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METHOD FOR WEIGHTS CALCULATION BASED ON INTERVAL MULTIPLICATIVE PAIRWISE COMPARISON MATRIX IN DECISION-MAKING MODELS

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ABSTRACT

Context. The pairwise comparison method is a component of several decision support methodologies such as the analytic hierarchy and network processes (AHP, ANP), PROMETHEE, TOPSIS and other. This method results in the weight vector of elements of decision-making model and is based on inversely symmetrical pairwise comparison matrices. The evaluation of the elements is carried out mainly by experts under conditions of uncertainty. Therefore, modifications of this method have been explored in recent years, which are based on fuzzy and interval pairwise comparison matrices (IPCMs).

Objective. The purpose of the work is to develop a modified method for calculation of crisp weights based on consistent and inconsistent multiplicative IPCMs of elements of decision-making model.

Method. The proposed modified method is based on consistent and inconsistent multiplicative IPCMs, fuzzy preference programming and results in more reliable weights for the elements of decision-making model in comparison with other known methods. The differences between the proposed method and the known ones are as follows: coefficients that characterize extended intervals for ratios of weights are introduced; membership functions of fuzzy preference relations are proposed, which depend on values of IPCM elements. The introduction of these coefficients and membership functions made it possible to prove the statement about the required coincidence of the calculated weights based on the "upper" and "lower" models. The introduced coefficients can be further used to find the most inconsistent IPCM elements.

Results. Experiments were performed with several IPCMs of different consistency level. The weights on the basis of the considered consistent and weakly consistent IPCMs obtained using the proposed and other known methods have determined the same rankings of the compared objects. Therefore, the results using the proposed method on the basis of such IPCMs do not contradict the results obtained for these types of IPCMs using other known methods. Rankings by the proposed method based on the considered highly inconsistent IPCMs are much closer to rankings based on the corresponding initial undisturbed IPCMs in comparison with rankings obtained using the known FPP method. The most inconsistent elements in the considered IPCMs are found.

Conclusions. The developed method has shown its efficiency, results in more reliable weights and can be used for a wide range of decision support problems, scenario analysis, priority calculation, resource allocation, evaluation of decision alternatives and criteria in various application areas.

KEYWORDS: interval multiplicative pairwise comparison matrix, consistency, expert judgements, fuzzy preference programming, decision support systems.

ABBREVIATIONS

DM is a decision-making;

PCM is a pairwise comparison matrix;

IPCM is an interval pairwise comparison matrix;

AHP is an analytic hierarchy process;

FPP is a fuzzy preference programming.

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NOMENCLATURE

A is an interval pairwise comparison matrix;

n is a dimension of matrix A, the number of pairwise compared objects;

a_{ij} is an interval number, an element of matrix A;

l_{ij} is a left end of interval a_{ij};

u_{ij} is a right end of interval a_{ij};

w is a weight vector;

w_i is an element of weight vector, $i = 1,...,n$;

P is a matrix describing a system of inequalities, $P \in R^{q \times n}$;

q is a number of rows in matrix P, $q = n(n-1)$;

P_k is the k-th row of matrix P;

$\mu_k(P_k w)$ is a membership function of a fuzzy preference relation;

d_k is a parameter in function $\mu_k(P_k w)$;

$\mu_{ij}(w)$ is a convex membership function of a fuzzy relation;

d_{ij} is a parameter in function $\mu_{ij}(w)$;

m_{ij} is the most probable predominance value from the interval $[l_{ij}, u_{ij}]$;

$\tilde{A}$ is a fuzzy feasible area;

T^{n-1} is a $(n-1)$-dimensional simplex;

w^* is a maximizing solution, the resulting vector of weights;

λ is an auxiliary variable;

$\lambda^* = \mu_{\tilde{A}}(w^*)$ is the degree of overall satisfaction of the decision maker with the optimal decision w^*;

λ_{ij}^* are coefficients, which characterize extended intervals for weights ratios.

INTRODUCTION

The problems of evaluating decision alternatives in semi-structured and unstructured subject areas have their own characteristics [1, 2]: the uniqueness, complexity, lack of optimality in the classical sense, multicriteria, data uncertainty, incompleteness of quantitative input information and the need to take into account qualitative judgements of a decision maker. Semi-structured and unstructured decision support problems are solved using the expert estimates and the principle of decomposition of a complex problem into subproblems [2–4]. Thus, a typical hierarchical or network model for a practical decision-making (DM) problem contains decision criteria, sub-criteria, goals, sub-goals and decision alternatives [4]. By performing pairwise comparisons of the criteria and alternatives, preference relations on the set of compared elements are built, and the pairwise comparison matrices (PCMs) are formed. Moreover, the pairwise comparison method represents a natural, general way of thinking of a person when making decisions [5, 6]. Based on PCMs, the weights of criteria and the priorities of alternatives in terms of each criterion are calculated. These priorities form partial solutions to the initial problem and are further aggregated into the resulting vector of global priorities or weights of the alternatives. In recent decades, considerable attention is given to development of the theory and applications of the Analytic Hierarchy Process (AHP) [7 – 10]. The rating, prioritization and resource allocation problems, problems of choosing the best decision alternatives, planning, scenario analysis and sustainable development problems are solved using the AHP.

One of the main elements of both the basic AHP and its modifications is the calculation of local priorities or weights using the pairwise comparison method and expert judgements. The complex psychological process of a comparative analysis made by a person in the presence of multiple criteria and decision alternatives is significantly influenced by various kinds of uncertainties [11]. The input information in the majority of multi-criteria DM problems is uncertaint and imprecise [12]. The uncertainty level of pairwise comparison results can increase with an increase of the number of compared elements [5, 13]. Therefore, many modifications of the AHP have been proposed for calculating weights based on uncertain estimates of pairwise comparisons: stochastic [14], fuzzy [15–35], or their combinations. Fuzzy multiplicative PCMs [15–18], interval multiplicative PCMs [19–29] and different types of fuzzy preference relations [30–35] on a set of pairwise compared alternatives are investigated. The weights are calculated on the basis of interval fuzzy preference relations [30–32] and interval multiplicative preference relations [18, 19, 34, 35].

The resulting local priorities (weights) of the decision criteria and alternatives on the basis of PCMs are significantly influenced by the consistency level of the initial expert pairwise comparison judgements. Methods for assessing the consistency level of PCMs are developed in [23, 26, 29, 31–35], methods for increasing the consistency level of PCMs are proposed and discussed in [27, 36], but this issue requires further research.

The object of study is an interval multiplicative PCM of elements of DM model.

The subject of study is a method for calculation of priorities (weights) of elements of DM model on the basis of interval multiplicative PCM and fuzzy preference programming.

The purpose of this work is to develop a modified method for calculation of crisp weights (priorities) based on consistent and inconsistent interval multiplicative PCM of elements of DM model.

1 PROBLEM STATEMENT

Let $A = \{a_{ij} = [l_{ij}, u_{ij}] \mid i, j = 1,...,n\}$ be an interval multiplicative pairwise comparison matrix (IPCM) of objects, for example, decision alternatives regarding their common characteristic (decision criterion), $0 < l_{ij} \le u_{ij}$, $l_{ij} = 1/u_{ji}$, $l_{ii} = u_{ii} = 1$. In is necessary to find the vector $w = \{w_i \mid i = 1,...,n\}$ of weights of the objects, such that $w_i \in R^+$, $\sum_{i=1}^{n} w_i = 1$.

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2 REVIEW OF THE LITERATURE

The first modifications of the pairwise comparison method using fuzzy set theory appeared in the early 1980s [37, 38]. Using extended binary arithmetic operations, a method for calculating fuzzy weights was proposed based on the PCMs with triangular fuzzy numbers [37]. This approach was further developed in [15–17], where the approximation of PCM elements a_{ij} by the (L–R)-type fuzzy numbers $\tilde{a}_{ij}$ was performed. Fuzzy numbers $\tilde{a}_{ij}$ represent the approximate value of preference of one alternative over the other and form a fuzzy PCM $\tilde{A}$. On the basis of the fuzzy matrix $\tilde{A}$, fuzzy weights are calculated, which approximate $\tilde{a}_{ij}$: $\tilde{a}_{ij} \approx \tilde{w}_i / \tilde{w}_j$.

Another approach, presented in [18, 20–22, 39], is based on the discretization of fuzzy numbers – elements of fuzzy PCM. Decomposition representation of fuzzy PCM using level sets results in interval PCMs. Models of least logarithmic squares [39], FPP [18, 19], goal programming [22], lower and upper approximation models [20], two-stage models [21, 40] and other have been proposed, which operate with IPCMs.

The first stage of the two-stage model [21] consists in finding the minimum deviations of the expert IPCM from the unknown consistent PCM. In the second stage, the weight vector is directly calculated based on the founded deviations. Another two-stage method for weight vector calculation is based on the interval additive PCM [40]. At the first stage of this method, programming models are built to obtain crisp consistent PCMs based on inconsistent interval PCMs. At the second stage, the resulting weights are calculated with different degrees of confidence.

The FPP method for calculation of crisp weights is based on the IPCM and fuzzy mathematical programming [18]. The FPP method requires the setting of additional parameters, which characterize the level of satisfaction of an expert or a decision maker with the calculated vector of weights. The problem is transformed to a classical fuzzy programming problem using the Bellman-Zade principle. The FPP method [18, 19] has significant limitation, which casts doubt on the validity of obtained results. It is the sensitivity of results by the FPP method to the renumbering of the compared objects.

The analysis of modifications of the pairwise comparison method, analytical hierarchy and network methods, which use the fuzzy sets, has shown that in these methods little attention is given to methods for increasing the consistency level of expert pairwise comparison judgements represented by interval and fuzzy PCMs.

3 MATERIALS AND METHODS

IPCM A is inversely symmetric: $a_{ji} = 1/a_{ij}$, $\forall i, j = 1,...,n$. This is equivalent to the fulfillment of the condition $l_{ij} = 1/u_{ji}$ for $\forall i, j = 1,...,n$. Thus, the elements of the upper triangular and lower triangular parts of the IPCM carry the same information about the values of preference on a set of compared objects.

Definition. IPCM A is called consistent, if a weight vector w exists, which satisfies the conditions $w_i \in R^+$, $\sum_{i=1}^{n} w_i = 1$ [20, 21]:

$$l_{ij} \le w_i / w_j \le u_{ij}, \quad i = 1,2,...,n-1, \; j = 2,3,...,n, \; i < j. \tag{1}$$

A weaker concept of a fuzzy consistent IPCM is also used, such that violation of inequalities (1) is allowed to some extent. According to the FPP method, a vector of weights w is calculated that satisfies inequalities (1) approximately.

Definition. IPCM A is called fuzzy consistent if a weight vector w exists, such that $w_i \in R^+$, $\sum_{i=1}^{n} w_i = 1$ [18, 19]:

$$l_{ij} \lesssim w_i / w_j \lesssim u_{ij}, \quad i = 1,2,...,n-1, \; j = 2,3,...,n, \; i < j, \tag{2}$$

where $\lesssim$ is a fuzzy preference relation.

Inequalities (2) are transformed in order to formulate a linear optimization problem for calculating the weight vector w:

$$w_i - u_{ij} w_j \lesssim 0, \quad -w_i + l_{ij} w_j \lesssim 0, \; i < j. \tag{3}$$

System (3), which contains $n(n-1)$ inequalities, can be written in the equivalent matrix form:

$$Pw \lesssim 0, \tag{4}$$

where $P \in R^{q \times n}$, $q = n(n-1)$.

In the initial FPP method [18], the following piecewise continuous membership function was used, representing the kth row of the inequality (4), for which $P_k w \lesssim 0$, $k = 1,2,...,q$:

$$\mu_k(P_k w) = \begin{cases} 1, & P_k w \le 0, \\ 1 - \dfrac{P_k w}{d_k}, & 0 < P_k w \le d_k, \\ 0, & P_k w > d_k; \end{cases} \tag{5}$$

where d_k is a parameter specifying the interval of approximate fulfillment of a crisp inequality $P_k w \le 0$, subscript k corresponds to the number of one-sided inequality in the constraint system (3).

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The membership function $\mu_k(P_k w)$ (5) describes the degree of satisfaction of the decision maker with some weight vector, according to the *k*th one-sided inequality (3). According to (5), the $\mu_k(P_k w)$ value:

– is zero, if the corresponding crisp constraint $P_k w \le 0$ is strongly violated;

– grows linearly, is positive and less than one, if the constraint $P_k w \le 0$ is approximately satisfied;

– is equal to one, if the constraint $P_k w \le 0$ is fully satisfied.

Since both constraints in (3) correspond to the same interval for a_{ij} given by the pair (i, j), a modified FPP model is proposed [19], such that functions (5) are represented in the form of the following convex membership functions:

$$\mu_{ij}(w_i, w_j) = \begin{cases} 1 - \dfrac{(-w_i + l_{ij} w_j)}{d_{ij}}, \dfrac{w_i}{w_j} \le m_{ij}, \\ 1 - \dfrac{(w_i - u_{ij} w_j)}{d_{ij}}, \dfrac{w_i}{w_j} \ge m_{ij}, \end{cases} \quad (5')$$

where d_{ij} is a parameter for interval $[l_{ij}, u_{ij}]$, m_{ij} is the most probable value of the preference from the interval $[l_{ij}, u_{ij}]$, namely the middle of this interval. The functions (5′) are as follows: $\mu_{ij}(w) : R^{n+} \to [-\infty, \mu^{\max}]$.

The Bellman-Zadeh mathematical programming approach is uses in [18] for calculating the weight vector on the basis of IPCM $A = \{a_{ij} = [l_{ij}, u_{ij}] \mid i, j = 1,...,n\}$. Let $\mu_k(P_k w)$, $k = 1,2,...,q$ be the membership functions (5) of fuzzy constraints $P_k w \lessapprox 0$ on the $(n-1)$-measuring simplex $T^{n-1} = \left\{(w_1,...,w_n) \mid w_i \in R^+, \sum_{i=1}^{n} w_i = 1\right\}$.

Definition. A fuzzy feasible region $\tilde{A}$ on T^{n-1} is a fuzzy set, which is an intersection of fuzzy constraints (4) [18]:

$$\mu_{\tilde{A}}(w) = \left\{ \min_{k=1,...,q} \{\mu_k(P_k w)\} \mid w_i \in R^+, \sum_{i=1}^{n} w_i = 1 \right\}. \quad (6)$$

The parameters d_k values in (5) should be chosen large enough to obtain a non-empty area $\tilde{A}$ (6). In this case, fuzzy set $\tilde{A}$ (6) on T^{n-1} is convex. If fuzzy constraints (3) are defined using membership functions (5′), the requirement of non-emptiness of the set $\tilde{A}$ (6) can be weakened, and $\tilde{A}$ is defined as follows:

$$\mu_{\tilde{A}}(w) = \min_{\substack{i=1,2,...,n-1, \\ j=2,3,...,n}} \{\mu_{ij}(w)\}. \quad (6')$$

Obviously, the fuzzy set $\tilde{A}$ (6′) is convex. In contrast to the function (6), the function (6′) can take negative values for a strongly inconsistent IPCM in a case of too small values of the parameter d_k.

The area $\tilde{A}$ defined by (6) or (6′) indicates the overall satisfaction for the decision maker with a certain crisp weight vector. The maximizing solution is the resulting weight vector.

Definition. The maximizing solution of the problem is the vector [18]:

$$w^* = \arg \max_{w \in T^{n-1}} \min_k \mu_{\lessapprox_k}(w), \quad (7)$$

where $\lessapprox_k$ presents the *k*-th fuzzy inequality in (4), namely $P_k w \lessapprox 0$.

Depending on the choice of the membership function $\mu_{\lessapprox_k}(w)$ in (7), the maximizing solution can be calculated in one of the following ways:

$$w^* = \arg \max_w \left\{ \min_{k=1,...,q} \{\mu_k(P_k w)\} \mid w_i \in R^+, \sum_{i=1}^{n} w_i = 1 \right\}, \quad (8)$$

where values $\mu_k(P_k w)$ are defined as in (5), or

$$w^* = \arg \max_{w \in T^{n-1}} \min_{i,j} \{\mu_{ij}(w)\}, \quad (8')$$

where $\mu_{ij}(w)$ are defined as in (5′).

In case when all fuzzy constraints are determined using the membership functions (5), at least one point w^* is present in T^{n-1}, which has the maximum degree of membership in the set $\tilde{A}$. However, the solution to problem (8) will not necessarily be unique. The optimization problem (8′), in turn, has a unique solution.

By introducing a variable λ, the problem (7) for resulting weight vector calculation is presented as a following fuzzy mathematical programming problem:

$$\max \lambda \quad (9)$$

under constraints

$\lambda \le \mu_k(P_k w)$, $k = 1,2,...,q$,

where values $\mu_k(P_k w)$ are defined as in (5).

Problem (9) can also be written as

$$\max \lambda \quad (9')$$

under constraints

$\lambda \le \mu_{ij}(w)$, $i = 1,2,...,n-1$, $j = 2,3,...,n$, $i < j$,

where $\mu_{ij}(w)$ are defined as in (5′).

The membership functions (5′) are linear with respect to variables $w_1,...,w_n$, so (9′) can be written as a following linear programming problem [18]:

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$$\max \lambda \tag{10}$$

under constraints

$d_{ij}\lambda + w_i - u_{ij} w_j \le d_{ij}$,

$d_{ij}\lambda - w_i + l_{ij} w_j \le d_{ij}$,

$i = 1,2,...,n-1,\ j = 2,3,...,n$, $i < j$,

$\sum_{k=1}^{n} w_k = 1$, $w_k > 0$, $k = 1,2,...,n$.

The pair (w^*, λ^*) forms the solution to problem (10), where w^* is the resulting weight vector, and $\lambda^* = \mu_{\tilde{A}}(w^*)$ is the value of maximum membership in the aggregated set $\tilde{A}$. The value λ^* measures the degree of expert's (decision maker's) overall satisfaction with the optimal solution w^*. Therefore $\lambda^* = \mu_{\tilde{A}}(w^*)$ is defined as an indicator of consistency level of expert (decision maker) judgments [18].

If IPCM is consistent, then $\lambda^* \ge 1$. Indeed, according to the definition of consistent IPCM, there is a vector w, $w_i \in R^+$, $\sum_{i=1}^{n} w_i = 1$, satisfying $l_{ij} \le w_i / w_j \le u_{ij}$, $i < j$. Therefore, $P_k w \le 0$ and $\mu_k(P_k w) \ge 1$ are carried out for every $k = 1,2,...,q$. Consequently, $\mu_{\tilde{A}}(w) = \left\{ \min_{k=1,...,m} (\mu_k(P_k w)) \mid w_i \in R^+, \sum_{i=1}^{n} w_i = 1 \right\} \ge 1$, and $\lambda^* \ge 1$, where $w^* = w$.

Next, let us consider an inconsistent IPCM and a case when a weight vector w exists, such that the system of inequalities (2) is satisfied. Then, there is such $k = 1,2,...,q$ that $P_k w > 0$ and $\mu_k(P_k w) < 1$. Consequently, $\mu_{\tilde{A}}(w) < 1$ and $\lambda^* < 1$. By choosing large enough values of d_k parameter, a positive value λ^* can be achieved. It can be shown that $\lambda^* > 0$ if $d_k \ge 1$. Thus, for an inconsistent IPCM we have $\lambda^* \in (0,1)$, and λ^* depends on the inconsistency level of IPCM and d_k values.

Let us consider problems (9), (9′) and (10) for calculating the resulting weight vector. In these problems, the weights are defined based on expert's estimates presented in the upper triangular part of the IPCM. As noted above, the IPCM has the property of inverse symmetry. So, the sets of elements of the IPCM, which form the upper and lower triangular parts of the IPCM, carry the same information about the unknown weights. Therefore, the solution based on problem (10) and judgments $[l_{ij}, u_{ij}]$, $i < j$ must coincide with the solution of the same problem based on judgments $[l_{ij}, u_{ij}]$, $i > j$. However, as shown in examples below, the weight vectors according to the "upper" and "lower" FPP models [18, 19] do not coincide with each other.

"Upper" FPP model: $\max \lambda$

under constraints

$\lambda \le \mu_{ij}(w)$, $i = 1,2,...,n-1,\ j = 2,3,...,n$, $i < j$.

"Lower" FPP model: $\max \lambda$

under constraints

$\lambda \le \mu_{ji}(w)$, $j = 1,2,...,n-1,\ i = 2,3,...,n$, $j < i$.

The above "upper" and "lower" FPP models have additional constraints $\sum_{k=1}^{n} w_k = 1$, $w_k > 0$, $k = 1,2,...,n$.

The difference in the results based on the "upper" and "lower" FPP models means that the FPP solution [18, 19] depends on how the compared elements are numbered. In this paper, a new method is proposed that does not have such drawback.

Consider membership functions (5′), where the middle of the interval $[l_{ij}, u_{ij}]$ is chosen as the most probable preference value m_{ij}. Indeed, an expert or a decision maker is not equally satisfied with all the ratios of the resulting weights inside or outside the intervals (2) for $a_{ij}, i < j$. Obviously, he/she would prefer a solution around the middle of each interval than a solution on the given boundaries of the intervals for a_{ij}. On the other hand, in case of inconsistent IPCM, any ratio of resulting weights w_i / w_j that is outside the interval a_{ij} close to the boundaries of this interval is more preferable than the solution far from these boundaries. Therefore, the degree of satisfaction of an expert or a decision maker with the ratio of resulting weights should be presented as a monotone continuous function, gradually increasing towards the middle of the interval.

However, the use of the middle of interval $m_{ij} = \frac{l_{ij} + u_{ij}}{2}$ seems to be justified only for the case $1 \le l_{ij} \le u_{ij}$. The middle of interval as the most probable preference value is not acceptable for all intervals in IPCMs. For example, if $l_{ij} \le u_{ij} \le 1$ then the value $m_{ij} = (\frac{(l_{ij})^{-1} + (u_{ij})^{-1}}{2})^{-1} = \frac{2}{(l_{ij})^{-1} + (u_{ij})^{-1}}$ should be considered as the most probable value of preference. In the third possible case $l_{ij} \le 1 \le u_{ij}$, it is reasonable to choose the value $m_{ij} = \frac{u_{ij} + 1}{(l_{ij})^{-1} + 1}$ as the most probable preference value.

Thus, it is proposed to determine the value m_{ij} depending on $[l_{ij}, u_{ij}]$ as follows:

– if $1 \le l_{ij} < u_{ij}$, then $m_{ij} = \frac{l_{ij} + u_{ij}}{2}$, (11)

– if $l_{ij} < u_{ij} \le 1$, then $m_{ij} = \frac{2}{(l_{ij})^{-1} + (u_{ij})^{-1}}$, (12)

– if $l_{ij} < 1 < u_{ij}$, then $m_{ij} = \frac{u_{ij} + 1}{(l_{ij})^{-1} + 1}$. (13)

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According to the new values m_{ij} (11) – (13), the membership functions (5′) are changed as follows:

1) if $1 \le l_{ij} < u_{ij}$, then

$$\mu_{ij}(w_i, w_j) = \begin{cases} 1 - \frac{(-w_i + l_{ij} w_j)}{d_{ij}}, \frac{w_i}{w_j} \le m_{ij}, \\ 1 - \frac{(w_i - u_{ij} w_j)}{d_{ij}}, \frac{w_i}{w_j} \ge m_{ij}, \end{cases} \quad (14)$$

where m_{ij} is calculated as in (11),

2) if $l_{ij} < u_{ij} \le 1$, then

$$\mu_{ij}(w_i, w_j) = \begin{cases} 1 - \frac{u_{ij}(-w_i + l_{ij} w_j)}{d_{ij}}, \frac{w_i}{w_j} \le m_{ij}, \\ 1 - \frac{l_{ij}(w_i - u_{ij} w_j)}{d_{ij}}, \frac{w_i}{w_j} \ge m_{ij}, \end{cases} \quad (15)$$

where m_{ij} is calculated as in (12),

3) if $l_{ij} < 1 < u_{ij}$, then

$$\mu_{ij}(w_i, w_j) = \begin{cases} 1 - \frac{(-w_i + l_{ij} w_j)}{(l_{ij} - 1)(u_{ij} - 1) + d_{ij}}, \frac{w_i}{w_j} \le m_{ij}, \\ 1 - \frac{(w_i - u_{ij} w_j)}{(l_{ij} - 1)(u_{ij} - 1) + d_{ij}}, \frac{w_i}{w_j} \ge m_{ij}, \end{cases} \quad (16)$$

where m_{ij} is calculated as in (13), and the value of d_{ij} parameter is proposed to be equal to $d_{ij} = u_{ij} - l_{ij}$ for all the above cases 1) – 3).

In order to preserve the structure of expert (decision-maker) preferences, to further assess and improve the consistency level of interval expert pairwise comparison judgments, a modified method is proposed for calculating a crisp vector of priorities or weights for elements of DM model. The method includes the following two models:

Proposed "upper" model:

$$\max \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \lambda_{ij}, \quad (17)$$

– under constraints

$$\lambda_{ij} \le \mu_{ij}(w),\ i = 1,2,...,n-1,\ j = 2,3,...,n,\ i < j,$$

where $\mu_{ij}(w)$ values are calculated as in (14)–(16).

Proposed "lower" model:

$$\max \sum_{j=1}^{n-1} \sum_{i=j+1}^{n} \lambda_{ij} \quad (18)$$

– under constraints

$$\lambda_{ji} \le \mu_{ji}(w),\ j = 1,2,...,n-1,\ i = 2,3,...,n,\ j < i,$$

where $\mu_{ij}(w)$ are calculated as in (14) – (16).

The "upper" and "lower" models (17) and (18) have additional constraints $\sum_{k=1}^{n} w_k = 1$, $w_k > 0$, $k = 1,2,...,n$. A similar approach to constructing the objective function as a sum $\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (p_{ij} + q_{ij})$ is used in another TLGP method [21], where p_{ij}, q_{ij} are variables that form unknown extended intervals. TLGP is a two-stage method and results in optimization problems with non-linear constraints.

Statement. The weight vector based on the "lower" model (18) is equal to the weight vector based on the "upper" model (17).

Proof. Suppose that the constraints $\lambda_{ij} \le \mu_{ij}(w)$, $\forall i < j$ of the "upper" model are satisfied. Let us show that for each of the conditions 1–3 for the ends of the interval $[l_{ij}, u_{ij}]$ in (14)–(16), the objective functions of the "upper" and "lower" models coincide and the constraints of the "upper" and "lower" models are also coincide.

Consider $l_{ij} \lesssim a_{ij} \lesssim u_{ij}$ and the coresponding condition $\lambda_{ij} \le \mu_{ij}(w)$ for $i < j$. Let $1 \le l_{ij} < u_{ij}$ be satisfied for $i < j$. Then the constraint $\lambda_{ij} \le \mu_{ij}(w)$, $i < j$ in (17) is equivalent to the fulfillment of two inequalities:

$$d_{ij}\lambda_{ij} + w_i - u_{ij} w_j \le d_{ij},\ i < j, \quad (19)$$

$$d_{ij}\lambda_{ij} - w_i + l_{ij} w_j \le d_{ij},\ j < i. \quad (20)$$

Consider $\frac{1}{u_{ij}} \lesssim a_{ji} \lesssim \frac{1}{l_{ij}}$ and the coresponding condition $\lambda_{ji} \le \mu_{ji}(w)$ for $j < i$. In this case, $l_{ji} < u_{ji} \le 1$, $j < i$, and inequality $\lambda_{ji} \le \mu_{ji}(w)$, $j < i$ is equivalent to the following two:

$$d_{ji}\lambda_{ji} + l_{ji} w_j - l_{ji} u_{ji} w_i \le d_{ji},\ j < i, \quad (19')$$

$$d_{ji}\lambda_{ji} - \frac{1}{l_{ij}} w_j + \frac{1}{l_{ij}} \frac{1}{u_{ij}} w_i \le d_{ji},\ j < i. \quad (20')$$

The inequality (19′) is written as

$$d_{ji}\lambda_{ji} + \frac{1}{u_{ij}} w_j - \frac{1}{u_{ij}} \frac{1}{l_{ij}} w_i \le d_{ji} \iff$$

$$d_{ji} u_{ij} l_{ij} \lambda_{ji} + l_{ij} w_j - w_i \le d_{ji} u_{ij} l_{ij}. \quad (19'')$$

The inequality (20′) is written as

$$d_{ji} u_{ij} l_{ij} \lambda_{ji} - u_{ij} w_j + w_i \le d_{ji} u_{ij} l_{ij}. \quad (20'')$$

Note that inequality (19″) is equivalent to (20), and (20″) is equivalent to (19), if $d_{ij} = d_{ji} u_{ij} l_{ij}$ and $\lambda_{ij} = \lambda_{ji}$.

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For example, when choosing $d_{ij} = u_{ij} - l_{ij}$, equality $d_{ij} = d_{ji} u_{ij} l_{ij}$ is satisfied, since $d_{ji} = u_{ji} - l_{ji} = \frac{u_{ij} - l_{ij}}{u_{ij} l_{ij}}$.

The proof is similar in other two cases: $l_{ij} < u_{ij} \le 1$, if $i < j$, and $l_{ij} < 1 < u_{ij}$, if $i < j$.

4 EXPERIMENTS

Let us consider several matrices of different consistency levels (Tables 1 – 4), which other researchers have analyzed using other methods. IPCM *A*1 [19] (Table 1) is not consistent by definition, and indicator $\lambda^* = 0.9583$. However, the IPCM *A*1 is weakly consistent, since $(a_{12} > 1) \wedge (a_{23} > 1) \Rightarrow (a_{13} > 1)$.

IPCMs *A*2 [21, 23, 26, 29] (Table 2) and *A*3 [22] (Table 3) are consistent by definition. Consistency is also confirmed by values $\lambda^* = 1.0435$ and $\lambda^* = 1.0313$ of these IPCMs. Consistency is a stronger concept than weak consistency. So, IPCMs *A*2 and *A*3 are weakly consistent.

PCM *B* (Table 4) has proposed in [10] to solve the sprint planning problem. PCM *B* is not consistent by definition, since the transitivity condition $b_{ij} = b_{ik} b_{kj}$ is not satisfied for all $i, j, k = 1, ..., n$. However, the consistency ratio CR = 0.083 is less than its threshold value, so PCM *B* is admissibly inconsistent and can be used for weights calculation. PCM *B* is weakly consistent, since $(b_{ik} > 1) \wedge (b_{kj} > 1) \Rightarrow (b_{ij} > 1)$ for all $i, j, k = 1, ..., 5$.

IPCM A4 (Table 5) is the result of *B* fuzzification for the application of the proposed method. IPCM *A*4 is consistent, since $\lambda^* = 1.0244$.

Let us disturb individual elements of IPCM *A*4 in order to increase the inconsistency level of this matrix. Inconsistent IPCMs often occur in practice. Therefore, it is interesting to investigate the implementation of the proposed method also on IPCMs of this class. So, only the element $a_{21} := [2, 4]$ of IPCM *A*4 and the symmetric element are changed. Resulting IPCM *A*5 is shown in Table 6. Such IPCM can be, for example, the result of an accidental expert error. The value $\lambda^* = 0.9500$ of IPCM *A*5 indicates an increase in inconsistency compared to *A*4. The same conclusion is based on the consistency ratio CR = 0.650 (after *A*5 defuzzification). In addition, IPCM *A*5 is not weakly consistent, which means it has a cycle and an undesirable violation of ordinal transitivity on the set of its elements.

The next IPCM A6 (Table 7) coincides with *A*4 except for the element $a_{24} := [3, 5]$ and symmetrical to it $a_{42} := [1/5, 1/3]$. Values $\lambda^* = 0.8611$ and CR = 1.002 (after defuzzification of *A*6) for IPCM *A*6 mean that this matrix is the most strongly inconsistent in comparison with *A*4, *A*5 and *A*1 – *A*3.

Table 1 – Weakly consistent IPCM *A*1 [19]

1	[1, 2]	[8, 9]
[1/2, 1]	1	[2, 3]
[1/9, 1/8]	[1/3, 1/2]	1

Table 2 – Consistent IPCM *A*2 [21, 23 , 26, 29]

1	[2, 5]	[2, 4]	[1, 3]
[1/5, 1/2]	1	[1, 3]	[1, 2]
[1/4, 1/2]	[1/3, 1]	1	[1/2, 1]
[1/3, 1]	[1/2, 1]	[1, 2]	1

Table 3 – Consistent IPCM *A*3 [22]

1	[1, 3]	[3, 5]	[5, 7]	[5, 9]
[1/3, 1]	1	[1, 4]	[1, 5]	[1, 4]
[1/5, 1/3]	[1/4, 1]	1	[1/5, 5]	[2, 4]
[1/7, 1/5]	[1/5, 1]	[1/5, 5]	1	[1, 2]
[1/9, 1/5]	[1/4, 1]	[1/4, 1/2]	[1/2, 1]	1

Table 4 – PCM *B* [10]

1	3	3	1/2	2
1/3	1	1/2	1/4	1/2
1/3	2	1	1/3	1/2
2	4	3	1	2
1/2	2	2	1/2	1

Table 5 – Consistent IPCM *A*4

[1, 1]	[2, 4]	[2, 4]	[1/3, 1]	[1, 3]
[1/4, 1/2]	[1, 1]	[1/3, 1]	[1/5, 1/3]	[1/3, 1]
[1/4, 1/2]	[1, 3]	[1, 1]	[1/4, 1/2]	[1/3, 1]
[1, 3]	[3, 5]	[2, 4]	[1, 1]	[1, 3]
[1/3, 1]	[1, 3]	[1, 3]	[1/3, 1]	[1, 1]

Table 6 – Weakly inconsistent IPCM *A*5

[1, 1]	**[1/4, 1/2]**	[2, 4]	[1/3, 1]	[1, 3]
[2, 4]	[1, 1]	[1/3, 1]	[1/5, 1/3]	[1/3, 1]
[1/4, 1/2]	[1, 3]	[1, 1]	[1/4, 1/2]	[1/3, 1]
[1, 3]	[3, 5]	[2, 4]	[1, 1]	[1, 3]
[1/3, 1]	[1, 3]	[1, 3]	[1/3, 1]	[1, 1]

Table 7 – The most strongly inconsistent IPCM *A*6

[1, 1]	[2, 4]	[2, 4]	[1/3, 1]	[1, 3]
[1/4, 1/2]	[1, 1]	[1/3, 1]	**[3, 5]**	[1/3, 1]
[1/4, 1/2]	[1, 3]	[1, 1]	[1/4, 1/2]	[1/3, 1]
[1, 3]	**[1/5, 1/3]**	[2, 4]	[1, 1]	[1, 3]
[1/3, 1]	[1, 3]	[1, 3]	[1/3, 1]	[1, 1]

5 RESULTS

In the following Tables 8–13, weights are shown, which are calculated using different methods based on the above-considered IPCMs. The λ^*_{ij} values for these IPCMs (Table 14) are further used to find the most inconsistent elements in the IPCMs. Calculated weights are also compared with weights by the Saaty's eigenvector method (EM) on the basis of defuzzified IPCMs *A*5 and *A*6 (Tables 12 and 13).

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6 DISCUSSION

As follows from Tables 8–13, the weights based on the "lower" FPP model [18, 19] differ from the weights based on the "upper" FPP model [18, 19] for all the considered IPCMs. The "upper" and "lower" models proposed in this paper lead to the same resulting weight vectors.

Table 8 – Weights obtained using different methods based on IPCM *A*1 from Table 1

	Weights			
	FPP model		Proposed model	
	"upper"	"lower"	"upper"	"lower"
w_1	0.6250	0.6154	0.5604	0.5604
w_2	0.2917	0.2967	0.3736	0.3736
w_3	0.0833	0.0879	0.0659	0.0659

Table 9 – Weights obtained using different methods based on consistent IPCM *A*2 from Table 2

	Interval weights			
	Li&Tong's method [26]	TLGP [21]	Liu's method [23]	Kuo's method [29]
w_1	[1.5540, 2.5329]	[1.6818 2.4495]	[1.4142, 2.7832]	0.4499
w_2	[0.7348, 1.1977]	[0.7598 1.1067]	[0.8409, 1.0466]	0.2127
w_3	[0.5105, 0.7442]	[0.5000 0.8409]	[0.5373, 0.7071]	0.1398
w_4	[0.7219, 1.0525]	[0.6866 1.0000]	[0.6389, 1.1892]	0.1977

Table 9 continuation – Weights obtained using different methods based on consistent IPCM *A*2 from Table 2

	Weights			
	FPP model		Proposed model	
	"upper"	"lower"	"upper"	"lower"
w_1	0.4783	0.4675	0.4000	0.4000
w_2	0.2174	0.2078	0.2667	0.2667
w_3	0.1304	0.1429	0.1333	0.1333
w_4	0.1739	0.1818	0.2000	0.2000

Table 10 – Weights obtained using different methods based on consistent IPCM *A*3 from Table 3

	Interval weights		
	LUAM-1	LUAM-2	GPM
w_1	[0.2909, 0.4091]	[0.4225, 0.5343]	0.4527
w_2	[0.1364, 0.2909]	[0.1781, 0.2817]	[0.1396, 0.3320]
w_3	[0.0273, 0.1818]	0.1409	[0.0818, 0.2097]
w_4	[0.0364, 0.1364]	[0.0763, 0.0845]	[0.0591, 0.1347]
w_5	[0.0455, 0.1364]	0.0704	0.0633

Table 10 continuation – Weights obtained using different methods based on consistent IPCM *A*3 from Table 3

	Weights			
	FPP model		Proposed model	
	"upper"	"lower"	"upper"	"lower"
w_1	0.5000	0.5089	0.4855	0.4855
w_2	0.1875	0.1809	0.2428	0.2428
w_3	0.1563	0.1583	0.1214	0.1214
w_4	0.0937	0.0840	0.0809	0.0809
w_5	0.0625	0.0679	0.0694	0.0694

Table 11 – Weights obtained using different methods based on consistent IPCM *A*4 from Table 5

	Weights					
			FPP model		Proposed model	
	g_i [10]	Normalised g_i	"upper"	"lower"	"upper"	"lower"
w_1	1.55	0.2660	0.2683	0.3000	0.2647	0.2647
w_2	0.46	0.0790	0.0976	0.1000	0.0882	0.0882
w_3	0.64	0.1110	0.1220	0.1250	0.1176	0.1176
w_4	2.17	0.3720	0.3659	0.3250	0.3529	0.3529
w_5	1.00	0.1720	0.1463	0.1500	0.1765	0.1765

Table 12 – Weights obtained using different methods based on weakly inconsistent IPCM *A*5 from Table 6

	Weights				
		FPP model		Proposed model	
	EM	"upper"	"lower"	"upper"	"lower"
w_1	0.1810	0.1167	0.1149	0.1935	0.1935
w_2	0.1290	0.1333	0.1609	0.0968	0.0968
w_3	0.1170	0.0833	0.1264	0.1290	0.1290
w_4	0.3920	0.5000	0.4138	0.3871	0.3871
w_5	0.1810	0.1667	0.1839	0.1935	0.1935

Table 13 – Weights obtained using different methods based on the most strongly inconsistent IPCM *A6* from Table 7

	Weights				
		FPP model		Proposed model	
	EM	"upper"	"lower"	"upper"	"lower"
w_1	0.1810	0.2500	0.2500	0.3000	0.3000
w_2	0.1290	0.1944	0.2143	0.1000	0.1000
w_3	0.1170	0.1944	0.1250	0.1000	0.1000
w_4	0.3920	0.1111	0.1607	0.3000	0.3000
w_5	0.1810	0.2500	0.2500	0.2000	0.2000

Table 14 – Values λ_{ij}^* for IPCMs of different consistency

IPCM	Values λ_{ij}^*
Weakly consistent *A*1 from Table 1	[1.1868, 1.0330, 0.8242]
Consistent *A2* from Table 2	[0.9556, 1.0667, 1.1000, 1.0667, 1.0667, 1.0667]
Consistent *A3* from Table 3	[1.1214, 1.0607, 1.0405, 1.0347, 1.0405, 1.0405, 1.0116, 1.0354, 0.9913, 1.0116]
Consistent *A4* from Table 5	[1.0441, 1.0147, 1.0441, 1.0441, 1.0147, 1.0441, 1.0441, 1.0588, 1.0294, 1.0882]
Weakly inconsistent *A5* from Table 6	[**0.8548**, 0.9677, 1.0968, 1.0000, 1.0161, 1.0484, 1.0484, 1.0645, 1.0323, 1.0968]
Weakly inconsistent *A6* from Table 7	[1.05, 1.05, 1.00, 1.05, 1.00, **0.6**, 1.05, 1.05, 1.05, 1.05]

Analysis of Tables 8–11 shows that the weights obtained by different methods on the basis of the consistent IPCMs *A*2–*A*4 (Tables 9–11) and weakly consistent IPCM *A*1 (Table 8) provide the same rankings of the compared objects. Therefore, the results obtained by the proposed models do not contradict the results for such IPCMs obtained by other known methods [10, 19, 21, 22, 23, 26, 29]. The modeling shows that solutions to the problem of choosing one "best" object, obtained by different methods on the basis of a consistent IPCMs, are generally coincide. The same is true for solutions to the

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problems of ranking and rating of objects based on weakly consistent IPCMs.

When highly inconsistent expert estimates of paired comparisons (IPCMs *A*5 and *A*6, see Tables 6 and 7) are the input data, then different considered methods lead not only to different weights, but also to different rankings of compared objects (Tables 12 and 13). For example, for IPCM *A*5, the "upper" FPP model specifies the ranking $w_4 > w_5 > w_2 > w_1 > w_3$, the "lower" FPP model specifies the ranking $w_4 > w_5 > w_2 > w_3 > w_1$, and the proposed method specifies another ranking $w_4 > w_5 = w_1 > w_3 > w_2$ (Table 12). For IPCM *A*6, the "upper" FPP model determines $w_1 = w_5 > w_2 = w_3 > w_4$, the "lower" FPP model determines $w_1 = w_5 > w_2 > w_4 > w_3$, and the proposed method determines the ranking $w_1 = w_4 > w_5 > w_2 = w_3$ (Table 13).

The result $w_1 = w_4 > w_5 > w_2 = w_3$ (Table 13) by the proposed method based on the inconsistent IPCM *A*6 is significantly closer to the ranking $w_4 > w_1 > w_5 > w_3 > w_2$ (Table 11) by the proposed method based on the initial unperturbed IPCM *A*4 in comparison with the FPP rankings. Therefore, the resulting vector of weights by the proposed method based on the inconsistent IPCMs with outliers, like IPCM *A*6, is not significantly sensitive to individual strongly perturbed element in this matrix.

Another advantage of the proposed method in comparison with FPP is that it becomes possible to find the most inconsistent elements in highly inconsistent IPCMs based on values λ_{ij}^*, in order to their further correction to reduce the inconsistency level of the entire IPCM. So, the element a_{12} of IPCM *A*5 (Table 6) is the most inconsistent. This element corresponds to the element λ_{12}^*, which is equal to 0.8548 and is the smallest element of corresponding λ_{ij}^* (Table 14). In IPCM *A*6 (Table 7), the most inconsistent element is a_{24}. It corresponds to the element λ_{24}^*, which is equal to 0.6000 and is the smallest element of these λ_{ij}^* (Table 14). In the consistent IPCMs *A*2 – *A*4, the elements λ_{ij}^* generally take on values greater than one.

Solutions to problems (9) and (9′) [18, 19] depend on the choice of values of parameters d_k and d_{ij} of membership functions $\mu_k(P_k w)$ (5) and $\mu_{ij}(w)$ (5′), respectively. In the general case, this choice has to be made by a decision maker. In this paper, the values d_{ij} are calculated without the role of a decision maker: $d_{ij} = u_{ij} - l_{ij}$.

The proposed method consists in solving a linear programming problem with $n + n(n-1)/2$ variables. For comparison, the linear programming problem of the FPP method has $(n + 1)$ variables. The linear programming problems of the GPM [22] and LUAM [20] have $6n$ and $4n$ variables, respectively. Additional $n(n-1)/2$ variables in proposed method are needed to preserve the prevalence values determined by an expert, and are also used to find the most inconsistent expert pairwise comparison judgments.

The TLGP [21], Liu's [23], Li&Tong's [26], Nedashkovskaya's [13] and Kuo's [29] methods result in interval and non-normalized weights on the basis of IPCM. So, the question of choosing a method for normalizing interval weights arises. The weights obtained by the proposed method are crisp and therefore do not require special methods for their normalization and ranking. However, interval resulting weights provide more information to the decision maker and are more flexible when applied in the DM process.

The method can be used for a wide range of decision support problems, planning, prioritization and rating, resource allocation problems, evaluating decision alternatives and criteria in various applied areas.

CONCLUSIONS

The scientific novelty of the obtained results consists in suggestion of a modified method for calculating a crisp weight vector for elements of DM model based on consistent and inconsistent multiplicative IPCMs. The differences between the proposed method and the known ones are as follows: coefficients that characterize extended intervals for ratios of weights are introduced; membership functions of fuzzy preference relations are proposed, which depend on values of IPCM elements. The introduction of these coefficients and membership functions made it possible to prove the statement about the required coincidence of the calculated weights based on the "upper" and "lower" models. The introduced coefficients can be further used to find the most inconsistent IPCM elements. Therefore, the proposed method results in more reliable weight vectors on the basis of inconsistent multiplicative IPCMs compared to another known methods.

The results show **the practical significance** of the method for solving prioritization problems. Resulting weight vectors, priorities and rankings based on them are not sensitive to the renumbering of the compared elements in contrast to the known FPP method. The most inconsistent elements of IPCM, which are a by-product of the proposed method, are further used for adjusting the elements of IPCM to improve the quality of decisions based on these matrices. In the proposed method, experts or a decision-maker are not required to set the parameters of the membership functions, these parameters are calculated without the participation of experts. Modeling has shown that the weights and rankings obtained by the proposed method on the basis of consistent and weakly consistent IPCMs do not contradict the weights and rankings for such IPCMs calculated using other known methods. For strongly inconsistent studied IPCMs, the proposed method gave more reliable weights compared to the other known methods, since the obtained weights were practically insensitive to individual strongly perturbed during the simulation elements in these matrices.

Prospects for further research consists in investigation of other types of fuzzy preference relations on a set of compared decision alternatives, other types of interval

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and fuzzy PCMs, in particular, type-2 fuzzy PCMs; investigation of aggregation of fuzzy local weights of decision alternatives according to multiple criteria; application of the developed method for solving practical problems of decision support in various applied areas.

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МЕТОД РОЗРАХУНКУ ВАГ ЕЛЕМЕНТІВ МОДЕЛІ ПІДТРИМКИ ПРИЙНЯТТЯ РІШЕНЬ НА ОСНОВІ ІНТЕРВАЛЬНИХ МУЛЬТИПЛІКАТИВНИХ МАТРИЦЬ ПАРНИХ ПОРІВНЯНЬ

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АНОТАЦІЯ

Актуальність. Метод парних порівнянь – складова кількох методологій підтримки прийняття рішень, таких як PROMETHEE, TOPSIS, аналізу ієрархій і мереж. Його суть полягає в розрахунку вектора пріоритетів (ваг) елементів моделі прийняття рішень на основі обернено симетричних матриць парних порівнянь. Оцінювання елементів моделі здійснюється здебільшого експертами в умовах невизначеності. Тому в останні роки досліджуються модифіковані методи розрахунку ваг з використанням нечітких та інтервальних матриць парних порівнянь (ІМПП).

Мета. Розробка модифікованого методу розрахунку ваг на основі узгоджених і неузгоджених мультиплікативних ІМПП елементів моделі прийняття рішень.

Метод. Запропоновано модифікований метод на основі узгоджених і неузгоджених мультиплікативних ІМПП та нечіткого програмування переваг, який призводить до більш достовірних ваг елементів моделі прийняття рішень порівняно з іншими відомими методами. Розроблений метод відрізняється від інших наступними особливостями: введено коефіцієнти, які характеризують розширені інтервали для відношень невідомих ваг; запропоновано функції належності нечітких відношень нестрогої переваги залежно від значень елементів ІМПП. Введення вказаних коефіцієнтів і функцій належності дозволило довести твердження про несуперечливість результуючих ваг на основі «верхньої» та «нижньої» моделей. Пропоновані коефіцієнти в подальшому використовуються для пошуку найбільш неузгоджених елементів ІМПП.

Результати. Виконано експерименти з кількома ІМПП різного рівня узгодженості. Ваги, отримані пропонованим та іншими відомими методами на основі розглянутих узгоджених та слабко узгоджених ІМПП, визначили однакові ранжування порівнюваних об'єктів. Результати, отримані пропонованим методом, не суперечать результатам для таких ІМПП за іншими відомими методами. Ранжування пропонованим методом на основі розглянутих сильно збурених ІМПП суттєво ближчі до ранжувань на основі відповідних початкових незбурених ІМПП порівняно з ранжуваннями відомим методом FPP. Знайдено найбільш неузгоджені елементи в розглянутих ІМПП.

Висновки. Розроблений метод показав свою ефективність і може використовуватися для широкого кола задач підтримки прийняття рішень, сценарного аналізу, розрахунку пріоритетів, розподілу ресурсів, оцінювання варіантів та критеріїв рішень у різних прикладних областях.

КЛЮЧОВІ СЛОВА: інтервальна мультиплікативна матриця парних порівнянь, узгодженість, експертні оцінки, нечітке програмування переваг, системи підтримки прийняття рішень.

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МЕТОД РАСЧЕТА ВЕСОВ ЭЛЕМЕНТОВ МОДЕЛИ ПОДДЕРЖКИ ПРИНЯТИЯ РЕШЕНИЙ НА ОСНОВЕ ИНТЕРВАЛЬНЫХ МУЛЬТИПЛИКАТИВНЫХ МАТРИЦ ПАРНЫХ СРАВНЕНИЙ

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АННОТАЦИЯ

Актуальность. Метод парных сравнений – составляющая нескольких методологий поддержки принятия решений, таких как PROMETHEE, TOPSIS, анализа иерархий и сетей. Его суть заключается в расчете вектора приоритетов или весов элементов модели принятия решений на основе обратно симметричных матриц парных сравнений. Оценка элементов модели осуществляется в основном экспертами в условиях неопределенности. Поэтому в последние годы исследуются модификации методов расчета весов с использованием нечетких и интервальных матриц парных сравнений (ИМПС).

Цель. Разработка модифицированного метода расчета приоритетов на основе согласованных и несогласованных мультипликативных ИМПС элементов модели принятия решений.

Метод. Предложен модифицированный метод на основе согласованных и несогласованных мультипликативных ИМПС и нечеткого программирования предпочтений, который приводит к более достоверным весам элементов модели принятия решений по сравнению с другими известными методами. Разработанный метод отличается от других следующими особенностями: введены коэффициенты, характеризующие расширенные интервалы для отношений неизвестных весов; предложены функции принадлежности нечетких отношений нестрогого предпочтения в зависимости от значений элементов ИМПС. Введение указанных коэффициентов и функций принадлежности позволило доказать утверждение о непротиворечивости результирующих весов на основе «верхней» и «нижней» моделей. Предлагаемые коэффициенты в дальнейшем используются для поиска наиболее несогласованных элементов ИМПС.

Результаты. Выполнены эксперименты с несколькими ИМПС разного уровня согласованности. Веса, полученные предлагаемым и другими известными методами на основе рассмотренных согласованных и слабо согласованных ИМПС, определили одинаковые ранжирования сравниваемых объектов. Результаты, полученные предлагаемым методом, не противоречат результатам для таких ИМПС по другим известным методам. Ранжирования предлагаемым методом на основе рассмотренных сильно несогласованных ИМПС существенно ближе к ранжированиям на основе соответствующих начальных невозмущенных ИМПС по сравнению с ранжированиями известным методом FPP. Найдены наиболее несогласованные элементы в рассмотренных ИМПС.

Выводы. Разработанный метод показал свою эффективность и может использоваться для широкого круга задач поддержки принятия решений, сценарного анализа, расчета приоритетов, распределения ресурсов, оценки вариантов и критериев решений в разных прикладных областях.

КЛЮЧЕВЫЕ СЛОВА: интервальная мультипликативная матрица парных сравнений, согласованность, экспертные оценки, нечеткое программирование предпочтений, системы поддержки принятия решений.

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