УПРАВЛІННЯ У ТЕХНІЧНИХ СИСТЕМАХ

CONTROL IN TECHNICAL SYSTEMS

УПРАВЛЕНИЕ В ТЕХНИЧЕСКИХ СИСТЕМАХ

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FREQUENCY FEATURES OF THE NUMERICAL METHOD SAMPLING OF DIGITAL CONTROL SYSTEMS

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ABSTRACT

Context. The studies of the frequency properties of the explicit multistep numerical integrators which use for sampling of continuous transfer function in the digital control systems, are conducted in this article. Numerical integrators in such systems implement as an integral parts of the digital regulators.

Objective. The goal of this research is an analysis of the behavior of explicit numerical integrators of different orders, which are used to discretize continuous systems, in order to study their impact on the properties of the synthesized digital system.

Method. Numerical methods of integration are considered as digital filters, the behavior of which is studied by the frequency characteristics method. To do this, the z-transform apparatus was used. Integrators' discrete transfer functions were found for frequency analysis using the Control Systems Toolbox package of the mathematical application MATLAB. For further analysis, two closed feedback test structures were used: with integrators in the forward channel and in the feedback loop. Both variants of structures were studied by the frequency characteristics method for sampling using numerical integrators of 1st–6th orders.

Results. The inefficiency of using high-order numerical integrators for continuous systems' discretization is shown. Given the behavior of the frequency characteristics of test systems, the most rational is the use of low-order integrators, namely – the first and second orders. Establishing the cause of this phenomenon requires additional research, in particular, to identify the possible impact of additional zeros and poles of discrete transfer functions of the numerical integrators.

Conclusions. The use of low-order integrators, namely the first and second orders, is the most rational for sampling of digital control systems and the inefficiency of using high-order numerical integrators to sample continuous systems is proven.

KEYWORDS: Bode diagram, control theory, digital control system, discrete transfer function, linear system, numeric integrators, structure models, z-transform.

ABBREVIATIONS

PID controller is proportional-integral-derivative controllers.

NOMENCLATURE

$X(s)$ is an input signal of the tested structure;

$Y(s)$ is an output signal of the tested structure;

$\frac{1}{s}$ is an ideal integrator's continuous transfer function;

a_k is a k-th denominator's coefficient of the continuous transfer function;

$b_\mathbf{k}$ is a k-th numerator's coefficient of the continuous transfer function;

y_{i+1} is a numeric integrator's output variable of i+1-th step (next sample);

y_i is a numeric integrator's output variable of i-th step (current sample);

x_k is an input variable of k-th step (sample) in the recurrent formula of the numeric integrator;

h is a sampling time (integration step);

j is an imaginary unit;

s is an operator in Laplace domain;

z is an operator in Z-domain (z-transform);

ω is an angular frequency;

ω_0 is an angular sampling frequency;

$A(\omega)$ is a magnitude of the continuous transfer function;

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$A^*(\omega)$ is a magnitude of the discrete transfer function;
$\varphi(\omega)$ is a phase of the continuous transfer function;
$\varphi^*(\omega)$ is a phase of the discrete transfer function;
$W(s)$ is a continuous transfer function in Laplace domain;
$W^*(z)$ is a discrete transfer function in Z-domain;
$W_n^*(z)$ is a n-order discrete transfer function of the numeric integrator.

INTRODUCTION

Modern digital control systems can be meet in almost all technological systems, from home appliances to complex technological complexes. Any digital system has two interconnected components: hardware and software, which cannot function without each other. The capabilities of the hardware usually determined by one of its few manufacturers and can be slightly corrected by adding some technical components; otherwise, the software fully meets the digital system requirements and largely determines its capabilities. In particular, the same controller used in home appliances, and industrial controllers, and to control the hard disk drive. Less noticeable, however, the defining part of the software is to provide mathematical – software implementation of numerical methods in control algorithms.

Means of the process automation of developing algorithmic software for digital systems have been widely used in engineering practice. The mathematical applications Mathcad, MATLAB (in particular, together with the Control System Toolbox library) etc. can be examples, which simplify the process of synthesizing the mathematical part of control algorithms for following software implementation in a digital control system. The presence in mathematical applications of ready-made implementations of the discretization typical methods for continuous systems makes it possible to some extent to automate the entire process of the mathematical component synthesis for digital system software. On the one hand, this is good because the developer gets a number of advantages:

– reduction of development time due to the avoidance of a large number of symbolical transformations in the discretization process (synthesis of the control algorithm);

– reducing the number of possible human errors in mathematical expression through the use of mentioned mathematical applications;

– the developer often no longer needs a deep understanding of discretization processes, in particular, in physical processes relation in the designed system, which significantly accelerates the actual development process.

On the other hand, these same advantages are to some extent a "Trojan horse" for the following consideration:

– The developer does not analyze the advantages or disadvantages of a method, and usually uses the default method (this option is available in almost all mathematical applications). This is due to the use of the existing mentioned means of automating the process of continuous systems sampling and it is not always suitable;

– the lack of need to understand the mathematical basis of discretization processes reduces the professional level of the developer, limiting its activities only to ready-made (however proven) standard solutions;

– availability of ready-made automated solutions does not encourage the digital systems developer to look for non-standard or optimal (rational) solution and encourages the use of rather limited standard methods (default methods).

The basic element of the implementation of most control algorithms is the integration operation. Therefore, the use of numerical methods of integration is one of the basic methods of discretization of continuous systems with the subsequent introduction of the obtained dependencies in the software of digital systems. It is clear that the choice of a numerical integrator under such conditions will affect the behavior of the obtained discretized (sampled-data) system. Thus, it is necessary to investigate such an action to improve the efficiency of digital systems.

The object of study is the process of the continuous systems' sampling.

The subject of study is the properties of the sampling methods that based on the numerical integrators.

The purpose of the work is the studies of the frequency properties of the numeric integration methods for sampling to obtain the digital transfer function and increase the efficiency of the synthesized digital systems.

1 PROBLEM STATEMENT

Suppose given the continuous transfer function $W(s) = \dfrac{b_n s^n + \ldots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0}$ that can be described by the structure of observer canonical form using n continuous integrators. Such structure used as a prototype for discretization of the digital control systems.

For the continuous transfer function $W(s)$ the problem of their digital transfer function $W^*(z)$ finding can be presented as the problem of finding n-order digital approximation $W_n^*(z)$ of the used continuous integrators $\dfrac{1}{s}$ in observer canonical form. Frequency properties of the transfer functions can be found using substitution $s = j\cdot\omega$ for continuous systems and $z = \exp(j\cdot\omega\cdot h)$ for sampled-data systems by the next expressions:

– for magnitude: $A(\omega) = |W(j\cdot\omega)|$ (for continuous transfer function) and $A^*(\omega) = |W^*(\exp(j\cdot\omega\cdot h)|$ (for discrete transfer function);

– for phase: $\varphi(\omega) = \arg(W(j\cdot\omega))$ (for continuous transfer function) and $\varphi^*(\omega) = \arg(W^*(\exp(j\cdot\omega\cdot h))$ (for discrete transfer function).

Thus, the problem of the finding the best discrete approximation $W_n^*(z)$ of a continuous integrator can be focused to finding the minimum of the integral standard deviation between the frequency characteristics of the continuous system and the sampled by the selected nu-

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merical integrator $\int_0^{\omega_0}\left(A(j\omega)-A^*(j\omega h)\right)^2 d\omega \Rightarrow \min$ and $\int_0^{\omega_0}\left(\varphi(j\omega)-\varphi^*(j\omega h)\right)^2 d\omega \Rightarrow \min$.

2 REVIEW OF THE LITERATURE

The most popular developers' methods of the continuous prototype discretization (for example, an analog regulator) are traditional engineering methods: Tustin's method, z-transform etc., that applicable during the synthesis of the control algorithm of digital systems [1, 2, 3, 4]. This is due, firstly, to the well-known and proven methods of automatic control theory for continuous systems, which, moreover, are well implemented in mathematical applications (it is enough to mention the MATLAB's Control System Toolbox library again [5, 6]). Secondly, simple engineering methods of continuous systems discretization have already been developed and they do not require significant symbolical work from the developer (the same method of Tustin), and are also implemented in the applicable programs. Thus, there are all the prerequisites for the wide application of known traditional methods of discretization continuous systems by a wide range of engineers.

Most mathematical models of control systems can be reduced to block diagrams in which the basic element is an integrator [2]. For example, it applies to such well-known and popular regulators as the PID controller, which, according to the IEEE, used in more than 90% of industrial applications [7]. Conversion to the digital systems in the case of using this method of describing the model of the controller involves the transformation from analog (continuous) integrator to its discrete analog – a numerical integrator, which allows you to leave the controller structure unchanged. Accordingly, in this case, the main difference in the behavior of the obtained digital controller and its continuous prototype will be the difference in the behavior of the numerical integrator compared to the continuous.

One of the main tools for the analysis of control systems and their elements in the classical theory of automatic control is the use the frequency characteristics method, which allows a quite clearly analysis of the system behavior by its frequency response. Thus, considering the numerical integrator as a digital filter, it is possible to analyze its behavior by the control theory methods.

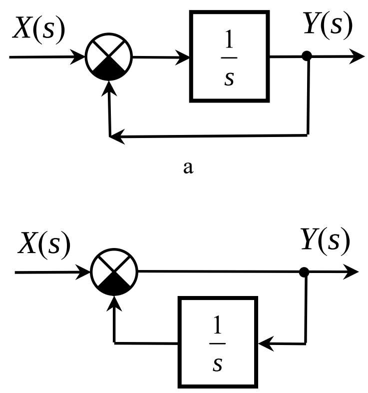
Analysis of the frequency characteristics of traditional numerical integrators using the z-transform method proposed in the classic (old but still actual) works by Elijah Jury [3, 4] and Julius Tou [8], although this method was used to only a few known numerical methods at the time: rectangles, trapezoids and Simpson. At the same time, the use of applied mathematics apparatus of frequency characteristics for the study of numerical methods for integrating ordinary differential equations (as proposed in the above-mentioned publications) has not been used before [9] and is not used in the future [10]. This is due to ignorance, misunderstanding or even non-acceptance by mathematicians the methods of automatic control's classical theory, for which the method of frequency response is basic. This case is because applied mathematics does not involve the consideration of numerical integrators as digital filters, although this method allows to obtain additional information about their behavior and to determine the possible impact on the digital control system [3, 4, 8].

The use of numerical integrators for continuous systems' discretization is not of interest in the case of modern methods of discrete systems synthesis too. This is due to the major use of state space and z-transform methods for this purpose [1, 2, 11]. The use of the simplest digital integrators to discretize continuous integrators in computer systems considered only as a partial case and a potential implementation [12].

3 MATERIALS AND METHODS

At the first stage of research, the analysis provided using two elementary structure (Fig. 1), that allows investigating the numerical integration method impact on forward channel and feedback of closed-loop systems.

This use, at first look, of primitive structures has its advantages: it allows minimizing the effects of the actual structure and complexity of the test system on the behavior of a continuous prototype discretized by a numerical integrator. This helps to abstract from the possible variants of structural transformations, highlighting only two basic combinations (Fig. 1), and allows us to focus only on the influence of the applied methods of discretization.



b

Figure 1 – Block diagrams of the tested structure:
a – integrator in forward channel; b – integrator in feedback channel

The practical implementation of the numerical integration in the real-time control system or real-time digital model (for prediction analysis of operating mode for power and energy systems [13] or electromechanical systems [14]) is possible for using explicit multi-step formulas only, that is due to the following factors:

– It is not possible to obtain information about the value of a function or controlled coordinate in the interval between the samples (reason to exclude from review the single-step methods).

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– It is not possible to obtain information of the next (future) sample of controlled coordinate (reason to exclude from review the multi-step implicit methods).

At present, Adam's formulas are still the most effective among the whole family of the explicit multi-step formulas [9, 10], so which was their choice for further analysis.

Obtaining process of the discrete transfer function for the digital integrator and fixed sampling time (integration step) h shown by this example of the 3rd-order explicit Adams formula [9, 10]

$$y_{i+1} = y_i + \frac{h}{12}\left(23x_i - 16x_{i-1} + 5x_{i-2}\right)$$

using method [3, 4, 15].

By the shifting theorem [3, 4, 8] $\begin{cases} y_i z = y_{i+1}; \\ y_i z^{-1} = y_{i-1}; \end{cases}$ a discrete transfer function of this integrator has a form

$$W_3^*(z) = \frac{\frac{h}{12}\left(23 - 16z^{-1} + 5z^{-2}\right)}{z-1} = \frac{\frac{h}{12}\left(23z^2 - 16z + 5\right)}{z^3 - z^2}$$

and may use for next analysis. Applying mentioned methods [3, 4, 8, 15], we can find discrete transfer functions for all of 1st–6th order digital integrators based on continuous integration by explicit Adams formulas. Obtained discrete transfer functions summarized in Table 1.

4 EXPERIMENTS

The founded discrete transfer functions (see Table 1) investigated by means of Control System Toolbox (MATLAB) [5, 6] for further analysis of frequency responses using standard function **bode**. The resulted frequency characteristics (Bode plots) are shown in Fig. 2 for mentioned 1st–6th orders explicit Adams formulas and sampling time $h = 0.1$ s with the continuous ideal

Table 1 – Discrete transfer functions of numerical integration operators

Integrator's order	Numerical integrator / Integrator's digital transfer function
1	$y_{i+1} = y_i + hx_i$
	$W_1^*(z) = \frac{h}{z-1}$
2	$y_{i+1} = y_i + \frac{h}{2}\left(3x_i - x_{i-1}\right)$
	$W_2^*(z) = \frac{\frac{h}{2}\left(3z-1\right)}{z^2 - z}$
3	$y_{i+1} = y_i + \frac{h}{12}\left(23x_i - 16x_{i-1} + 5x_{i-2}\right)$
	$W_3^*(z) = \frac{\frac{h}{12}\left(23z^2 - 16z + 5\right)}{z^3 - z^2}$
4	$y_{i+1} = y_i + \frac{h}{24}\left(55x_i - 59x_{i-1} + 37x_{i-2} - 9x_{i-3}\right)$
	$W_4^*(z) = \frac{\frac{h}{24}\left(55z^3 - 59z^2 + 37z - 9\right)}{z^4 - z^3}$
5	$y_{i+1} = y_i + \frac{h}{720}\left(1901x_i - 2774x_{i-1} + 2616x_{i-2} - 1274x_{i-3} + 251x_{i-4}\right)$
	$W_5^*(z) = \frac{\frac{h}{720}\left(1901z^4 - 2774z^3 + 2616z^2 - 1274z + 251\right)}{z^5 - z^4}$
6	$y_{i+1} = y_i + \frac{h}{1440}\left(4277x_i - 7923x_{i-1} + 9982x_{i-2} - 7298x_{i-3} + 2877x_{i-4} - 475x_{i-5}\right)$
	$W_6^*(z) = \frac{\frac{h}{1440}\left(4277z^5 - 7923z^4 + 9982z^3 - 7298z^2 + 2877z - 475\right)}{z^6 - z^5}$

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integrator to comparing. Note, that ideal continuous integrator has a descending magnitude characteristic with – 20 dB/decade and constant phase –90 deg.

It is worth noting two things:

1. The big difference in the frequency characteristics (note, both magnitude and phase shift) of high-order digital integrators compared to the ordinary continuous integrator on highest frequency especial.

2. The form of frequency characteristics is practically independent of the sampling period (see Fig. 3, Fig. 4).

The conclusions of the second point are quite expected, but the first point (regarding the behavior of higher-order methods) came as a surprise, as it is traditionally believed that the higher order of the integration formula causes the higher the accuracy of such an operation. However, the analysis of the obtained frequency characteristics shows the opposite – the higher order of the numerical integrator has the greater the amplitude and phase errors at high frequencies. Moreover, the nature of such errors becomes more and more nonlinear, with increasing order of the integration method. This does not give possibility add correction.

The next stage of the research is to study the behavior of numerical integrators using the method of frequency characteristics in the case of their use of their digital equivalents instead of the continuous integration for two case of test structural models. They show in Fig. 1 – to place the integrator in the direct channel of the closed system and in the case of placing the integrator in the feedback circuit. In this part of the researches influence of the feedback control system’s structure and the location of the integrator on the frequency characteristics of a closed system with a digital integrator is studied.

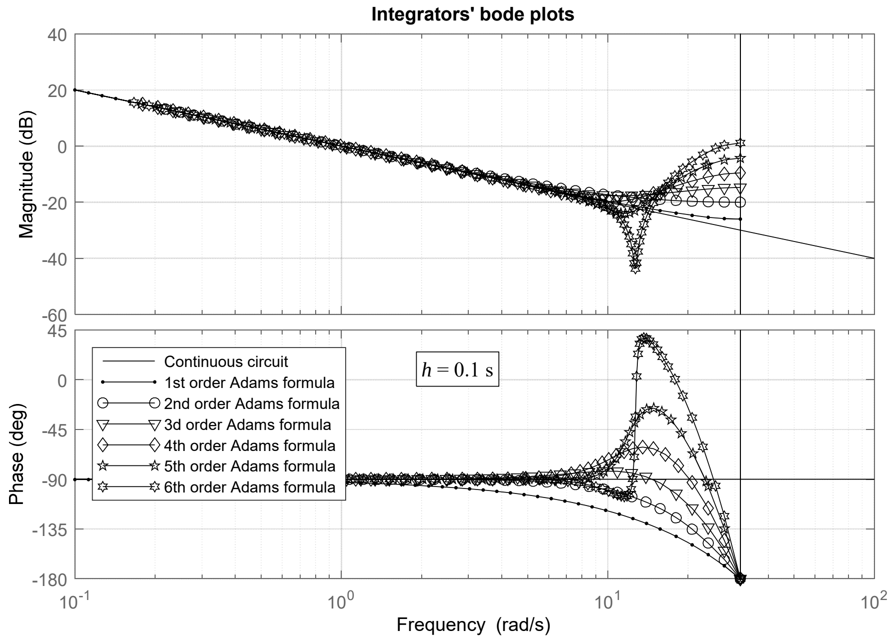

Figure 2 – Bode plots for 1st–6th-order numerical explicit integrators and sample time h = 0.1 s

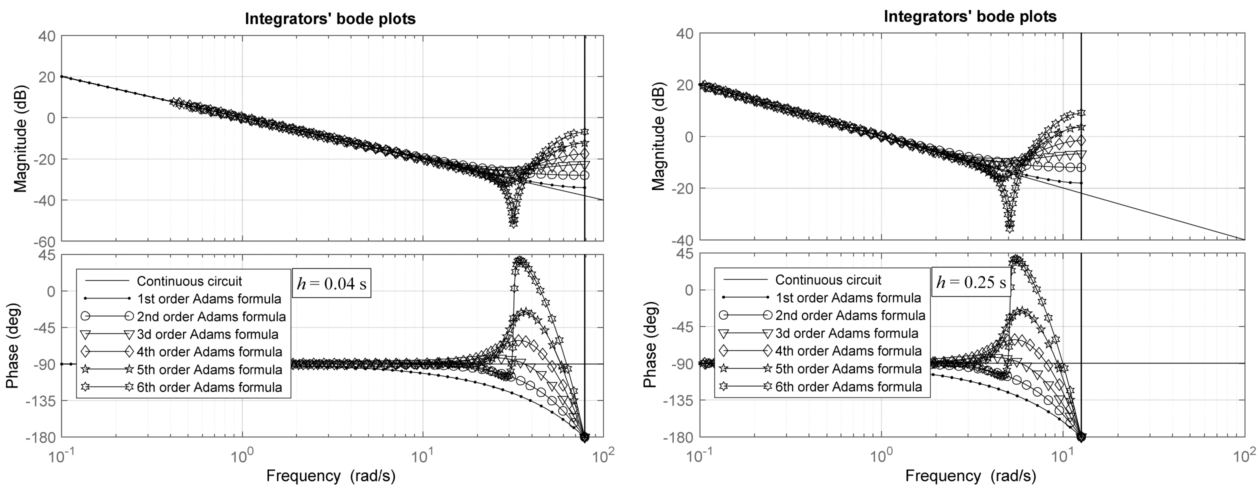

Figure 3 – Bode plots for 1st–6th-order numerical explicit integrators and sample times h = 0.04 s and h = 0.25 s

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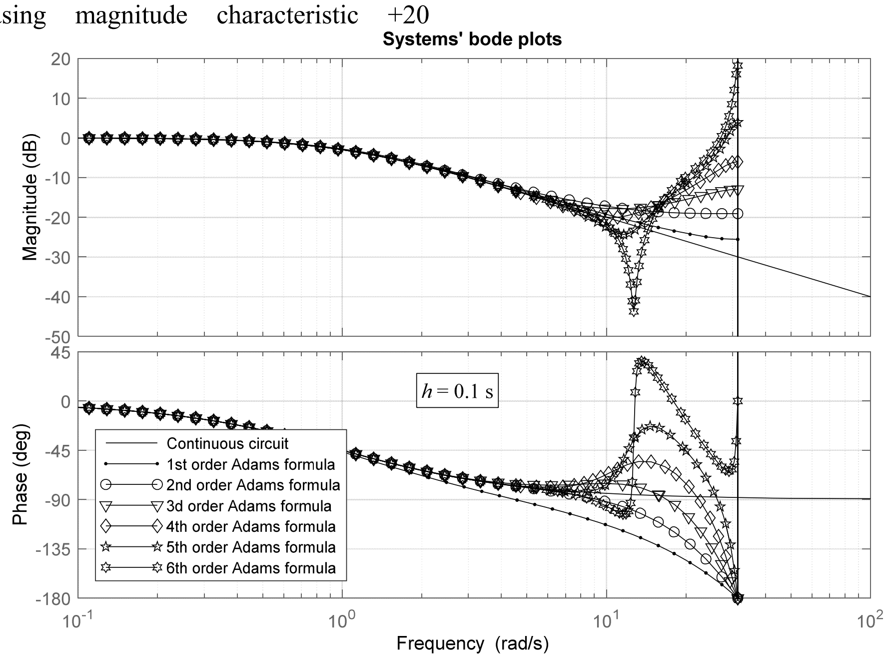
5 RESULTS

The obtained discrete transfer functions of integrators from Table 1 implemented to the digital models of structures Fig. 1 using again Control System Toolbox (MATLAB) [5, 6] for further analysis of frequency responses. Bode plots build for frequency workspace of the digital systems (1/10 of the sampling frequency [4, 16]).

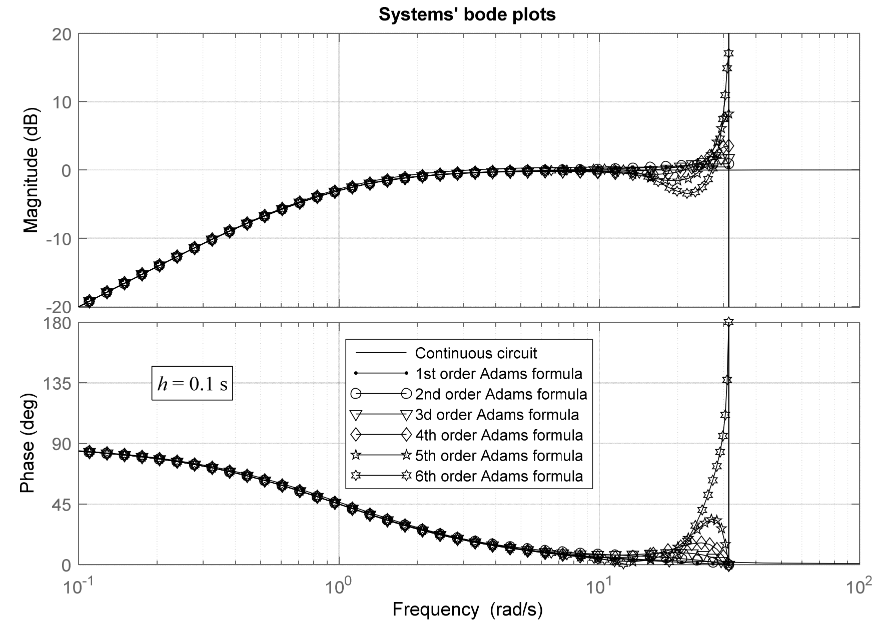
From Fig. 1, a – numerical integrator placed in forward channel. This is correspond to ordinary first-order inertial circuit with a descending magnitude characteristic with –20 dB/decade after the cutting frequency and smooth varying phase from zero to –90 deg.

Form Fig. 1, b – numerical integrator placed in feedback loop, that correspond to real differential operation with an increasing magnitude characteristic +20 dB/decade before the cutting frequency and smooth varying phase from +90 deg. to zero. Note that in this case, the negative effect of numerical integrators' high-order is unexpectedly reduced.

It is important that in the case of the second structure (Fig. 1, b) – the inclusion of a digital integrator in the feedback loop, the amplitude and phase errors in the use of higher-order formulas are smaller than in the case of placing a digital integrator in a direct channel. In addition, as noted earlier, it is possible to note a rather unexpected fact that given the behavior of both types of frequency response characteristics of test systems (Fig. 1) it is the formulas of high-order integration bring in errors in a closed system.



a



b

Figure 4 – Frequency characteristics for circuit Fig. 1:
a – using numerical integrators; b – using numerical integrators

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5 DISCUSSION

A somewhat unexpected result is that high-order numerical integrators make larger errors at higher frequencies range after discretization of continuous system. To some point, this opposes our understanding from applied mathematics about the higher accuracy of the high-order methods. Moreover, the use of low-order integrators (first and second orders) surprise turned out to be the most rational given the frequency errors.

What is the next step of research? The digital transfer functions of the synthesized digital systems get additional discrete zeros and poles comparing to the continuous systems (analog prototype). This is caused as a result of sampling continuous transfer function by digital integrator [17, 18]. In this case, additional zeros and poles from discrete transfer functions of numeric integrators add to obtained discrete transfer functions of discretized continuous system. It is clear, that a system with more zeros and poles (digital system) will behave differently than a system with fewer zeros and poles (continuous system).

CONCLUSIONS

Studies using the frequency response have shown:

– The behavior of the frequency characteristics of test systems (Fig. 1) showed that the use of low-order integrators, namely the first and second orders, is the most rational. In this case, the amplitude and phase's errors by discretization process will be the smallest, regardless of the structure of the synthesized digital system and the location of the numerical integrator.

– The inefficiency of using high-order numerical integrators to sample continuous systems is proven. Establishing the cause of this phenomenon requires additional research, in particular, to identify the possible impact of additional zeros and poles of the transfer functions of discrete integrators.

The scientific novelty of obtained results is the based on two simple circuits the researches of frequency properties of the numerical integrators in the digital control systems was made.

The practical significance of obtained results is the possibility to improve efficiency of the synthesized digital control systems.

Prospects for further research are the researches of the impact of the zeros and poles of the numerical integrators causes to additional zeros and poles of the obtained discrete transfer function.

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ЧАСТОТНІ ХАРАКТЕРИСТИКИ ЧИСЛОВИХ МЕТОДІВ ДИСКРЕТИЗАЦІЇ ЦИФРОВИХ СИСТЕМ КЕРУВАННЯ

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АНОТАЦІЯ

Актуальність. У статті проведено дослідження частотних властивостей явних багатокрокових числових інтеграторів, які використовують для дискретизації неперервних передавальних функцій в цифрових системах керування. Числові інтегратори в таких системах виступають як складові частини цифрових регуляторів.

Метою дослідження є аналіз поведінки явних числових інтеграторів різних порядків, які використовують для дискретизації неперервних систем, з метою вивчення їхнього впливу на властивості синтезованої цифрової системи.

Метод. Методи числового інтегрування розглянуто як цифрові фільтри, поведінку яких досліджено методом частотних характеристик. Для цього з використанням апарату z-перетворення знайдено їхні дискретні передавальні функції для частотного аналізу з використанням пакету Control Systems Toolbox математичного застосунку MATLAB. Для подальшого аналізу використано дві замкнені зворотним зв'язком тестові структури: з інтеграторами в прямому каналі та в колі зворотного зв'язку. Обидва варіанти структур досліджено методом частотних характеристик для дискретизації за допомогою числових інтеграторів 1–6 порядків.

Результати. Показана неефективність застосування числових інтеграторів високого порядку для дискретизації неперервних систем. З огляду на поведінку частотних характеристик тестових систем найраціональнішим є використання інтеграторів невисокого порядку, а саме – першого і другого. Встановлення причини такого явища потребує додаткових досліджень, зокрема, виявлення можливого впливу додаткових нулів та полюсів дискретних передавальних функцій числових інтеграторів.

Висновки. Використання інтеграторів низького порядку, а саме – першого та другого порядків, є найбільш раціональним для вибірки цифрових систем керування, також доведена неефективність використання чисельних інтеграторів високого порядку для дискретизації неперервних систем.

КЛЮЧОВІ СЛОВА: дискретні передавальні функції, лінійні системи, структурні моделі, теорія автоматичного керування, цифрові системи керування, частотні характеристики, числові інтегратори, z-перетворення.

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ЧАСТОТНІ ХАРАКТЕРИСТИКИ ЧИСЛОВИХ МЕТОДІВ ДИСКРЕТИЗАЦІЇ ЦИФРОВИХ СИСТЕМ КЕРУВАННЯ

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АННОТАЦИЯ

Актуальность. В статье проведено исследование частотных свойств явных многошаговых численных интеграторов, которые используются для дискретизации непрерывных передаточных функций в цифровых системах управления. Численные интеграторы в таких системах выступают как составные части цифровых регуляторов.

Целью исследования является анализ поведения явных численных интеграторов разных порядков, которые используют для дискретизации непрерывных систем, с целью изучения их влияния на свойства синтезируемой цифровой системы.

Метод. Методы численного интегрирования рассмотрены как цифровые фильтры, поведение которых исследовано методом частотных характеристик. Для этого с использованием аппарата z-преобразования найдены дискретные передаточные функции для частотного анализа с использованием пакета Control Systems Toolbox математического приложения MATLAB. Для дальнейшего анализа использованы две замкнутые обратной связью тестовые структуры: с интеграторами в прямом канале и в цепи обратной связи. Оба варианта структур исследованы методом частотных характеристик для дискретизации с помощью численных интеграторов 1–6 порядков.

Результаты. Показана неэффективность использования численных интеграторов высокого порядка для дискретизации непрерывных систем. Учитывая поведение частотных характеристик тестовых систем, наиболее рациональным является использование интеграторов невысокого порядка, а именно – первого и второго. Установление причины такого явления требует дополнительных исследований, в частности, выявления возможного влияния дополнительных нулей и полюсов дискретных передаточных функций численных интеграторов.

Выводы. Использование интеграторов низкого порядка, а именно – первого и второго порядков, наиболее рационально для выборки цифровых систем управления, также доказана неэффективность использования численных интеграторов высокого порядка для дискретизации непрерывных систем.

КЛЮЧЕВЫЕ СЛОВА: дискретные передаточные функции, линейные системы, структурные модели, теория автоматического управления, цифровые системы управления, частотные характеристики, численные интеграторы, z-преобразование.

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