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DEVELOPMENT OF MATHEMATICAL MODELS OF GROUP DECISION SYNTHESIS FOR STRUCTURING THE ROUGH DATA AND EXPERT KNOWLEDGE

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ABSTRACT

Context. The problem of aggregating the decision table attributes values formed out of group expert assessments as the classification problem was solved in the context of structurally rough set notation. The object of study is the process of the mathematical models synthesis for structuring and managing the expert knowledge that are formed and processed under incompleteness and inaccuracy (roughness).

Objective. The goal of the work is to develop a set of mathematical models for group expert assessments structuring for classification inaccuracy problem solving.

Method. A set of mathematical models for structuring the group expert assessments based on the methods of the theory of evidence has been proposed. This techniques allow to correctly manipulate the initial data formed under vagueness, imperfection, and inconsistency (conflict). The problems of synthesis of group decisions has been examined for two cases: taking into account decision table existing data, only, and involving additional information, i.e. subjective expert assessments, in the process of the aggregation of the experts' judgments.

Results. The outcomes gained can become a foundation for the methodology allowing to classify the groups of expert assessments with using the rough sets theory. This make it possible to form the structures modeling the relationship between the classification attributes of the evaluated objects, the values of which are formed out of the individual expert assessments and their belonging to the certain classes.

Conclusions. Models and methods of the synthesis of group decisions in context of structuring decision table data have been further developed. Three main tasks of structuring decision table data gained through the expert survey has been considered: the aggregation of expert judgments of the values of the decision attributes in the context of modeling of the relationship between the universe element and certain class; the aggregation of expert judgments of the values of the condition attributes; the synthesis of a group decision regarding the belonging of an object to a certain class, provided that the values of the condition attributes are also formed through the expert survey. The proposed techniques of structuring group expert assessments are the theoretical foundation for the synthesis of information technologies for the solution of the problems of the statistical and intellectual (classification, clustering, ranking and aggregation) data analysis in order to prepare the information and make the reasonable and effective decisions under incompleteness, uncertainty, inconsistency, inaccuracy and their possible combinations.

KEYWORDS: theory of evidence, rough set theory, aggregation, classification, inaccuracy, experts' judgments.

ABBREVIATIONS

bpa is a basic probability assignment;
DST is the Dempster-Shafer theory;
RST is the rough set theory;
DT is a decision table;
EP is an expert profile.

NOMENCLATURE

U is a non-empty, finite set of objects (the universe);
A is a set of primitive attributes;
C is a set of classification attributes;
E is a set of experts;
P is a set of profiles of expert preferences;
B is a set that reflects the judgments of all experts regarding the affiliation of the *j*-th object to a given class;
O is a set that reflects the preferences of all experts regarding the affiliation of the *j*-th object to a given class;
H is a set that reflects the judgments of all experts regarding the values of condition attributes for all objects;
Γ is a set of condition attributes values formed based on subjective and objective data for all objects;
B^{gr} is a set of profiles of group expert preferences in relation to the decision attributes values;
H^{gr} is a set of profiles of group expert preferences in relation to the condition attributes values;
Bel() is a belief function of corresponding subset;
Pl() is a plausibility function of corresponding subset;
B_i is a profile of the assessments of the *i*-th expert;
B_j^* is a set of expert judgments for the *j*-th object;
B_j^{**} is a set of unique expert judgments for the *j*-th object;
B_j^{comb} is a combined set of experts' evidences for *j*-th object by all experts;
M_j^* is a vector of mass functions (*bpa's*) formed on the judgments of all experts for *j*-th object;
M_j^{comb} is a vector of mass functions formed through the combination of corresponding mass functions by all experts for *j*-th object;

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N is a maximum limit value of using scale;

R_j^* is a vector that contains a number of identical expert preferences regarding the belonging of the j-th object to a certain class;

$Y_k \subseteq b_j^i$ contains a number / name / marker of some class, to which j-th object was referred by the i-th expert in case when expert can refer the j-th object either to several classes or subgroups of classes with different degree of preference;

X_S is a subsets of the universe;

Z_k is a degree of preference of $Y_k \subseteq b_j^i$, $Y_k \succ a_q$ defined by the i-th expert for the j-th object;

$a_l^i(u_j)$ is a value of relevant condition attribute a_l formed by i-th expert in relation to j-th object;

a_q is a set of numbers, definitions, markers of the given classes;

$a_q^{gr}(u_j)$ is a group assessment regarding the belonging of the j-th object to a certain class;

b_j^i contains a number / name / marker of some class k_p, to which j-th object was referred by the i-th expert;

d is a total amount of the subsets (groups of elements) highlighted by the i-th expert for the j-th analyzed object;

d_j is the distance measure between its arguments;

k_p is a certain class, to which the j-th object was referred by the i-th expert;

$m()$ is a *bpa* of corresponding subset;

m_j^{**} is a vector that contains the values of *bpa's* of corresponding subsets;

n is a total number of experts;

o_j^i is the expert's subjective assessment (numeric value) proving that the element j-th object can be referred to a class k_p or a group of classes;

t is a total number of condition attributes;

z is a total number of the elements of the universe;

Ω is the frame of discernment;

θ_i is weighting coefficient (competence coefficient) of the i-th expert;

$[\pi]$ is some operator for processing the composite (group) expert assessment such as methods, algorithms;

2^Ω is a power-set of all possible subsets of Ω, including the empty set;

agr is an aggregation operator;

min is the function that gives the minimum value of its arguments;

$|\cdot|$ is a cardinality of the corresponding subset.

INTRODUCTION

The basic elements of the artificial intelligence systems such as pattern recognition systems, expert systems, decision support systems, etc. are knowledge bases formed out of such two approaches as object-oriented approach and object-structural approach [1, 2].

It is worth mentioning that, besides, the basic operation, which is realized as both above-mentioned approaches are used, is structuring the knowledge through their adjustment and classification as well as the typification of the highlighted classes. The currently mentioned procedures based on the generating specifications (functions) such as sum, difference, product, augmentation, etc., allow to form the families of subsets $\{X_1, X_2, ...X_n\}$ such that $X_i \subset X$, $X_i \neq \varnothing$, $X_i \cap X_j = \varnothing$ and $\cup X_i = \mathrm{X}$ ($i \neq j$, $i, j = \overline{1, n}$), which are based on the initial set of elements of knowledge X. Therefore, that allows to describe the knowledge by highlighting their properties and attributes or criteria. The currently described abstraction makes a base for choosing the basic concepts of knowledge processing such as production rules, predicate logic, and so on, as the artificial intelligence system design is done.

As a matter of fact, in the circumstances of real life, we quite often have to tackle the problems of getting the knowledge out of arrays of unstandardized, unprocessed, rough data and knowledge. The knowledge gained in the currently-mentioned process cannot be considered accurate, so it is not able to accurately classify them and to define a category of classification. Thus, it is connected, first of all, with the fact that the inflexibility of the existing models of knowledge presentation makes the analysts either unify or abridge the factual knowledge of the experts.

Thereupon, it is advisable to use the *RST*, the mathematical mechanism of which makes an inaccurate classification possible, which can be more factual than an accurate classification is, in practice [3]. Thus, according to the *RST*, a classification problem is formed as it is described beneath [4]. There are a set of multiple samples such as, for instance, a set of expert assessments of various types of objects, phenomena, events, and so on. Such initial set is called a learning set or universe. It is widely known that each sample belong to a class highlighted out of the given set of classes. Each sample possesses a typical set of classification attribute values. Taking that into account, the *RST* allows to model the relationship between the sample classified attribute values and sample membership in a certain class.

The object of study is the process of the mathematical models synthesis for structuring and managing the expert knowledge that are formed and processed under incompleteness and inaccuracy (roughness).

The subject of study is the models and methods of the group expert assessment analysis and structuring in the context of multi-alternative, incompleteness and inaccuracy (roughness).

The purpose of the work is to develop a set of mathematical models for group experts' assessments structuring for classification inaccuracy problem solving, based on the system application of methods of evidence theory and rough set theory.

1 PROBLEM STATEMENT

Assume that the given bounded set of analyzed objects (universe elements) is $U \neq \varnothing$. On the basis of U, it is pos-

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sible to highlight the subset of universe elements X_S $X_S \subseteq U$ (a concept or category in U). Then, any family of concepts in U is considered to be abstract knowledge on U. Thus, the concepts form the division or classification of the currently-mentioned universe U. To put it in other words, in U, it is possible to highlight the family $C = \{X_S \mid s = \overline{1,n}\}$, whereas $X_s \subseteq U$, $X_s \neq \varnothing$, $X_s \cap X_t = \varnothing$ for $s \neq t$, $s, t = \overline{1,n}$. A family of classifications in U form a knowledge data base in U. Such knowledge data base is a set of aspects in the classification of universe objects.

Therefore, the existing knowledge system can be presented in a form of knowledge data base $K = (U, R)$, whereas $U = \{u_j \mid j = \overline{1,z}\}$ is a nonempty bounded set of elements (universe), R is equivalence relation, on the base of which one can form the equivalence classes (categories) of U-elements. Each category contains the elements possessing the common properties (attributes); within each category, the elements are considered indiscernible. The goal set $X_S \subseteq U$ is R-definite (R-accurate) if it is a unification of categories highlighted in U on the basis of R-relation. Otherwise, $X_S \subseteq U$ can be considered R-indefinite (R-inaccurate, R-rough).

The random knowledge data base K can correspond to the information system $S = (U, A, V, f)$, whereas $A = \{a_l \mid l = \overline{1,q}\}$ is a nonempty bounded set of primitive attributes; $V = \bigcup_{a_l \in A} V_{a_l}$, V_{a_l} is a set of attribute values a_l; $f \wedge U \times A \to V$ is an data function such that $\forall a_l \in A, x \in U, f(x, a_l) \in V_{a_l}$.

To model the situation in which the element $u \in U$ can belong to a preliminarily-defined class based on the given set of attributes, the information system can be represented in a form of a DT $T = (U, A)$, whereas $A = C \cup D$ is a set of multiple condition attributes C ($|C| > 1$, $C = A \backslash \{a_q\}$ (classification attribute set) and a single-element subset D ($|D| = 1$, $D = \{a_q\}$) is a set of multiple decision attributes, the value of which describes the possible classes (a_q is a set of numbers, definitions, markers of the given classes), to which one can refer the elements of the initial universe. The relevant initial data of the information system and DT can be gained in different ways, i.e. both on the basis of objective and subjective initial information.

In the process of the analysis and partitioning the DT, during the group expertise, one can highlight the problems given below:

1) the problem of the aggregation of the appropriate values of the relevant decision attributes, i.e. the subjective expert values in relation to the values of $a_q(u_j)$, $a_q \in D$ formed out of the given set $a_l(u_j)$, $a_l \in C$, and synthesis of the group assessment in relation to the values of $a_q^{gr}(u_j)$, $u_j \in U$;

2) the problem of the aggregation of the appropriate values of the relevant condition attributes, i.e. the subjective expert values in relation to the values $a_l(u_j)$, $a_l \in C$, and synthesis of the group assessment in relation to the values of $a_l^{gr}(u_j)$, $u_j \in U$;

3) the problem of the group decision synthesis in relation to the membeship of the element $u_j \in U$ in the certain class: $u_j \to k_p$, $k_p \in a_q$ provided the relevant values of $a_l(u_j)$, ($a_l \in C$, $u_j \in U$) are also formed on the basis of the group expertise.

2 REVIEW OF THE LITERATURE

The *RST*, which was introduced by Z. Pawlak [5], allows to manipulate the initial data, which are considered rough as far as they are inaccurate and vague. The currently mentioned theory is for modeling the vagueness related with the universe elements belonging to the given goal set. To quantify such vagueness in [4, 5] measures of approximation accuracy and quality has been defined.

The theory is peculiar due to its mathematical mechanism helping to process the implicit arrays of unstandardized, i.e. inaccurate, rough, or unprocessed data and knowledge, and, thus, get the new knowledge. The theory is based on the fact that the knowledge is deeply involved in the human capability of classifying the subjects, phenomena, objects, situations, and so on, and so forth. They are reflected in the division (classification) of the relevant elements [3, 4, 5]. Such kind of division can be considered the knowledge presentation semantics. As a matter of fact, knowledge consists of the classification patterns of the application environment that is examined [3].

At the same time, knowledge is a kind of systematized information (objective or expert data) gained provided meeting the set criteria and structured for the solution of the set problem. In case if a sufficient amount of objective data, i.e. statistical, analytical, experimental, and empirical information, which can be gained my means of the methods of observation (registration), measurements (experiments, tests), is missing, it is advisable to involve a group of specialists (experts) in the certain application environment who form their judgments on the basis of the opinions and personal experience based on the interview, survey, focus-group with the methods of expert assessments. In such case, we can face a problem of aggregated expert assessment obtaining.

The analysis of a number of conventional methods of obtaining the expert assessments helps to arrive at the conclusion that different techniques of direct expert assessment averaging as well as the methods based on the various procedures of the comparison of the analyzed objects such as pairwise and multiple comparisons have become the most spread [6–9]. However, they are not deprived of a number of disadvantages. Obtaining the averaged assessment will be justified only if there is a high expert assessment consistency (proximity). In case if, there are several group supporting different opinions in the expert commission, it

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will be no use simply averaging all the expert assessments. The main disadvantage of the methods based on the procedures of pairwise comparison is that they can be used for a small amount of compared elements. As the number of the latter grows, it is quite often difficult to achieve a high level of consistency of local priorities.

A favourable decision for the problems mentioned above should be provided by applying the advanced methods of the management of the indeterminacies, which have appeared in the last years. Therefore, we refer *DST*, i.e. evidence theory [10–12], and Theory of Plausible and Paradoxical Reasoning [13] to such methods. The mathematical apparatus of those theories allows to get the aggregated expert assessments using the technique of their combination. The choice of the combination rule depends on the study model (Dempster-Shafer model or Dezert-Smarandache model); the information on the conflicts between the expert evidence, which are combined; a structure of expert evidence; degree of consistency of expert evidences. In the works [13, 14], a number of recommendations for the choice of combination technique has been proposed.

3 MATERIALS AND METHODS

Let us consider the problem of aggregation of group expert assessments of decision attributes. Let us assume that the values of the *C*-subset elements are formed on the basis of the data gained through the objective studies based on the independent measurements, calculations and so on (objective data), and the values of the attribute a_q are formed out of the subjective data, i.e. data gained through the expert surveys.

Let, a group of experts $\mathrm{E} = \{E_i \mid i = \overline{1,n}\}$, taking into consideration the data of the given DT based on the values of the given set of *C*-tokens, formed the profiles of expert preferences $\mathrm{P} = <\mathrm{B}>$, whereas $\mathrm{B} = \{B_i \mid i = \overline{1,n}\}$. B_i-profile formed by the expert reflects its priorities in relation to the $u_j \in U$ ($j = \overline{1,z}$) element's membership in the given class $k_p \in a_q$ ($p = \overline{1,r}$, $r < z$). Thus, $B_i = \{b_j^i \mid j = \overline{1,z}\}$, whereas b_j^i contains a number / name / marker of some class $k_p \in a_q$, to which the object $u_j \in U$ was referred by the expert E_i.

The task is to synthesize the composite (group) profile $\mathrm{B}^{gr} = \{b_j^{gr} \mid j = \overline{1,z}\}$, $\underset{i=\overline{1,n}}{agr}(b_j^i) \to b_j^{gr}$, each b_j^{gr} element of which reflects a group solution, has a number, name or marker of some class $k_p \in a_q$, to which the object $u_j \in U$ was referred.

On the basis of the gained values of the B^{gr} composite profile for each object $u_j \in U$ that is examined, it will be able to set a class, to which it belongs: $\forall u_j \in U, j = \overline{1,z} : (u_j, b_j^{gr})$. The pair (u_j, k_p) sets the u_j object appurtenance to some class $k_p \in a_q$, the marker of which is preserved in b_j^{gr}.

A generalized scheme of the synthesis of the composite profile $\mathrm{B}^{gr} = \{b_j^{gr} \mid j = \overline{1,z}\}$ can be represented as follows, $i = \overline{1,n}$; $j = \overline{1,z}$:

$$B = \begin{pmatrix} B_1 \\ \dots \\ B_i \\ \dots \\ B_n \end{pmatrix} = \begin{pmatrix} b_1^1 & \dots & b_z^1 \\ \dots & \dots & \dots \\ b_1^i & \dots & b_z^i \\ \dots & \dots & \dots \\ b_1^n & \dots & b_z^n \end{pmatrix} \begin{matrix} \Rightarrow \\ \Uparrow \\ [\pi] \end{matrix} \begin{pmatrix} b_1^{gr} \\ \dots \\ b_j^{gr} \\ \dots \\ b_z^{gr} \end{pmatrix}^{-1} = \mathrm{B}^{gr}. \quad (1)$$

For the synthesis of group (composite) expert assessments in modeling the relation "the element of the universe – the defined class", the mathematical notation of *DST* was used. While modeling the dependence "*U*-element – the *DT*-class", the following situations were studied:

1. $\forall u_j \in U$ whereas u_j element belongs to the only one class: $u_j \to k_p$;

2. $\exists u_j \in U$, which, according to the expert choice, can be referred to several classes: $u_j \to \{k_p,\dots,k_s\}$, $p \neq s$, $p,s = \overline{1,r^*}$, $r^* < r$, $\forall p,s = \overline{1,r^*} : \{k_p \sim k_s\}$; in the result of modeling, $u_j \in U$ can be referred only to one class.

3. $\exists u_j \in U$, for which E_i cannot define a reference to any of the set classes: $u_j \to a_q$, $\forall p,s = \overline{1,r} : \{k_p \sim k_s\}$; as a result, $u_j \in U$ can be referred to the only one class.

The constraints, which are imposed on, and the conditions of the procedure of the expert survey can result in the following:

1. Using only the existing data and knowledge of the *DT* in the process of the expert assessment aggregation.

Let us examine the set a_q as the *DST* regards it. Let us assume, a_q is a frame of discernment, then, in the result of the expert survey, a system of subsets $B_i = \{b_j^i \mid j = \overline{1,z}\}$ will be formed, whereas b_j^i reflects E_i judgments in relation to the $u_j \in U$-membership either in some $k_p \in a_q$ class, or in several classes (provided the expert defines a subgroup of classes, to one of which the $u_j \in U$-object can be referred; the classes are equivalent inside the mentioned group). Thus, taking into account the *DST*-notation, b_j^i shall be regulated by a system of rules:

$$1.\ b_j^i = \{\varnothing\}; \quad (2)$$

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2. $\left|b_j^i\right| = 1$ – the expert has chosen and evaluated one element $k_p \in a_q$.

3. $\left|b_j^i\right| = h$, $h < \left|a_q\right|$ – the expert has highlighted h of the elements $k_p \in a_q$.

4. $b_j^i = a_q$ – it was difficult for the expert to assess / choose as far as all the elements of the set a_q are equivalent.

Aggregating the expert judgments is done according to the following suggested procedure:

1.1 Problem structuring. Let us highlight a subset of expert judgments $B_j^* = \{b_j^i\}$, $i = \overline{1,n}$ for each $u_j \in U$ and form a subset of the unique elements on the basis of those values: $B_j^{**} = \{b_t^*\}$, $t \le n$.

1.2 Define the vector $R_j^* = \{r_t^*\}$, whereas for $\forall t = \overline{1, | B_j^{**} |}$: $r_t^* = \text{count}(B_j^*(b_t^*))$ corresponds the number of the B_j^*-component that are equal to some value of $b_t^* \in B_j^{**}$.

1.3 Calculate the *bpa* masses for each subset B_j^{**}, taking into consideration the equation (formula):

$$m\{b_t^*\} = r_t^* \big/ | B_j^* |. \tag{3}$$

Thus, for each B_j^{**}-subset, it is possible to draw a vector $\mathrm{m}_j^{**} = \{m_t^* \mid t = \overline{1, | B_j^{**} |}\}$, the elements of which are in accord with the following constraints [10–12]:

$$0 \le m(X_j) \le 1, \quad m(\varnothing) = 0, \ \sum_{X_j \in \Lambda} m(X_j) = 1, \tag{4}$$

whereas Λ corresponds to 2^{Ω}; $m : \Lambda \to [0,1]$.

1.4 The calculation of the upper and lower limits of the probability for each $k_p \in a_q$, which correspond the values of the belief function $Bel : \Lambda \to [0,1]$, [10–12]:

$$Bel(B) = \sum_{X_j \subseteq B,\, X_j \in \Lambda} m(X_j) \tag{5}$$

and plausibility function $Pl : \Lambda \to [0,1]$:

$$Pl(B) = \sum_{X_j \cap B \ne \varnothing,\, X_j \in \Lambda} m(X_j) \tag{6}$$

1.5 Forming the intervals $[Bel(\{k_p\}), Pl(\{k_p\})]$ for the subsets $k_p \in a_q$.

1.6 Choosing the optimal solution $b_{opt}^* \in a_q$ is done by means of the comparison of the intervals $[Bel(\{k_p\}), Pl(\{k_p\})]$, $\forall p = \overline{1, | a_q |}$ formed through the belief function and plausibility functions. The maximal interval, in which the lower value and upper value of the interval limits are the highest among the similar values of all the other intervals, corresponds the optimal solution: $b_{opt}^* = k_p : \max_p [Bel(\{k_p\}), Pl(\{k_p\})]$, $\forall p = \overline{1, r}$, $b_j^{gr} = b_{opt}^*$. Comparing all the embedded intervals, one can go from the interval values to crisp values. Thus, $b_j^{gr} = b_{opt}^*$, on the assumption that $b_{opt}^* \in a_q$.

2. Involving the additional information, i.e. subjective assessments, in the process of the expert judgment aggregation.

Situation 2.a. Expert E_i can refer the object $u_j \in U$ only either to a single class $k_p \in a_q$ or one subgroup of classes (the classes are considered equivalent within a highlighted subgroup, so the object $u_j \in U$ can be referred only to one of those classes).

Let us assume that, a group of experts $\mathrm{E} = \{E_i \mid i = \overline{1,n}\}$, based on the data of a given *DT*, constructed on the basis of the values of the organized set of *C*-tokens, formed the set of *EP's* $\mathrm{P} =< \mathrm{B}, \mathrm{O} >$. The P-set forms a tuple consisting of two components such as:

1) a set $\mathrm{B} = \{B_i \mid i = \overline{1,n}\}$, each element of which is $B_i = \{b_j^i \mid j = \overline{1,z}\}$ reflecting the reference mentioned by expert E_i regarding the affiliation of the element $u_j \in U$ ($j = \overline{1,z}$) either to a class $k_p \in a_q$ or several classes provided the expert can define a subgroup of classes, to one of which one can refer the object $u_j \in U$:

$$\forall u_j \in U \ \text{ expert } E_i : \begin{cases} u_j \to k_p \in a_q; \\ u_j \to \{k_p, ..., k_s\} \subseteq a_q. \end{cases} \tag{7}$$

2) a set $\mathrm{O} = \{O_i \mid i = \overline{1,n}\}$, each element of which is $O_i = \{o_j^i \mid j = \overline{1,z}\}$ reflecting the assessment of the E_i-expert's belief in the fact that the element $u_j \in U$ ($j = \overline{1,z}$) can be referred either to a class $k_p \in a_q$ or a subgroup of classes.

Thus, under the *DST* notation, b_j^i shall meet the standards of a system of rules (2); in its turn, o_j^i is expert's subjective assessment (probability) proving that the element $u_j \in U$ can be referred to a class $k_p \in a_q$ or a group of classes. The assessment of o_j^i can be repre-

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sented within a set scale with using a range from 0 to N ($N > 0$). Under the assumption that $N \neq 1$, then the value o_j^i shall be normalized to a unit interval, i.e. $o_j^i \in [0;1]$.

Aggregating the expert judgments is done according to the following suggested procedure:

2.1 Problem structuring (partitioning). For each $u_j \in U$, let us highlight a set of expert judgements $B_j^* = \{b_j^i\}$, $i = \overline{1,n}$ and a set of assessments $O_j^* = \{o_j^i\}$, $i = \overline{1,n}$; let us form a subset of unique elements $B_j^{**} = \{b_t^*\}$, $t \le n$ of the $B_j^* = \{b_j^i\}$ on the basis of the obtained values.

2.2 According to the *DST*-notation, let us consider a set, i.e. the frame of discernment $\Omega = \{\omega_1, \omega_2\}$, whereas ω_1 for each E_i corresponds to the value of $b_j^i \in B_j^*$; $\omega_2 = a_q$ represents a complete lack of knowledge of the expert as to his choice. Under the assumption that $m(\omega_1)$ is the probability of the fact that the element $u_j \in U$ really belongs to the mentioned class, $m(\omega_1)=o_j^i$, in case of $o_j^i \in [0;1]$, $o_j^i \in O_j^*$; then the probability of the fact that the element can belong to some other class can be represented as $m(\omega_2)=1-m(\omega_1)$.

Thus, for each B_j^*, one will be able to get a set $\mathrm{M}_j^* = \{\mathrm{m}_j^i \mid i = \overline{1,n}\}$, whereas $\mathrm{m}_j^i = \{m(\omega_1), m(\omega_2)\}$ is a *bpa* vector of, as the expert E_i thinks, either right or wrong classification of the element $u_j \in U$, the elements of m_j^i satisfy (4).

2.3 Defining a procedure of the expert evidence aggregation (combination). For combining, one should choose a pair of expert evidences $b_j^i, b_j^h \in B_j^*$, such that under $i \neq h$: $\min d_J(\mathrm{m}_j^i, \mathrm{m}_j^h) \in [0;1]$ in compliance to one of the metric [15–18].

2.4 Aggregation of expert assessments is done through a combination of corresponding mass functions (*bpa's*) $\mathrm{M}_j^* = \{\mathrm{m}_j^i \mid i = \overline{1,n}\}$ and $B_j^* = \{b_j^i\}$, by all the experts E_i, $(i = \overline{1,n})$ for each $u_j \in U$ individually. In the result of the combination, a vector $B_j^{comb} = \{b_j^i \mid i = \overline{1,v}\}$, $v = 2^{\left|B_j^{**}\right|}$ and a vector $\mathrm{M}_j^{comb} = \{\mathrm{m}_j^i \mid i = \overline{1,v}\}$ can be obtained accordingly.

2.5. Calculation of the upper and lower bound of the plausibility for each $k_p \in a_q$ in compliance with (5) and (6) on the basis of the obtained B_j^{comb} and M_j^{comb}. Forming the intervals $[Bel(\{k_p\}), Pl(\{k_p\})]$ for the subsets $k_p \in a_q$.

2.6. The choice of an optimal $b_{opt}^* \in a_q$ is done through the comparison of the intervals $[Bel(\{k_p\}), Pl(\{k_p\})]$, $\forall p = \overline{1,\left|a_q\right|}$ formed with the belief and plausibility functions. The maximal interval corresponds the optimal solution. Thus, $b_j^{gr} = b_{opt}^*$ under the assumption that $b_{opt}^* \in a_q$.

Situation 2.b. Expert E_i can refer the object $u_j \in U$ either to several classes $k_p \in a_q$ or subgroups of classes with different degree of confidence (belief) in one's own choice. As far as the classes are considered equivalent within a highlighted subgroup, the object $u_j \in U$ can be referred only either to one class or a group of classes.

Let us assume that, analyzing the data of a given *DT*, constructed on the basis of the values of the organized set of *C*-tokens, a group of experts $\mathrm{E} = \{E_i \mid i = \overline{1,n}\}$ formed the set of *EP's* $\mathrm{P} = < \mathrm{B}, \mathrm{O} >$. A set of the *EP's* creates a tuple consisting of two components.

The first tuple component is $\mathrm{B} = \{B_i \mid i = \overline{1,n}\}$, each $B_i = \{b_j^i \mid j = \overline{1,z}\}$ element of which reflects the priorities mentioned by the expert E_i as to the membership of the element $u_j \in U$ ($j = \overline{1,z}$) in a class $k_p \in a_q$, or several classes. At the same time, $b_j^i = \{Y_k \mid k = \overline{1,d}\}$, $d \le 2^{\left|a_q\right|}$ is more than one value (several aimed classes or groups of classes). The second tuple component is $\mathrm{O} = \{O_i \mid i = \overline{1,n}\}$, each $O_i = \{o_j^i \mid j = \overline{1,z}\}$ element of which reflects the assessment of the E_i-expert belief in the fact that $u_j \in U$ ($j = \overline{1,z}$) is a member of the certain class $k_p \in a_q$ or a subgroup of classes. At the same time, $o_j^i = \{Z_k \mid k = \overline{1,d}\}$, $d \le 2^{\left|a_q\right|}$, $\forall i,j : \left|o_j^i\right| = \left|b_j^i\right|$, $i = \overline{1,n}$, $j = \overline{1,z}$.

Thus, taking into consideration, the *DST* notation, each element $Y_k \subseteq b_j^i$ shall meet the standards of the system of rules (2); in its turn, each element $Z_k \in o_j^i$ can create a probability, according to the expert's subjective assessment / belief), that the element $u_j \in U$ belongs to the certain class $k_p \in a_q$ or a group of classes. The assessment $Z_k \in o_j^i$ can be represented within the first given scale, using a range from 0 to the certain given N ($N > 0$).

Aggregation of the expert judgements is done in compliance with the suggested procedure, such as

2.1. Problem structuring (partitioning). Let us highlight a set of the expert's judgments $B_j^* = \{b_j^i\}$,

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$i=\overline{1,n}$, and a set of expert's assessments $O_j^*=\{o_j^i\}$, $i=\overline{1,n}$ for each $u_j \in U$.

2.2 Defining the mass functions that correspond the highlighted subsets $Y_k \subseteq b_j^i$, $\forall b_j^i \in B_j^*$. For each formed system of subsets $b_j^i=\{Y_k \mid k=\overline{1,d}\}$, it will be possible to get a vector $\mathrm{m}_j^i=\{m_k \mid k=\overline{1,d+1}\}$, the elements of which correspond (4) and are calculated by the formulae, such as [10]:

$$m_k(Y_k)=\frac{Z_k\cdot\theta_i}{\sum_{k=1}^{d} Z_k\cdot\theta_i+\sqrt{d}},$$
$$m_{d+1}(a_q)=\frac{\sqrt{d}}{\sum_{k=1}^{d} Z_k\cdot\theta_i+\sqrt{d}}. \quad (8)$$

The value equaling $m_{d+1}(a_q)$ can reflect a degree of complete ignorance of E_i in relation to the membership of the object $u_j \in U$ in any class $k_p \in a_q$.

2.3. Defining the aggregation (combination) procedure of the expert judgments. For the combination, one can choose a pair of $b_j^i, b_j^h \in B_j^*$ such that under $i \neq h$: $\min d_J(\mathrm{m}_j^i,\mathrm{m}_j^h)\in[0;1]$ in accordance with one of metrics [15–18].

2.4. The aggregation of the *EP's* is done by the combination of the obtained *bpa's* $\mathrm{M}_j^*=\{\mathrm{m}_j^i \mid i=\overline{1,n}\}$ and $B_j^*=\{b_j^i\}$, by all the experts E_i, $(i=\overline{1,n})$, for each $u_j \in U$ individually, as well. The combination results are a vector $B_j^{comb}=\{Y_k^{comb} \mid k=\overline{1,v}\}$, $v \le 2^{|a_q|}$ and vector $\mathrm{M}_j^{comb}=\{\mathrm{m}(Y_k^{comb}) \mid k=\overline{1,v}\}$, accordingly.

2.5. The calculation of the upper and lower bound for each $k_p \in a_q$ is done in compliance with (5) and (6), and on the basis of the obtained B_j^{comb} and M_j^{comb}, as well. The formation of the intervals $[Bel(\{k_p\}),Pl(\{k_p\})]$ for the subsets $k_p \in a_q$.

2.6. Choosing an optimal solution $b_{opt}^* \in a_q$ is done through the comparison of the intervals $[Bel(\{k_p\}),Pl(\{k_p\})]$, $\forall p=\overline{1,|a_q|}$. The maximal interval corresponds to the optimal solution. Thus, $b_j^{gr}=b_{opt}^*$ ($b_{opt}^* \in a_q$).

Let us consider the problem of aggregation of group expert assessments of condition attributes. For the *DT*, it is supposed that a *A*-set of primitive attributes is a union of two subsets $A=\{a_l \mid l=\overline{1,q-1}\}\cup a_q$, i.e. subsets of independent condition attributes $C=\{a_l \mid l=\overline{1,q-1}\}$ and one-element set of decision attribute $D=\{a_q\}$. Let us assume that, on the set *C,* it is possible to highlight a subset $C^* \subseteq C$, the elements of which are formed on the basis of the subjective data, i.e. data obtained by means of the expert survey. Let us enter the token $t=|C^*|$.

Let us assume that, examining the universe of discourse, a group of experts $\mathrm{E}=\{E_i \mid i=\overline{1,n}\}$ has formed the set of *EP's* such as $\mathrm{P}=<\mathrm{H}>$ or $\mathrm{P}=<\mathrm{H},\mathrm{O}>$, whereas $\mathrm{H}=\{H_i \mid i=\overline{1,n}\}$, $\mathrm{O}=\{O_i \mid i=\overline{1,n}\}$. Each element $H_i=\{H_j^i \mid j=\overline{1,z}\}$, $H_j^i=\{a_l^i(u_j) \mid l=\overline{1,t}\}$ of the first component of the E_i-profile reflects its preferences in relation to the values of the relevant condition attributes $a_l^i(u_j)$ of the element $u_j \in U$ ($j=\overline{1,z}$). The second component $\mathrm{O}=\{O_i \mid i=\overline{1,n}\}$ of the E_i-profile represents the expert's assessment of the belief in the correctness of his / her judgements, $O_i=\{O_j^i \mid j=\overline{1,z}\}$, $O_j^i=\{o_l^i(u_j) \mid l=\overline{1,t}\}$, whereas $o_l^i(u_j)$ is the assessment of the degree of confidence of the E_i in the set value of the attribute a_l for the element $u_j \in U$.

The task is to synthesize a group profile $\mathrm{H}^{gr}=\{H_j^{gr} \mid j=\overline{1,z}\}$, each element of which, i.e. $H_j^{gr}=\{a_l^{gr}(u_j) \mid l=\overline{1,t}\}$, represents a group solution and contains the aggregated values of the condition attributes $a_l^{gr}(u_j)$ of the $u_j \in U$ ($j=\overline{1,z}$), which are formed on the basis of the individual *EP's* $H_i=\{H_j^i \mid j=\overline{1,z}\}$, $\forall i=\overline{1,n}$, Fig. 1.

Taking into account the values of the H^{gr} composite profile, one can do the further examination and *DT*-data structuring.

The synthesis of the group decision is done according to the following procedure:

1. Problem structuring. Let a set of judgements $H_j^*=\{H_j^i \mid i=\overline{1,n}\}$, $\forall u_j \in U$ will be formed.

2. Aggregation of the group expert assessments $H_j^i \in H_j^*$.

The aggregation of the group expert's assessments is done individually for each attribute $a_l^i(u_j)$ by all the experts E_i, $i=\overline{1,n}$, i.e. $\forall l=\overline{1,t}: \underset{i}{agr}(a_l^i(u_j)) \to H_j^{gr}$.

As an operator for processing the group expert's assessments of the relevant condition attributes can be one of the above schemes used.

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U	$C^* \subseteq C, \lvert C^* \rvert = t$								
	E_1				…	E_n			
	a_1^1	…	a_t^1	H_1	…	a_1^n	…	a_t^n	H_n
…	…	…	…	…	…	…	…	…	…
u_j	$a_1^1(u_j)$	…	$a_t^1(u_j)$	H_j^1	…	$a_1^n(u_j)$	…	$a_t^n(u_j)$	H_j^n
…	…	…	…	…	…	…	…	…	…
u_z	$a_1^1(u_z)$	…	$a_t^1(u_z)$	H_z^1	…	$a_1^n(u_z)$	…	$a_t^n(u_z)$	H_z^n

$$\mathrm{H}^{gr} = \{H_j^{gr} \mid j = \overline{1,z}\}, \quad H_j^{gr} = \{a_l^{gr}(u_j) \mid l = \overline{1,t}\}.$$

U	C^*			H^{gr}
	a_1^{gr}	…	a_t^{gr}	
…	…	…	…	…
u_j	$a_1^{gr}(u_j)$	…	$a_t^{gr}(u_j)$	H_j^{gr}
…	…	…	…	…
u_z	$a_1^{gr}(u_z)$	…	$a_t^{gr}(u_z)$	H_z^{gr}

Figure 1 – The procedure for H^{gr} profile synthesis

The generalized scheme of the aggregation of H_j^i, $i = \overline{1,n}$, and construction of H_j^{gr} as a group expert assessment of the values of condition attributes a_l by the j-th object, can be represented in the following way:

$$H_j^* = \begin{pmatrix} a_1^1(u_j) & \dots & a_t^1(u_j) \\ \dots & \dots & \dots \\ a_1^i(u_j) & \dots & a_t^i(u_j) \\ \dots & \dots & \dots \\ a_1^n(u_j) & \dots & a_t^n(u_j) \end{pmatrix} \begin{matrix} \\ \\ \Rightarrow \\ \Uparrow \\ [\pi] \end{matrix} \begin{pmatrix} a_1^{gr}(u_j) \\ \dots \\ a_l^{gr}(u_j) \\ \dots \\ a_t^{gr}(u_j) \end{pmatrix}^{-1} = H_j^{gr}. \quad (9)$$

Similarly, the formation of the aggregated attribute values is made for each $u_j \in U$, $j = \overline{1,z}$.

Let us examine the problem of the group decision synthesis in relation to the membership of the element $u_j \in U$ in the given class provided the certain values of the relevant condition attributes of the $u_j \in U$ are also formed on the basis of the group expert evaluation.

In such a case, the expert evaluation shall be divided in two stages.

Stage 1. Solving the task of aggregation of the group expert assessments of condition attributes.

At that stage, a group of experts $\mathrm{E} = \{E_i \mid i = \overline{1,n}\}$ forms the set of *EP's* of $\mathrm{P} =< \mathrm{H} >$ or $\mathrm{P} =< \mathrm{H}, \mathrm{O} >$–types, whereas $\mathrm{H} = \{H_i \mid i = \overline{1,n}\}$, $\mathrm{O} = \{O_i \mid i = \overline{1,n}\}$. In the first case, the *EP* $H_i = \{H_j^i \mid j = \overline{1,z}\}$, $H_j^i = \{a_l^i(u_j) \mid l = \overline{1,t}\}$ formed by E_i reflects its preferences in relation to the values of the relevant condition attributes $a_l^i(u_j)$ of the $u_j \in U$ ($j = \overline{1,z}$). In the second case, the *EP* formed by the E_i contains an additional set $\mathrm{O} = \{O_i \mid i = \overline{1,n}\}$, $O_i = \{O_j^i \mid j = \overline{1,z}\}$, $O_j^i = \{o_l^i(u_j) \mid l = \overline{1,t}\}$ whereas $o_l^i(u_j)$ is an assessment of the degree of the E_i belief in the correctness of the fixed value of the attribute a_l for the $u_j \in U$.

The synthesis of a set of composite *EP's* $\mathrm{H}^{gr} = \{H_j^{gr} \mid j = \overline{1,z}\}$, each $H_j^{gr} = \{a_l^{gr}(u_j) \mid l = \overline{1,t}\}$ element of which reflects a group decision and contains the aggregated values of the relevant condition attributes $a_l^{gr}(u_j)$ of the $u_j \in U$ ($j = \overline{1,z}$) obtained on the basis of the individual *EP's* $H_i = \{a_l^i(u_j) \mid l = \overline{1,t}\}$, $\forall i = \overline{1,n}$, is carried out in accordance with the above-given scheme, i.e. a problem of aggregation of group expert assessments of condition attributes.

Stage 2. Solving the task of aggregation of the group expert assessments of decision attributes.

At the second stage, taking into account the values $\Gamma = \{\gamma_j \mid j = \overline{1,z}\}$, $\gamma_j = \{a_l(u_j) \mid l = \overline{1,|C|}\}$ of the given set of tokens (attributes) $A = \{a_l \mid l = \overline{1,q}\}$, $C = \{a_l \mid l = \overline{1,q-1}\}$, a group of experts $\mathrm{E} = \{E_i \mid i = \overline{1,n}\}$ forms the set of *EP's* $\mathrm{P} =< \mathrm{B} >$ whereas $\mathrm{B} = \{B_i \mid i = \overline{1,n}\}$.

We assume that each subset γ_j is formed on the basis of both:

1. The initial subjective data, i.e. under the group expert evaluation, the subjective values of the γ_j are formed out of the obtained at the first stage values of the relevant condition attributes $\mathrm{H}^{gr} = \{H_j^{gr} \mid j = \overline{1,z}\}$, $H_j^{gr} = \{a_l^{gr}(u_j) \mid l = \overline{1,t}\}$ such that $H_j^{gr} \subseteq \gamma_j$, $\forall j = \overline{1,z} : \left|H_j^{gr}\right| \le \left|\gamma_j\right|$.

2. The initial objective data. Under the assumption that $\forall j = \overline{1,z} : \left|H_j^{gr}\right| < \left|\gamma_j\right|$, the $\gamma_j = \{a_l(u_j)\}$, $\forall j = \overline{1,z} : \gamma_j \setminus H_j^{gr}$, are the values $a_l(u_j)$ formed on the basis of the objective data.

The E_i profile $B_i = \{b_j^i \mid j = \overline{1,z}\}$ represents its preferences in relation to the membership of the $u_j \in U$ ($j = \overline{1,z}$) in the given class $k_p \in a_q$ ($p = \overline{1,r}$, $r < z$), and the value b_j^i contains a number / name / or a marker of some class $k_p \in a_q$, to which the object $u_j \in U$ was referred by the E_i. The set of *EP's* can be represented in a form of $\mathrm{P} =< \mathrm{B}, \mathrm{O} >$. The second tuple component is a set $\mathrm{O} = \{O_i \mid i = \overline{1,n}\}$, each element of which, i.e.

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$O_i = \{O_j^i \mid j = \overline{1,z}\}$, represents the degree of the E_i belief in the fact that the $u_j \in U$ ($j = \overline{1,z}$) is a member of either a certain class $k_p \in a_q$ or a subgroup of classes whereas $o_j^i = \{Z_k \mid k = \overline{1,d}\}$, $d \le 2^{|a_q|}$, $\forall i, j : |o_j^i| = |b_j^i|$, $i = \overline{1,n}$, $j = \overline{1,z}$.

The task is to synthesize the composite profile $\text{B}^{gr} = \{b_j^{gr} \mid j = \overline{1,z}\}$ whereas b_j^{gr} represents a group decision in relation to the $u_j \in U$ membership in some $k_p \in a_q$ in accordance with the above-given scheme.

4 EXPERIMENTS

Let us demonstrate the above-suggested approaches, taking as a sample the solution of the problem of the group decisions synthesis in relation to the values of the relevant decision attributes. Let us assume that, taking into account the values of the formed set of tokens C, a group of experts $\text{E} = \{E_i \mid i = \overline{1,5}\}$ evaluated the membership of the elements of the universe $u_j \in U$ ($j = \overline{1,3}$) in the given set of classes $a_q = \{k_p \mid p = \overline{1,3}\}$.

Sample 1. In the process of forming a group expert assessment, the only existing *DT* data and knowledge are used. The results of the expert survey are given in Table 1.

Table 1 – Expert profiles (Sample 1)

Objects	E_1	E_2	E_3	E_4	E_5
u_1	$\{k_2\}$	$\{k_2, k_3\}$	$\{k_2\}$	$\{k_1, k_3\}$	$\{k_3\}$
u_2	$\{k_1\}$	$\{k_2\}$	$\{k_1\}$	$\{k_1\}$	$\{k_1, k_2\}$
u_3	$\{k_2\}$	$\{k_1\}$	$\{k_1, k_2\}$	$\{k_3\}$	$\{k_2\}$

Sample 2. In the process of forming a group *EP's*, the embedded additional expert information is used; the expert E_i can refer the object $u_j \in U$ only to either one class $k_p \in a_q$ or a one subset of classes. The results of the expert survey are given in Table 2.

Sample 3. In the process of forming a group *EP's*, the embedded additional expert information is used; the expert E_i can refer the $u_j \in U$ either to several classes $k_p \in a_q$ or several subsets of classes with different degree of belief in one's own choice. The results of the expert survey are given in Table 3.

Table 2 – Expert profiles (Sample 2)

Objects	E_1		E_2		E_3		E_4		E_5	
	B_1	O_1	B_2	O_2	B_3	O_3	B_4	O_4	B_5	O_5
u_1	$\{k_2\}$	6	$\{k_2, k_3\}$	8	$\{k_2\}$	9	$\{k_1, k_3\}$	7	$\{k_3\}$	7
u_2	$\{k_1\}$	7	$\{k_2\}$	9	$\{k_1\}$	7	$\{k_1\}$	7	$\{k_1, k_2\}$	8
u_3	$\{k_2\}$	8	$\{k_1\}$	6	$\{k_1, k_2\}$	8	$\{k_3\}$	8	$\{k_2\}$	9

Table 3 – Expert profiles (Sample 3)

Objects	E_1		E_2		E_3		E_4		E_5	
	$Y_k \subseteq b_j^1$	$Z_k \subseteq o_j^1$	$Y_k \subseteq b_j^2$	$Z_k \subseteq o_j^2$	$Y_k \subseteq b_j^3$	$Z_k \subseteq o_j^3$	$Y_k \subseteq b_j^4$	$Z_k \subseteq o_j^4$	$Y_k \subseteq b_j^5$	$Z_k \subseteq o_j^5$
u_1	$\{k_2\}$	6	$\{k_2, k_3\}$	8	$\{k_2\}$	9	$\{k_1, k_3\}$	7	$\{k_3\}$	7
	$\{k_1\}$	7	$\{k_1\}$	5	–	–	$\{k_2\}$	5	$\{k_1\}$	9
	$\{k_3\}$	3	–	–	–	–	–	–	–	–
u_2	$\{k_1\}$	8	$\{k_2\}$	9	$\{k_1\}$	7	$\{k_1\}$	7	$\{k_1, k_2\}$	8
	$\{k_2, k_3\}$	5	–	–	$\{k_3\}$	4	$\{k_2\}$	9	–	–
u_3	$\{k_2\}$	8	$\{k_1\}$	6	$\{k_1, k_2\}$	8	$\{k_3\}$	8	$\{k_2\}$	9

Tables 1–3 represents only the subjective judgments made by five experts as to the membership of the elements of the given universe in a fixed set of classes. In such a case, the values of the classified attributes of the universe elements are omitted on purpose as far as they do not matter, by any means, for the problem that is examined.

The elements of the set O_i (Table 2) and set Z_k (Table 3) were evaluated according to the ten-point scale (zero stands for the lowest degree of preference and ten stands for an absolute degree of preference).

5 RESULTS

Let us examine the practical realization of the above methods for synthesizing a group decision in relation to the membership of the element $u_1 \in U$ in a class $k_p \in a_q$.

In the process of analyzing the data from Table 1, it can be seen that, taking into account $a_q = \{k_p \mid p = \overline{1,3}\}$, for $u_1 \in U$, a group of experts formed a set $B_1^* = \{\{k_2\},\{k_2,k_3\},\{k_2\},\{k_1,k_3\},\{k_3\}\}$ on the basis of which it will be possible to form a set $B_1^{**} = \{\{k_2\},\{k_3\},\{k_1,k_3\},\{k_2,k_3\}\}$, and a vector $R_1^* = \{2,1,1,1\}$.

Let us calculate the basic probability assignment for each element of the set B_1^{**} according to equation (3):

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$m\{k_2\} = 2/5$; $m\{k_3\} = 1/5$;
$m\{k_1, k_3\} = 1/5$; $m\{k_2, k_3\} = 1/5$.

Let us calculate the value of the functions (5) and (6) for each element of the set a_q:

$$k_1 : \begin{cases} Bel(\{k_1\}) = m(\{k_1\}) = 0; \\ Pl(\{k_1\}) = 0.2; \end{cases}$$

$$k_2 : \begin{cases} Bel(\{k_2\}) = m(\{k_2\}) = 0.4; \\ Pl(\{k_2\}) = 0.6; \end{cases}$$

$$k_3 : \begin{cases} Bel(\{k_3\}) = m(\{k_3\}) = 0.2; \\ Pl(\{k_3\}) = 0.6. \end{cases}$$

After taking a look at the above calculations, one can see that the $b^*_{opt} = \{k_2\}$ is an optimal one. Thus, we obtain $u_1 \to k_2$ and $b_1^{gr} = \{k_2\}$, accordingly.

In the process of analysis the data given in Table 2, one can see that, taking into account $a_q = \{k_p \mid p = \overline{1,3}\}$, for $u_1 \in U$, the experts formed a set $B_1^* = \{\{k_2\},\{k_2,k_3\},\{k_2\},\{k_1,k_3\},\{k_3\}\}$. On the basis of the values of the latter, we can form a set $B_1^{**} = \{\{k_2\},\{k_3\},\{k_1,k_3\},\{k_2,k_3\}\}$ and a set $O_1^* = \{6,8,9,7,7\}$, as well.

The *bpa's* of the formed focal elements are given in Table 4.

Let us calculate the combined values of the *bpa's* of the highlighted subsets:

$m\{k_2\} = 0.44$; $m\{k_3\} = 0.47$;
$m\{k_2, k_3\} = 0.003$; $m\{k_1, k_3\} = 0.0863$.
$m\{k_1, k_2, k_3\} = 0.0007$;

Let us calculate the values of the functions (5) and (6) for each element of the set a_q:

$$k_1 : \begin{cases} Bel(\{k_1\}) = 0; \\ Pl(\{k_1\}) = 0.087; \end{cases}$$

$$k_2 : \begin{cases} Bel(\{k_2\}) = 0.44; \\ Pl(\{k_2\}) = 0.444; \end{cases}$$

$$k_3 : \begin{cases} Bel(\{k_3\}) = 0.47; \\ Pl(\{k_3\}) = 0.56. \end{cases}$$

Taking into account the above calculations, one can see that the $b^*_{opt} = \{k_3\}$ is an optimum choice. Thus, we obtain $u_1 \to k_3$ and $b_1^{gr} = \{k_3\}$, accordingly.

In the process of the analysis of the data from Table 3, one can see that the experts formed a set $B_1^* = \{b_1^i\}$ and a set of assessments $O_1^* = \{o_1^i\}$, $i = \overline{1,n}$ for $u_1 \in U$, on the basis of $a_q = \{k_p \mid p = \overline{1,3}\}$, whereas

$b_1^1 = \{\{k_1\},\{k_2\},\{k_3\}\}$; $o_1^1 = \{7,6,3\}$;
$b_1^2 = \{\{k_1\},\{k_2,k_3\}\}$; $o_1^2 = \{5,8\}$;
$b_1^3 = \{\{k_2\}\}$; $o_1^3 = \{9\}$;
$b_1^4 = \{\{k_2\},\{k_1,k_3\}\}$; $o_1^4 = \{5,7\}$;
$b_1^5 = \{\{k_1\},\{k_3\}\}$; $o_1^5 = \{9,7\}$.

The *bpa's* of the formed focal elements are given in Table 5.

Table 4 – The *bpa's* of the formed focal elements (Sample 2)

Objects	E_1		E_2		E_3		E_4		E_5	
	$m(\omega_1)$	$m(\omega_2)$	$m(\omega_1)$	$m(\omega_2)$	$m(\omega_1)$	$m(\omega_2)$	$m(\omega_1)$	$m(\omega_2)$	$m(\omega_1)$	$m(\omega_2)$
u_1	0.6	0.4	0.8	0.2	0.9	0.1	0.7	0.3	0.7	0.3
u_2	0.7	0.3	0.9	0.1	0.7	0.3	0.7	0.3	0.8	0.2
u_3	0.8	0.2	0.6	0.4	0.8	0.2	0.8	0.2	0.9	0.1

Table 5 – The *bpa's* of the formed focal elements (Sample 3)

Objects	E_1		E_2		E_3		E_4		E_5	
	$Y_k \subseteq b_j^1$	$m(Y_k)$	$Y_k \subseteq b_j^2$	$m(Y_k)$	$Y_k \subseteq b_j^3$	$m(Y_k)$	$Y_k \subseteq b_j^4$	$m(Y_k)$	$Y_k \subseteq b_j^5$	$m(Y_k)$
u_1	$\{k_2\}$	0.34	$\{k_2, k_3\}$	0.55	$\{k_2\}$	0.90	$\{k_1, k_3\}$	0.52	$\{k_3\}$	0.40
	$\{k_1\}$	0.39	$\{k_1\}$	0.35	$\{k_1, k_2, k_3\}$	0.10	$\{k_2\}$	0.37	$\{k_1\}$	0.52
	$\{k_3\}$	0.17	$\{k_1, k_2, k_3\}$	0.10	–	–	$\{k_1, k_2, k_3\}$	0.11	$\{k_1, k_2, k_3\}$	0.08
	$\{k_1, k_2, k_3\}$	0.10	–	–	–	–	–	–	–	–
u_2	$\{k_1\}$	0.55	$\{k_2\}$	0.9	$\{k_1\}$	0.56	$\{k_1\}$	0.40	$\{k_1, k_2\}$	0.89
	$\{k_2, k_3\}$	0.35	$\{k_1, k_2, k_3\}$	0.1	$\{k_3\}$	0.32	$\{k_2\}$	0.52	$\{k_1, k_2, k_3\}$	0.11
	$\{k_1, k_2, k_3\}$	0.10	–	–	$\{k_1, k_2, k_3\}$	0.12	$\{k_1, k_2, k_3\}$	0.08	–	–
u_3	$\{k_2\}$	0.89	$\{k_1\}$	0.86	$\{k_1, k_2\}$	0.89	$\{k_3\}$	0.89	$\{k_2\}$	0.90
	$\{k_1, k_2, k_3\}$	0.11	$\{k_1, k_2, k_3\}$	0.14	$\{k_1, k_2, k_3\}$	0.11	$\{k_1, k_2, k_3\}$	0.11	$\{k_1, k_2, k_3\}$	0.10

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Let us calculate the combined values of the *bpa's* of highlighted subsets in compliance with (8):

$m\{k_1\} = 0.331$; $m\{k_2\} = 0.438$;
$m\{k_3\} = 0.217$; $m\{k_1, k_3\} = 0.013$;
$m\{k_2, k_3\} = 0.0002$; $m\{k_1, k_2, k_3\} = 0.0008$.

Let us calculate values of the functions (5) and (6) for each element of the set a_q:

$$k_1 : \begin{cases} Bel(\{k_1\}) = 0.331; \\ Pl(\{k_1\}) = 0.3448. \end{cases}$$

$$k_2 : \begin{cases} Bel(\{k_2\}) = 0.438; \\ Pl(\{k_2\}) = 0.439. \end{cases}$$

$$k_3 : \begin{cases} Bel(\{k_3\}) = 0.217; \\ Pl(\{k_3\}) = 0.231. \end{cases}$$

Taking a look at the estimations described above, one can see that the $b^*_{opt} = \{k_2\}$ is an optimum. Thus, we obtain $u_1 \to k_2$ and $b_1^{gr} = \{k_2\}$, accordingly.

6 DISCUSSION

The problems of the group decisions synthesis while modeling the relationship between the element of universe and definite class either in case if one takes into consideration only the existing *DT* data or if the additional information, i.e. subjective expert assessments, is involved in the process of aggregating the expert judgment, have been studied. In solving the problem of the aggregation of the relevant *DT* attributes formed on the basis of the subjective assessments of the expert group, the situations are considered when an expert can either refer a universe object only to one class, i.e. a single subgroup of classes when the classes are considered equivalent within a highlighted subset, or define that a universe object can refer to several separate classes, i.e. subsets of classes, with different degree of confidence in one's own choice, i.e. the classes can be considered equivalent within a highlighted subset.

To make the aggregated expert assessments, a mathematical mechanism of evidence theory has been used. Unlike the existing techniques of the expert evidence aggregation, that allowed to synthesize the group decisions in the context of multiple alternatives, incompleteness, inaccuracy and inconsistency (conflict), as well as to model the uncertainty and not to strictly restrain the expert in his / her personal choice. To put it in other words, the expert can choose not only a single priority but also can form the clustered ranges of objects, setting a degree of confidence in his / her own choice.

CONCLUSIONS

The problems, which arise in the process of the analysis and structuring of *DT* data in the context of the group expert evaluation have been formulated. A set of mathematical models for structuring the *DT* data obtained out of the expert assessments, which are formed and processed in the context of inaccuracy (roughness), imperfection, and multiple alternatives in the process of solving the inaccurate classification problem, has been proposed.

The scientific novelty of obtained results is that the models and methods of synthesizing the group decisions and *DT* data structuring are received the further development. The next problems of synthesizing the group decisions and *DT* data structuring have been solved: the synthesis of the group assessments of the values of the relevant decision attributes, the synthesis of the group assessments of the values of the relevant condition attributes, and the synthesis of the group assessments concerning the membership of the universe object in the given class provided the appropriate values of the relevant condition attributes of the object are also formed on the basis of the group expert evaluation. The suggested techniques are based on the mathematical notation of the evidence theory. That allowed processing the group expert assessments under vagueness, imperfection, and inconsistency (conflict).

The practical significance of the obtained results implies that the suggested techniques form a theoretical substratum for plotting the methods, algorithms, and information technologies for intelligent support of the decision-making process, and its implementing in the automated decision-support systems for an inaccurate classification problem solving. The obtained results can be helpful in the formation of the bases of the expert's knowledge in different universes of discourse.

The prospects for further research imply the development of the procedure of reducing the *DT* knowledge formed on the basis of the individual expert judgments, especially in the context of the incomplete expert data.

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РОЗРОБКА МАТЕМАТИЧНИХ МОДЕЛЕЙ СИНТЕЗУ ГРУПОВИХ РІШЕНЬ СТРУКТУРИЗАЦІЇ ГРУБИХ ДАНИХ ТА ЕКСПЕРТНИХ ЗНАНЬ

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АНОТАЦІЯ

Актуальність. Розглянуті питання агрегування значень атрибутів таблиці рішень, сформованих на основі групових експертних оцінок при вирішенні задачі неточної класифікації в рамках нотації теорії грубих множин. Об'єктом дослідження є процеси синтезу математичних моделей структуризації та управління експертними знаннями, які формуються та обробляються в умовах неточності (грубості) та неповноти. Мета роботи – розробка математичних моделей структуризації групових експертних оцінок при вирішенні задачі «неточної класифікації».

Метод. Запропоновано комплекс математичних моделей структуризації групових експертних оцінок, в основу яких покладено методи теорії свідоцтв, які дозволяють коректно оперувати з вихідними даними, сформованими в умовах невизначеності, неповноти, неузгодженості (конфлікту). Розглянуті питання синтезу групових рішень для двох випадків: тільки на основі існуючих даних таблиці рішень, і з залученням додаткової інформації (суб'єктивних експертних оцінок) в процесі агрегування суджень експертів.

Результати. Отримані результати можуть бути покладені в основу методики, що дозволяє виконувати класифікацію групових експертних оцінок із застосуванням теорії грубих множин. Це дає можливість формувати структури, що моделюють залежність між класифікаційними атрибутами оцінюваних об'єктів, значення яких формуються на основі індивідуальних експертних оцінок, і їх приналежністю відповідним класам.

Висновки. Дістали подальшого розвитку моделі та методи синтезу групових рішень у контексті структурування даних таблиці рішень. Три основні задачі структурування даних таблиці рішень, одержаних у результаті експертного опитування, було розглянуто: агрегування експертних суджень щодо значень атрибутів рішень при моделюванні залежності «елемент універсуму – визначений клас»; агрегування експертних оцінок щодо значень атрибутів умов; синтез групового рішення щодо належності об'єкта до певного класу за умови, що значення атрибутів умов також формуються шляхом експертного опитування. Запропоновані техніки структуризації групових експертних оцінок становлять теоретичне підґрунтя для синтезу інформаційних технологій вирішення задач статистичного та інтелектуального (класифікація, кластеризація, ранжування, агрегування) аналізу даних з метою підготовки інформації для прийняття обґрунтованих та ефективних рішень в умовах неповноти, невизначеності, неузгодженості неточності та їх можливих комбінацій.

КЛЮЧОВІ СЛОВА: теорія свідоцтв, теорія грубих множин, агрегування, класифікація, неточність, експертні оцінки.

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РАЗРАБОТКА МАТЕМАТИЧЕСКИХ МОДЕЛЕЙ СИНТЕЗА ГРУППОВЫХ РЕШЕНИЙ СТРУКТУРИЗАЦИИ ГРУБЫХ ДАННЫХ И ЭКСПЕРТНЫХ ЗНАНИЙ

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АННОТАЦИЯ

Актуальность. Рассмотрены вопросы агрегирования значений атрибутов таблицы решений, сформированных на основе групповых экспертных оценок при решении задач неточной классификации в рамках нотации теории грубых множеств. Объектом исследования являлись процессы синтеза математических моделей структуризации и управления экспертными знаниями, которые формируются и обрабатываются в условиях неточности (грубости) и неполноты. Цель работы – разработка математических моделей структуризации групповых экспертных оценок при решении задачи «неточной классификации».

Метод. Предложено комплекс математических моделей структуризации групповых экспертных оценок, в основу которых положены методы теории свидетельств, позволяющие корректно оперировать с исходными данными, сформированными в условиях неопределенности, неполноты, несогласованности (конфликта). Рассмотрены вопросы синтеза групповых решений для двух случаев: только на основе существующих данных таблицы решений, и с привлечением дополнительной информации (субъективных экспертных оценок) в процессе агрегирования суждений экспертов.

Результаты. Полученные результаты могут быть положены в основу методики, позволяющей выполнять классификацию групповых экспертных оценок с применением теории грубых множеств. Это позволяет формировать структуры, моделирующие зависимость между классификационными атрибутами оцениваемых объектов, значения которых формируются на основе индивидуальных экспертных оценок, и их принадлежностью соответствующим классам.

Выводы. Получили дальнейшее развитие модели и методы синтеза групповых решений в контексте структурирования данных таблицы решений. Три основные задачи структурирования данных таблицы решений, полученных в результате экспертного опроса, были рассмотрены: агрегирование экспертных оценок значений атрибутов решений при моделировании зависимости «элемент универсума – определенный класс»; агрегирование экспертных оценок значений атрибутов условий; синтез группового решения о принадлежности объекта к некоторому классу при условии, что значения атрибутов условий также формируются за результатами экспертного опроса. Предложенные техники структуризации групповых экспертных оценок составляют теоретическую основу для синтеза информационных технологий решения задач статистического и интеллектуального (классификация, кластеризация, ранжирование, агрегирование) анализа данных с целью подготовки информации для принятия обоснованных и эффективных решений в условиях неполноты, неопределенности, несогласованности неточности их возможных комбинаций.

КЛЮЧЕВЫЕ СЛОВА: теория свидетельств, теория грубых множеств, агрегирование, классификация, неточность, экспертные оценки.

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