result of the QS – the average delay of requests in the queue, especially in the case of shifted distributions.

In this regard, it will be interesting to see the results obtained.

Next, consider a system $E_2/M/1$ formed by two flows given by functions of distribution densities:

– for the input flow

$$f_\lambda(t) = \lambda^2 t e^{-\lambda t}, \tag{1}$$

– for service times

$$f_\mu(t) = \mu e^{-\mu t}. \tag{2}$$

Let us write the Laplace transform of functions (1) and (2):

$$F_\lambda^*(s) = \left(\frac{\lambda}{\lambda+s}\right)^2,\ F_\mu^*(s) = \frac{\mu}{\mu+s}.$$

Then the spectral expansion of the LIE solution for the $E_2/M/1$ system takes the form:

$$\begin{aligned} F_\lambda^*(-s)F_\mu^*(s) - 1 &= \frac{\psi_+(s)}{\psi_-(s)} = \left(\frac{\lambda}{\lambda - s}\right)^2 \frac{\mu}{\mu+s} - 1 = \\ &= \frac{\lambda^2\mu - (\lambda-s)^2(\mu+s)}{(\lambda-s)^2(\mu+s)} = \\ &= -\frac{s[s^2 + (\mu-2\lambda)s + \lambda(\lambda-2\mu)]}{(\lambda-s)^2(\mu+s)} = \\ &= -\frac{s(s+s_1)(s-s_2)}{(\lambda-s)^2(\mu+s)}, \end{aligned} \tag{3}$$

since the quadratic equation $s^2 + (\mu-2\lambda)s + \lambda(\lambda-2\mu) = 0$ obtained from the expansion numerator has one negative root $-s_1 = -(\mu-2\lambda)/2 - \sqrt{\mu(\mu+4\lambda)}/2$ and one positive root $s_2 = (2\lambda-\mu)/2 + \sqrt{\mu(\mu+4\lambda)}/2$ in the case of a stable system with $\lambda < \mu$.

Therefore, we will take an expression $\psi_+(s) = \frac{s(s+s_1)}{s+\mu}$ as a function $\psi_+(s)$, since its zeros $s=0$, $s=-s_1$ and the pole $s=-\mu$ lie in the region $\mathrm{Re}(s) \le 0$, and we take an expression $\psi_-(s) = -\frac{(\lambda-s)^2}{s-s_2}$ as a function $\psi_-(s)$.

Finally, the components of the spectral expansion for the $E_2/M/1$ system will have the form

$$\psi_+(s) = \frac{s(s+s_1)}{s+\mu};\ \psi_-(s) = -\frac{(\lambda-s)^2}{s-s_2}. \tag{4}$$

Fig. 1 confirms the fulfillment of special conditions [1] where the zeros and poles of the fractional rational function on the complex s – plane are displayed to eliminate errors in constructing the spectral decomposition.

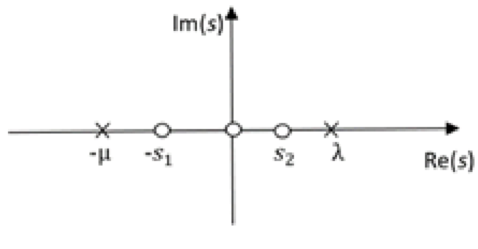

Figure 1 – Zeros and poles of the function $\psi_+(s)/\psi_-(s)$ for the $E_2/M/1$ system

In Fig. 1, the poles are marked with crosses, and the zeros are marked with circles.

Further, using the method of spectral decomposition, we find the constant K, which determines the probability that a demand entering the system finds it free:

$$K = \lim_{|s|\to 0} \frac{\psi_+(s)}{s} = \lim_{|s|\to 0} \frac{s+s_1}{s+\mu} = \frac{s_1}{\mu}.$$

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Let us construct a function $\Phi_+(s)=\dfrac{K}{\psi_+(s)}=\dfrac{s_1(s+\mu)}{\mu s(s+s_1)}$ through which we find the Laplace transform of the delay density function:

$$W^*(s)=s\cdot\Phi_+(s)=\frac{s_1(s+\mu)}{\mu(s+s_1)}. \qquad (5)$$

Derivative of a function $W^*(s)$ with a minus sign incl. $s=0$:

$$-\frac{dW^*(s)}{ds}\Big|_{s=0}=-\left[\frac{s_1\mu(s+s_1)-s_1(s+\mu)\mu}{\mu^2(s+s_1)^2}\right]\Big|_{s=0}=$$

$$=\frac{s_1\mu^2-s_1^2\mu}{\mu^2 s_1^2}=\frac{1}{s_1}-\frac{1}{\mu}.$$

Then the average delay of requests in the queue for the $E_2/M/1$ system:

$$\overline{W}=1/s_1-1/\mu, \qquad (6)$$

where $s_1=(\mu-2\lambda)/2+\sqrt{\mu(\mu+4\lambda)}/2$ is the absolute value of the negative root $-s_1$. After the expression for the average waiting time for the $E_2/M/1$ system has been found, we can proceed to the study of the $E_2/M/1$ system with a time lag.

We denote such a system $E_2^-/M^-/1$. For this system, the distributions of the arrival and service intervals are described by the following shifted density functions:

$$f_\lambda(t)=\lambda^2(t-t_0)e^{-\lambda(t-t_0)}, \qquad (7)$$

$$f_\mu(t)=\mu e^{-\mu(t-t_0)}. \qquad (8)$$

Statement 1. The spectral expansion of the LIE solution for the $E_2^-/M^-/1$ system and the final formula for the average delay have exactly the same form as for the $E_2/M/1$ system, but with changed parameters due to a shift in the distribution laws.

Proof. Laplace transforms of functions (7) and (8) have the form:

$$F_\lambda^*(s)=\left(\frac{\lambda}{\lambda+s}\right)^2 e^{-t_0 s},\ F_\mu^*(s)=\frac{\mu}{\mu+s}e^{-t_0 s}.$$

For the $E_2^-/M^-/1$ system, the spectral expansion will have the form:

$$F_\lambda^*(-s)F_\mu^*(s)-1=\left(\frac{\lambda}{\lambda-s}\right)^2 e^{t_0 s}\times\frac{\mu}{\mu+s}e^{-t_0 s}-1=$$

$$=\left(\frac{\lambda}{\lambda-s}\right)^2\times\frac{\mu}{\mu+s}-1.$$

Here, the powers of the exponentials in the spectral decomposition are also zeroed, and thus the time shift operation is leveled. The last expression after the transformations will result in the result (3). Thus, the spectral expansions of the LIE solution for both systems coincide. Consequently, all the above calculations for the $E_2/M/1$ system are also valid for the $E_2^-/M^-/1$ system. Statement 1 is proved.

To determine the unknown distribution parameters E_2^-, we use the Laplace transform of function (7). The average value of the interval between arrivals is given by the first derivative of the Laplace transform with a minus sign at the point s=0:

$$-\frac{dF_\lambda^*(s)}{ds}\Big|_{s=0}=\left[\frac{2\lambda^2 e^{-t_0 s}}{(\lambda+s)^3}+\frac{\lambda^2 t_0 e^{-t_0 s}}{(\lambda+s)^2}\right]\Big|_{s=0}=2/\lambda+t_0.$$

From here

$$\overline{\tau}_\lambda=2/\lambda+t_0. \qquad (9)$$

The second initial moment of the interval between arrivals is equal to

$$\frac{d^2F_\lambda^*(s)}{ds^2}\Big|_{s=0}=\frac{6}{\lambda^2}+4\frac{t_0}{\lambda}+t_0^2.$$

From here

$$\overline{\tau_\lambda^2}=\frac{6}{\lambda^2}+4\frac{t_0}{\lambda}+t_0^2.$$

Determine the square of the coefficient of variation

$$c_\lambda^2=\frac{\overline{\tau_\lambda^2}-(\overline{\tau_\lambda})^2}{(\overline{\tau_\lambda})^2}=\frac{2}{(2+\lambda t_0)^2}.$$

From here

$$c_\lambda=\sqrt{2}/(2+\lambda t_0). \qquad (10)$$

Note that for the distribution E_2: $\overline{\tau}_\lambda=2/\lambda$, $c_\lambda=1/\sqrt{2}$. Consequently, because of the shift of the distribution laws by the value $t_0>0$, the coefficient of variation c_λ for the distribution E_2^- decreases by $(1+\lambda t_0/2)$ a factor of comparison with c_λ for the distribution E_2.

It remains to determine the numerical characteristics for the shifted exponential distribution M^-.

$$-\frac{dF_\mu^*(s)}{ds}\Big|_{s=0}=-\frac{d}{ds}\left[\left(\frac{\mu}{\mu+s}\right)e^{-t_0 s}\right]\Big|_{s=0}=$$

$$=\left[\frac{\mu t_0 e^{-t_0 s}(s+\mu)+\mu e^{-t_0 s}}{(s+\mu)^2}\right]\Big|_{s=0}=1/\mu+t_0.$$

From here

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$$\overline{\tau}_\mu = 1/\mu + t_0 \; . \quad (11)$$

Using the second derivative of the Laplace transform for s=0, we define the second initial moment of service time

$$\left.\frac{d^2 F^*_\mu(s)}{ds^2}\right|_{s=0} = \frac{2}{\mu^2} + 2\frac{t_0}{\mu} + t_0^2 \; .$$

From here

$$c_\mu = 1/(1+\mu t_0) \; . \quad (12)$$

Now setting the values obtained above $\overline{\tau}_\lambda$, $\overline{\tau}_\mu$, c_λ, c_μ as input parameters for calculating for the $E_2^-/M^-/1$ system, as well as the shift parameter t_0, you can calculate the average delay using formula (6). Here, the ranges of variation of the variation coefficients $c_\lambda \in \left(0, 1/\sqrt{2}\right)$ and $c_\mu \in (0,1)$, are determined by relations (10) and (12), respectively, depending on the magnitude of the shift parameter $0 < t_0 < \overline{\tau}_\mu$.

Next, consider the M/E$_2$/1 system formed by two flows given by the functions of the distribution densities of the intervals:

– for the input flow

$$f_\lambda(t) = \lambda e^{-\lambda t} \; , \quad (13)$$

– for service time

$$f_\mu(t) = \mu^2 t e^{-\mu t} \; . \quad (14)$$

The spectral solution of the Lindley integral equation for this system takes the form

$$F^*_\lambda(-s) F^*_\mu(s) - 1 = \frac{\lambda}{\lambda - s} \cdot \left(\frac{\mu}{\mu + s}\right)^2 - 1 = $$
$$= \frac{s\left[s^2 + (2\mu - \lambda)s + \mu(\mu - 2\lambda)\right]}{(\lambda - s)(\mu + s)^2} .$$

The square trinomial $s^2 + (2\mu - \lambda)s + \mu(\mu - 2\lambda)$ in the numerator of the expansion in the case of a stable system has two real negative roots $-s_1$, $-s_2$:

$$-s_1 = -(2\mu - \lambda)/2 + \sqrt{\lambda(\lambda + 4\mu)}/2 \; ,$$

$$-s_2 = -(2\mu - \lambda)/2 - \sqrt{\lambda(\lambda + 4\mu)}/2 \; .$$

The final spectral solution will have the form

$$\frac{\psi_+(s)}{\psi_-(s)} = \frac{s(s + s_1)(s + s_2)}{(\lambda - s)(\mu + s)^2} \; .$$

Based on the rules for constructing functions $\psi_+(s)$ and $\psi_-(s)$ choose $\psi_+(s) = \frac{s(s+s_1)(s+s_2)}{(\mu + s)^2}$, $\psi_-(s) = \lambda - s$. The zeros and poles of this expansion are shown in Fig. 2, where the poles are marked with crosses and zeros are marked with circles.

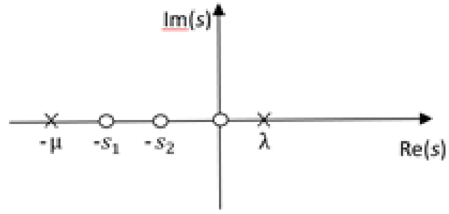

Figure 2 – Zeros and poles of the function $\psi_+(s)/\psi_-(s)$ for the M/E$_2$/1 system

The constant required to obtain a solution $K = \lim_{s \to 0} \frac{\psi_+(s)}{s} = \frac{s_1 s_2}{\mu^2}$. Next, we construct the function $\Phi_+(s) = \frac{K}{\psi_+(s)} = \frac{(1-\rho)(\mu + s)^2}{s(s+s_1)(s+s_2)}$. Whence it follows that the Laplace transform of the density function of the delay time in the M/E$_2$/1 system

$$W^*(s) = s \cdot \Phi_+(s) = \frac{(1-\rho)(\mu + s)^2}{(s+s_1)(s+s_2)} \; . \quad (15)$$

The first derivative of function (14) with a minus sign at point s=0 is

$$-\frac{dW^*(s)}{ds}\Big|_{s=0} = \frac{1}{s_1} + \frac{1}{s_2} - \frac{2}{\mu} \; .$$

Hence the average delay of requests in the queue

$$\overline{W} = \frac{1}{s_1} + \frac{1}{s_2} - \frac{2}{\mu} \; . \quad (16)$$

Comment. The Laplace transforms of the delay density functions (5) and (15) make it possible to obtain formulas not only for the average values of the delay, but also for the moments of higher orders for the delay. Considering the definition of jitter for telecommunications as the spread of the delay around the average value, then jitter can also be determined through the variance of the delay [17].

There is another way to obtain the formula for the average delay for the M/E$_2$/1 system. Because this system belongs to the class of M/G/1 systems, we will use the known result for this system by the Polyachek – Khinchin equation for the Laplace transform of the density function of the delay for the M/G/1 system [1]:

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$$W^*(s) = \frac{s(1-\rho)}{s-\lambda+\lambda F_\mu^*(s)}, \quad (17)$$

where $F_\mu^*(s) = \mu^2/(s+\mu)^2$ is the Laplace transform of the service time density function.

The Polyachek-Khinchin formula [1], gives the average delay of requests in the queue in the M/G/1 system:

$$\bar{W} = \frac{\lambda\,\overline{\tau_\mu^2}}{2(1-\rho)}, \quad (18)$$

where $0<\rho=\lambda/\mu<1$. For the distribution E_2, the second initial moment of service time, then from (18) we obtain the average delay in the M/G/1 system:

$$\bar{W} = \frac{3\rho}{2\mu(1-\rho)}. \quad (19)$$

Now it remains to verify that equalities (16) and (19) are identical. When substituting already calculated values s_1, s_2 in (16), and performing simple mathematical calculations, we get a complete coincidence with formula (19).

Let us begin to determine the average delay of requests in the queue for the $M/E_2/1$ system with delay. To do this, consider a system formed by two flows given by the functions of the distribution densities of the intervals:

– for the input flow

$$f_\lambda(t) = \lambda e^{-\lambda(t-t_0)}, \quad (20)$$

– for service times

$$f_\mu(t) = \mu^2(t-t_0)e^{-\mu(t-t_0)}. \quad (21)$$

We denote such a system $M^-/E_2^-/1$. Based on a similar statement 1, we conclude that for a pair of systems $M/E_2/1$ and $M^-/E_2^-/1$, their Laplace transforms, the delay density functions and formulas for the average delay of requests in the queue also coincide.

For the service time according to the law, we obtain similar expressions for the average service time and the coefficient of variation:

$$\bar{\tau}_\mu = 2/\mu + t_0, \quad (22)$$

$$c_\mu = \sqrt{2}/(2+\mu t_0). \quad (23)$$

To determine the unknown parameters of distributions (20) and (21), we use the corresponding moment equations:

$\bar{\tau}_\lambda = 1/\lambda + t_0$, $\bar{\tau}_\mu = 2/\mu + t_0$, $c_\lambda = (1+\lambda t_0)^{-1}$, $c_\mu = \sqrt{2}/(2+\mu t_0)$.

By specifying the values of the numerical characteristics and the shift parameter $t_0 > 0$ as input parameters for the system, and having determined the roots, we can calculate the average delay using formula (16).

4 EXPERIMENTS

Table 1–2 below shows the calculation data for systems $E_2^-/M^-/1$ and $M^-/E_2^-/1$ for cases of low, medium and high load.for cases of low, medium and high load $\rho = 0.1; 0.5; 0.9$ for a wide range of c_λ, c_μ and a shift parameter t_0. For comparison, the right-hand columns show data for conventional systems $E_2/M/1$ and $M/E_2/1$.

Table 1 – Results of experiments for QS $E_2^-/M^-/1$ and $E_2/M/1$

Input parameters				Average delay	
ρ	c_λ	c_μ	t_0	For QS $E_2^-/M^-/1$	For QS $E_2/M/1$
0.1	0.643	0.1	0.9	0.000	0.030
	0.672	0.5	0.5	0.005	
	0.700	0.9	0.1	0.023	
	0.706	0.99	0.01	0.029	
0.5	0.389	0.1	0.9	0.003	0.618
	0.530	0.5	0.5	0.132	
	0.672	0.9	0.1	0.491	
	0.704	0.99	0.01	0.605	
0.9	0.134	0.1	0.9	0.055	6.588
	0.389	0.5	0.5	1.609	
	0.643	0.9	0.1	5.322	
	0.701	0.99	0.01	6.456	

Table 2 – Results of experiments for QS $M^-/E_2^-/1$ and $M/E_2/1$

Input parameters				Average delay	
ρ	c_λ	c_μ	t_0	For QS $M^-/E_2^-/1$	For QS $M/E_2/1$
0.1	0.643	0.071	0.9	0.001	0.083
	0.950	0.354	0.5	0.021	
	0.990	0.636	0.1	0.068	
	0.999	0.700	0.01	0.082	
0.5	0.550	0.071	0.9	0.008	0.75
	0.750	0.354	0.5	0.188	
	0.950	0.636	0.1	0.608	
	0.995	0.700	0.01	0.735	
0.9	0.190	0.071	0.9	0.068	6.75
	0.550	0.354	0.5	1.688	
	0.910	0.636	0.1	5.468	
	0.991	0.700	0.01	6.616	

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Results for systems with a delay are compared with results for usual systems. It is obvious that the average waiting time in a system with a delay depends on the shift parameter t_0. The load factor ρ in both tables is determined by the ratio of average intervals $\rho = \overline{\tau}_\mu / \overline{\tau}_\lambda$. The calculations used the normalized service time $\overline{\tau}_\mu = 1$.

5 RESULTS

In this work, spectral expansions of the solution to the Lindley integral equation are obtained for the usual dual systems $E_2/M/1$ and $M/E_2/1$, as well as their analogs with shifted distribution laws, with the help of which the calculation formulas for the average delay of requests in the queue in a closed form are derived.

The same calculation formulas are valid for systems with time lag, respectively, taking into account changes in the numerical characteristics of their shifted distributions. The results of numerical calculations in Tables 1–2 are identical to the data obtained for the same systems with normalized Erlang distributions.

6 DISCUSSION

The average delay of requests in the queue in systems with latency is, as expected, less than in conventional systems, and as the value of the offset parameter decreases, it approaches the average waiting time in a conventional system. This fully confirms the adequacy of the constructed mathematical models.

Thus, the results of Table 1 and 2 confirm the complete adequacy of the constructed mathematical models for determining the average delay of requests in the queue both for ordinary dual systems and their analogs with shifted distribution laws.

In contrast to the conventional $E_2/M/1$ system, the $E_2^- / M^- / 1$ system with delay can be used for a range c_λ of 0 to $1/\sqrt{2}$ and c_μ 0 to 1. In the case of the $M/E_2/1$ system, the $M^- / E_2^- / 1$ system with delay allows a ramp range c_λ of 0 to 1, and c_μ from 0 to $1/\sqrt{2}$. Thus, the main advantage of introducing distributions shifted to the right from the zero point is to expand the range of variation coefficients of arrival intervals and service time.

Due to this, the scope of these QSs is expanding. Note that, in addition to the average delay of requests in the queue, it is possible to determine the variance and moments of higher orders of the delay time.

CONCLUSIONS

The problem of deriving formulas for the average delay of requests in the queue for two pairs of dual queuing systems with ordinary Erlang distributions in contrast to normalized distributions is solved.

The scientific novelty of the results is that spectral expansions of the solution of the Lindley integral equation for the systems under consideration are obtained and with their help the calculated formulas for the average delay in the queue for systems with delay in closed form are derived. These formulas complement and expands the well-known incomplete formula for the average waiting time in the G/G/1 systems with arbitrary laws of input flow distribution and service time.

The practical significance of the work lies in the fact that the obtained results can be successfully applied in the modern theory of teletraffic, where the delays of incoming traffic packets play a primary role. For this, it is necessary to know the numerical characteristics of the incoming traffic intervals and the service time at the level of the first two moments, which does not cause difficulties when using modern traffic analyzers.

Prospects for further research are seen in the continuation of the study of systems of type G/G/1 with other common input distributions and in expanding and supplementing the formulas for average waiting time.

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ДВІ ПАРИ ДВОЇСТИЙСИСТЕМ МАСОВОГО ОБСЛУГОВУВАННЯ ЗІ ЗВИЧАЙНИМИ І ЗСУНУТИМИ РОЗПОДІЛАМИ

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АНОТАЦІЯ

Актуальність. Актуальність дослідження систем G/G/1 пов'язана з тим, що вони потрібні для моделювання систем передачі різного призначення, а також з тим, що для них не існує рішення в кінцевому вигляді в загальному випадку. Розглянуто задачу виведення рішення для середньої затримки вимог у черзі в замкнутій формі для звичайних систем з ерлангівським і експонентним вхідними розподілами і для цих систем зі зсунутими вправо розподілами.

Мета роботи. Отримання рішення для основної характеристики системи – середньої затримки вимог у черзі для двох пар систем масового обслуговування зі звичайними і зі зсунутими ерлангівськими та експоненціальними вхідними розподілами, а також порівняння результатів для систем із нормованими ерлангівськими розподілами. Отримання рішення для основної характеристики системи – середнього часу очікування вимог в черзі для двох систем масового обслуговування типу G/G/1 зі зсунутими вхідними розподілами.

Метод. Для вирішення поставленого завдання був використаний метод спектрального рішення інтегрального рівняння Ліндлі, який дозволяє отримати рішення для середньої затримки в черзі для розглянутих систем в замкнутій формі. Для практичного застосування отриманих результатів було використаний відомий метод моментів теорії ймовірностей.

Результати. Отримано спектральні рішення інтегрального рівняння Ліндлі для двох пар систем, за допомогою яких виведені розрахункові формули для середньої затримки вимог у черзі в замкнутій формі. Порівняння отриманих результатів зі даними для систем зі нормованими ерлангівськими розподілами підтверджує їхню ідентичність.

Висновки. Введення параметра зсуву в часі в закони розподілу вхідного потоку і часу обслуговування для систем, що розглядаються, перетворює їх в системи запізненням з меншим часом очікування. Це пов'язано з тим, що операція зсуву у часі зменшує величину коефіцієнтів варіацій інтервалів між надходженнями вимог та його часу обслуговування, а як відомо з теорії масового обслуговування, середня затримка вимог пов'язана з цими коефіцієнтами варіацій квадратичною

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залежністю. Якщо система з ерлангівським і експонентним вхідними розподілами працює тільки при одній фіксованій парі значень коефіцієнтів варіацій інтервалів між надходженнями вимог та їх часу обслуговування, то ця ж система зі зрушеними розподілами дозволяє оперувати з інтервальними значеннями коефіцієнтів варіацій, що розширює сферу застосування цих систем. Аналогічно і зі зрушеними експонентними розподілами. Крім того, зрушений експонентний розподіл містить два параметри і дозволяє апроксимувати довільні закони розподілу з використанням перших двох моментів. Такий підхід дозволяє розрахувати середній час очікування та моменти вищих порядків для зазначених систем у математичних пакетах для широкого діапазону зміни параметрів трафіку. Метод спектрального вирішення інтегрального рівняння Линдлі для розглянутих систем дозволив отримати рішення у замкнутій формі, і ці отримані рішення публікується вперше.

КЛЮЧОВІ СЛОВА: ерлангівський і експонентний закони розподілу, інтегральне рівняння Ліндлі, метод спектрального розкладання, перетворення Лапласа.

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ДВЕ ПАРЫ ДВОЙСТВЕННЫХ СИСТЕМ МАССОВОГО ОБСЛУЖИВАНИЯ С ОБЫЧНЫМИ И СДВИНУТЫМИ ЗАКОНАМИ РАСПРЕДЕЛЕНИЙ

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АННОТАЦИЯ

Актуальность. Актуальность исследований систем G/G/1 связана с тем, что они востребованы для моделирования систем передачи данных различного назначения, а также с тем, что для них не существует решения в конечном виде в общем случае. Рассмотрена задача вывода решения для средней задержки требований в очереди в замкнутой форме для обычных систем с эрланговским и экспоненциальным входными распределениями и для этих же систем со сдвинутыми вправо распределениями.

Цель работы. Получение решения для основной характеристики системы – средней задержки требований в очереди для двух пар систем массового обслуживания с обычными и со сдвинутыми эрланговскими и экспоненциальными входными распределениями, а также сравнение результатов для систем с нормированными эрланговскими распределениями.

Метод. Для решения поставленной задачи использован метод спектрального решения интегрального уравнения Линдли, который позволяет получить решение для среднего времени ожидания для рассматриваемых систем в замкнутой форме. Для практического применения полученных результатов использован метод моментов теории вероятностей.

Результаты. Получены спектральные решения интегрального уравнения Линдли для двух пар систем, с помощью которых выведены расчетные формулы для средней задержки требований в очереди в замкнутой форме. Сравнение полученных результатов с данными для систем с нормированными эрланговскими распределениями подтверждает их идентичность.

Выводы. Введение параметра сдвига во времени в законы распределения входного потока и времени обслуживания для рассматриваемых систем, преобразует их в системы запаздыванием с меньшим временем ожидания. Это связано с тем, что операция сдвига во времени уменьшает величину коэффициентов вариаций интервалов между поступлениями требований и их времени обслуживания, а как известно из теории массового обслуживания, средняя задержка требований связана с этими коэффициентами вариаций квадратичной зависимостью. Если система с эрланговским и экспоненциальным входными распределениями работает только при одной фиксированной паре значений коэффициентов вариаций интервалов между поступлениями требований и их времени обслуживания, то эта же система со сдвинутыми распределениями позволяет оперировать с интервальными значениями коэффициентов вариаций, что расширяет область применения этих систем. Аналогично обстоит дело и со сдвинутыми экспоненциальными распределениями. Кроме того, сдвинутое экспоненциальное распределение содержит два параметра и позволяет аппроксимировать произвольные законы распределения с использованием двух первых моментов. Такой подход позволяет рассчитать среднее время ожидания и моменты высших порядков для указанных систем в математических пакетах для широкого диапазона изменения параметров трафика. Метод спектрального решения интегрального уравнения Линдли для рассматриваемых систем позволил получить решение в замкнутой форме и эти полученные решения публикуется впервые.

КЛЮЧЕВЫЕ СЛОВА: эрланговский и экспоненциальный законы распределения, интегральное уравнение Линдли, метод спектрального разложения, преобразование Лапласа.

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