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APPLICATION OF SPLINE FUNCTIONS AND WALSH FUNCTIONS IN PROBLEMS OF PARAMETRIC IDENTIFICATION OF LINEAR NONSTATIONARY SYSTEMS

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ABSTRACT

Context. In this article, a generalized parametric identification procedure for linear nonstationary systems is proposed, which uses spline functions and orthogonal expansion in a series according to the Walsh function system, which makes it possible to find estimates of the desired parameters by minimizing the integral quadratic criterion of discrepancy based on solving a system of linear algebraic equations for a wide class of linear dynamical systems. The accuracy of parameter estimation is ensured by constructing a spline with a given accuracy and choosing the number of terms of the Walsh series expansion when solving systems of linear algebraic equations by the A. N. Tikhonov regularization method. To improve the accuracy of the assessment, an algorithm for adaptive partitioning of the observation interval is proposed. The partitioning criterion is the weighted square of the discrepancy between the state variables of the control object and the state variables of the model. The choice of the number of terms of the expansion into the Walsh series is carried out on the basis of adaptive approximation of non-stationary parameters in the observation interval, based on the specified accuracy of their estimates. The quality of the management of objects with variable parameters is largely determined by the accuracy of the evaluation of their parameters. Hence, obtaining reliable information about the actual nature of parameter changes is undoubtedly an urgent task.

Objective. Improving the accuracy of parameter estimation of a wide class of linear dynamical systems through the joint use of spline functions and Walsh functions.

Method. A generalized parametric identification procedure for a wide class of linear dynamical systems is proposed. The choice of the number of terms of the expansion into the Walsh series is made on the basis of the proposed algorithm for adaptive partitioning of the observation interval.

Results. The results of modeling of specific linear non-stationary systems confirm the effectiveness of using the proposed approaches to estimating non-stationary parameters.

Conclusions. The joint use of spline functions and Walsh functions makes it possible, based on the proposed generalized parametric identification procedure, to obtain analytically estimated parameters, which is very convenient for subsequent use in the synthesis of optimal controls of real technical objects. This procedure is applicable to a wide class of linear dynamical systems with concentrated and distributed parameters.

KEYWORDS: linear non-stationary systems, spline functions, Walsh functions, operating matrix, Tikhonov regularization method, piecewise constant approximation.

NOMENCLATURE

$\bar{x}(t)$ – state vector;

$\dot{\bar{x}}(t)$ – derivative of state vector;

$\bar{u}(t)$ – control vector;

$A(t)$ – matrix parameters $a_{ij}(t)$;

$B(t)$ – matrix parameters $b_{ik}(t)$;

$\hat{a}_{ij}(t)$ – estimates of unknown parameters $a_{ij}(t)$;

$\hat{b}_{ik}(t)$ – estimates of unknown parameters $b_{ik}(t)$;

$\bar{S}_x(\tau)$ –vector of spline functions of the state vector;

$\bar{S}_u(\tau)$ – vector of spline functions of the vector of controls;

$<\tau_i>$ – discrete grid with increments Δ_N;

$\bar{\varphi}_R(t)$ – vector of Walsh functions $\varphi_i(t)$;

$P_{(NxN)}$ – operational integration matrix;

$\bar{g}_l^{(i)}$ – vector of estimates of all unknown parameters;

$Q(\bar{g}_l^{(i)})$ – an integral quadratic discrepancy criterion;

$d \oplus c$ – bitwise addition modulo 2.

INTRODUCTION

It is known that all real control objects are nonlinear and non-stationary to one degree or another. The analysis and synthesis of control systems for such objects is a complex mathematical problem, the solution of which has so far been obtained for some special cases [1–4]. However, most control objects make it possible to accept a non-stationary and linearized system of equations as a mathematical model and apply the developed mathematical apparatus for solving linear non-stationary differential equa synthesis of optimal equation systems for such objects remains a difficult task due to the non-stationarity of the parameters. Often this problem is complicated by the

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fact that the parameters of dynamic models of control objects are unknown in advance and their preliminary assessment is required. In this regard, the subject area of research in this article is limited to the class of continuous linear non-stationary systems with monotonic and sign-constant parameters that describe a significant number of control objects.

The object of research is the identification of parameters of a wide class of linear dynamical systems and its implementation for systems with non-stationary parameters.

The subject of the research is a generalized algorithm for parametric identification of the parameters of linear dynamic systems, in this case, systems with non-stationary parameters.

The aim of the research is to develop an efficient algorithm for the parametric identification of linear dynamic systems based on the combined use of spline functions and Walsh functions.

1 PROBLEM STATEMENT

The task of parametric identification in this case is as follows. For a model of a linear dynamical system described by a system of differential equations of the form

$$\dot{\bar{x}}(t) = A(t)\bar{x}(t) + B(t)\bar{u}(t), t \in \left[t_0, T_f\right], \bar{x}(t_0) = \bar{x}^{(0)}, \qquad (1)$$

where $A(t) = \{a_{ij}(t)\}, B(t) = \{b_{ik}(t)\}$, – matrices of size $n \times n$ and $n \times m$, respectively, whose elements are sign-constant

$$sign\left[a_{ij}(t)\right] = const, sign\left[b_{ik}(t)\right] = const, \qquad (2)$$

monotonous

$$sign\left[da_{ij}(t)/dt\right] = const, sign\left[db_{ik}(t)/dt\right] = const \qquad (3)$$

functions that have continuous first derivatives and bounded domains of definition on a time interval $[t_0, T_f]$, it is necessary to evaluate unknown parameters $a_{ij}(T_l), b_{ik}(T_l)$.

The parameters will be evaluated based on minimizing the square of the discrepancy

$$I = \min\left\{\int_{t_0}^{t_0+T_f} \left[\dot{\bar{x}}(t) - A(t)\bar{x}(t) - B(t)\bar{u}(t)\right]^2 dt\right\}. \qquad (4)$$

2 REVIEW OF THE LITERATURE

Currently, there are many methods for evaluating the parameters of control objects, which can be divided into two large classes: adaptive and non-adaptive [10–12]. When considering linear objects, the mathematical description of which is given in the state space, and the coefficients of differential equations provide complete information about the dynamic properties, estimates of unknown parameters can be obtained by both adaptive and non-adaptive methods. At the design stage, non-adaptive identification methods are usually used, which, although they require a large amount of calculations, also allow obtaining more accurate values of the estimated parameters over the entire observation interval. To obtain estimates of the variable coefficients of differential equations, various direct methods are used, among which the following methods have become most popular: least squares and its various variants, differential approximation, stochastic approximation, sequential integration, etc. [13–16]. Each of them has its advantages and disadvantages, but all of them are applicable if the assumption of quasi-stationarity of changing the parameters of the control object is accepted. In the case of non-stationarity of the parameters of the control object, orthogonal functions have found great practical application. Traditionally, approximation by finite sums of orthogonal functions has been used to evaluate such dynamic characteristics of objects as a transient function or an impulse transient function [17, 18]. In recent years, many papers have appeared on the use of orthogonal systems of functions for estimating the parameters of a mathematical model given by differential equations, both for stationary linear objects with distributed parameters and for linear non-stationary objects with concentrated parameters, and differing from each other mainly by the choice of one or another system of orthonormal functions. Here the basic approach is as follows: the initial model, represented by ordinary differential equations for concentrated systems or partial differential equations for distributed systems, is transformed into integral equations: all known and unknown functions are decomposed into finite series according to the selected orthogonal functions and then substituted into the transformed model; the so-called operational matrix [19, 20] is introduced to integrate the selected system of functions, which allows further obtaining an identification algorithm in the form of algebraic equations. A great interest in the theory of estimation has also arisen due to a significant change in the possibilities of applying the theory of estimation associated with the enormous capabilities of modern computers. Taking into account the last remark, the use of the apparatus of orthogonal Walsh basis functions is of undoubted interest for identification [20–22]. Firstly, this is due to the fact that Walsh functions take values only ±1 and represent an apparatus closely related to binary decomposition. And, since the decomposition of variables according to the Walsh function system requires their analytical representation, the paper uses the mathematical apparatus of polynomial approximation in the form of spline functions, in particular, cubic splines [23].

It is obvious that the use of functional (4) in parametric identification problems presupposes the presence of a well-known analytical expression for both the vector of state variables $\bar{x}(t)$ and its derivative $\dot{\bar{x}}(t)$. The known difficulties associated with the definition of these expressions are proposed in this article to overcome using the mathematical apparatus of spline functions [23]. The

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question of the possibility of decomposing functions into a series according to the Walsh function system boils down to finding out the possibility of approximating this function by a piecewise constant function.

3 MATERIALS AND METHODS

Since the observation intervals differ in duration, it is advisable to bring them to a normalized interval [0,1]. To do this, it is necessary to introduce a dimensionless time $\tau = \frac{t - t_0}{T_f - t}$, equal to and leading the control interval to the normalized interval [0,1].

In addition, to obtain analytical expressions of state variables taken at discrete time points, provided that the state vector is fully measurable and meets conditions (2) and (3), as mentioned earlier, it is advisable to use cubic splines. Considering the above, the following parametric identification procedure based on spline functions and Walsh functions is proposed:

Step 1. On the normalized interval, set a grid with a step $< \tau_i > (i = \overline{0, N}; t_N = 1)$. Determine the values of the state $\bar{x}(\tau_i)$ and control $\bar{u}(\tau_i)$ vectors.

Step 2. On the selected grid perform interpolation, obtain an analytical expression for evaluating the vector function of the state and control, respectively, in the form of cubic splines $\bar{S}_x(\tau)$ and $\bar{S}_u(\tau)$.

Step 3. For the found functions $\bar{S}_x(\tau)$ and $\bar{S}_u(\tau)$ for n unknown, time-normalized parameters of the system (1), we apply the orthogonal expansion into the Walsh series.
Step 4. Normalize the initial system (1) in time and bring it to an integral form.

Stage 5. Using the properties of Walsh functions, we replace the Walsh functions in the transformed form with a square integration matrix of the form $P_{(NxN)}$ of dimension $N = 2^n$ [20]. The essence of this property is that the integral of the Walsh function remains in the class of the Walsh function system, i.e.

$$\int_0^x \bar{\varphi}_N(x)dx \approx P_{(NXN)}\bar{\varphi}_N(x),$$

where $\bar{\varphi}_N(x) = \{\varphi_0(x)..\varphi_N(x)\}$ – a vector whose components are Walsh functions.

By reducing the left and right sides of the resulting equation by the vector of the selected system of Walsh functions, we obtain a system of algebraic equations.

Step 6. We solve the resulting algebraic system of equations with respect to unknown parameters represented by a set of coefficients of the interval [0,1], recalculating the model parameters found accordingly.

The procedure proposed above for parametric identification of a linear non-stationary system of the form (1) uses spline interpolation and orthogonal decomposition of functions into a Walsh series and allows us to obtain estimates of non-stationary parameters in the form of approximations by Walsh series. Let us show a practical implementation of this parametric identification procedure for system (1).

The state vector $\bar{x}(t)$ is defined on the interval [0,1] by its values $x^{-(i)} = \bar{x}(t_i)$ in a finite number of points $t_i \in [t_0, T_f](i = \overline{0, N})$. As before, to obtain an analytical expression for $\bar{x}(t)$ and $\dot{\bar{x}}(t)$, we use cubic spline functions $\bar{S}(t)$, making the transition from $\bar{x}(t_i)(i = \overline{0, N})$ to $\bar{S}(t), t \in [t_0, T_f]$.

Functions $u_k(t), S_i(t), \dot{S}_i(t)$ and estimate $\hat{a}_{ij}(t), \hat{b}_{ik}(t)(i, j = \overline{1, n}), (k = \overline{1, m})$ of unknown parameters $a_{ij}(t), b_{ik}(t)$ of matrices $A(t), B(t)$, assuming their integrability on a segment $[t_0, T_f]$, can be approximated by decomposition into a Walsh series of the following form:

$$\begin{aligned}
u_k(t) &\approx \sum_{r=0}^{R-1} u_r^{(k)}\varphi_r(t) = \bar{u}^{(k)T}\bar{\varphi}_R(t),\\
S_i(t) &\approx \sum_{r=0}^{R-1} s_r^{(i)}\varphi_r(t) = \bar{S}^{(i)T}\bar{\varphi}_R(t),\\
\dot{S}_i(t) &\approx \sum_{r=0}^{R-1} s^{\cdot(i)}\varphi_r(t) = \dot{\bar{s}}^{(i)T}\bar{\varphi}_R(t),\\
\hat{a}_{ij}(t) &\approx \sum_{r=0}^{R-1} a_r^{(ij)}\varphi_r(t) = \bar{a}^{(ij)T}\bar{\varphi}_R(t),\\
\hat{b}_{ik}(t) &\approx \sum_{r=0}^{R-1} b_r^{(ik)}\varphi_r(t) = \bar{b}^{(ik)T}\bar{\varphi}_R(t),
\end{aligned} \tag{5}$$

where $\bar{\varphi}_R^T(t) = \{\varphi_0(t), ..., \varphi_r(t), ..., \varphi_{R-1}(t)\}$ – R-dimensional vector of Walsh functions.
Here:

$$\begin{aligned}
\bar{u}^{(k)T} &= \{u_0^{(k)}, ..., u_r^{(k)}, ..., u_{R-1}^{(k)}\},\\
\bar{S}^{(i)T} &= \{S_0^{(i)}, ..., S_r^{(i)}, ..., S_{R-1}^{(i)}\},\\
\dot{\bar{S}}^{(i)T} &= \{\dot{\bar{S}}^{(i)}, ..., \dot{\bar{S}}^{(i)}, ..., \dot{\bar{S}}^{(i)}\}
\end{aligned} \quad –$$

R-dimensional vectors of constant Fourier coefficients of the Walsh series of functions $u_k(t), S_i(t), \dot{S}_i(t)$, respectively, whose elements are defined as

$$u_r^{(k)} = 1/(T_f - t_0)\int_{t0}^{Tf} u_k(t)\varphi_r(t)dt, s_r^{(i)} = 1/(T_f - t_0)\int_{t_0}^{T_f} S_i(t)\varphi_r(t)dt,$$

$$\dot{s}_r^{(i)} = 1/(T_f - t_0)\int_{t_0}^{T_f} \dot{S}_i(t)\varphi_r(t)dt (r = \overline{0, R-1});$$

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$\overline{a}^{(ij)T} = \{a_0^{(ij)},...,a_r^{(ij)},...a_{R-1}^{(ij)}\}, \overline{b}^{(ik)T} = \{b_0^{(ik)},...,b_r^{(ik)},...,b_{R-1}^{(ik)}\}$ – R-dimensional vectors of unknown constant coefficients of the Walsh series of estimated parameter functions $a_{ij}(t), b_{ik}(t)$.

Then, given the ratio (5), the model of the system (1) will have the form

$$\dot{\overline{s}}^{(i)T}\overline{\varphi}_R = \sum_{j=1}^{n}\left[\left(\overline{a}^{(ij)T}\overline{\varphi}_R\right)\left(\overline{s}^{(j)T}\overline{\varphi}_R\right)\right] + \sum_{k=1}^{m}\left[\left(\overline{b}^{(ik)T}\overline{\varphi}_R\right)\left(\overline{u}^{(k)T}\overline{\varphi}_R\right)\right](i=\overline{1,n}). \quad (6)$$

The vector of the desired parameters $\overline{g}^{(i)}(i=\overline{1,n})$ equations (6) can be written as a vector of estimates of the constant coefficients of the Walsh series

$$\overline{g}^{(i)T} = \{\hat{a}_0^{(i1)},...,\hat{a}_{R-1}^{(i1)},...,\hat{a}_0^{(ij)},...,\hat{a}_r^{(ij)},...,\hat{a}_{R-1}^{(ij)},...,\hat{a}_0^{(in)},...,\hat{a}_{R-1}^{(in)}, \hat{b}_0^{(i1)},...,\hat{b}_{R-1}^{(i1)},...,\hat{b}_0^{(ik)},...,\hat{b}_r^{(ik)},...,\hat{b}_{R-1}^{(ik)},...,\hat{b}_0^{(im)},...,\hat{b}_{R-1}^{(im)}\} \quad (7)$$

and the integral quadratic discrepancy criterion of the form (4) of the model (6) , taking into account (7), is represented as

$$Q_i(\overline{g}^{(i)}) = \int_{t_0}^{T_f}\left[\dot{\overline{s}}^{(i)T}\overline{\varphi}_R - \sum_{j=1}^{n}(\overline{a}^{(ij)T}\overline{\varphi}_R)(\overline{s}^{(j)T}\overline{\varphi}_R) - \sum_{k=1}^{m}(\overline{b}^{(ik)T}\overline{\varphi}_R)\times(\overline{u}^{(k)T}\overline{\varphi}_R)\right]^2 dt(i=\overline{1,n}). \quad (8)$$

The number of identifiable model parameters (6) is equal to $n \times (n+m) \times R$.

It is obvious that the task of parametric identification is to find estimates $\overline{g}^{(i)}\ (i=\overline{1,n})$ that provide a minimum of the functional (8). Using the necessary conditions for the minimum of criterion (8) for the desired parameters

$$\partial Q_i(\overline{g}^{(i)})/\partial\hat{a}_p^{(iz)} = 0,$$
$$\partial Q_i(\overline{g}^{(i)})/\partial\hat{b}_p^{(it)} = 0, \qquad ,$$
$$(z=\overline{1,n}),(t=\overline{1,m}),(p=\overline{0,R-1}),(i=\overline{1,n})$$

we obtain a system of equations

$$\int_{t_0}^{T_f}\left[\dot{\overline{s}}^{(i)T}\overline{\varphi}_R - \sum_{j=1}^{n}(\hat{\overline{a}}^{(ij)T}\overline{\varphi}_R)(\overline{s}^{(j)T}\overline{\varphi}_R) - \sum_{k=1}^{m}(\hat{\overline{b}}^{(ik)T}\overline{\varphi}_R)(\overline{u}^{(k)T}\overline{\varphi}_R)\right]\times\left[-\varphi_p(\overline{s}^{(z)T}\overline{\varphi}_R)\right]dt = 0;$$
$$\int_{t_0}^{T_f}\left[\dot{\overline{s}}^{(i)T}\overline{\varphi}_R - \sum_{j=1}^{n}(\hat{\overline{a}}^{(ij)T}\overline{\varphi}_R)(\overline{s}^{(j)T}\overline{\varphi}_R) - \sum_{k=1}^{m}(\hat{\overline{b}}^{(ik)T}\overline{\varphi}_R)(\overline{u}^{(k)T}\overline{\varphi}_R)\right]\times\left[-\varphi_p(\overline{s}^{(t)T}\overline{\varphi}_R)\right]dt = 0,$$

which we will write in a form convenient for the following transformations, namely:

$$\sum_{j=1}^{n}\int_{t_0}^{T_f}\left[(\sum_{r=0}^{R-1}\hat{a}_r^{(ij)}\varphi_r)(\varphi_p)(\sum_{r=0}^{R-1}s_r^{(i)}\varphi_r)\right]dt +$$
$$+\sum_{k=1}^{m}\int_{t_0}^{T_f}\left[\left(\sum_{r=0}^{R-1}\hat{b}_r^{(ik)}\varphi_r\right)\left(\varphi_p\right)\left(\sum_{r=0}^{R-1}u_r^{(k)}\varphi_r\right)\left(\sum_{r=0}^{R-1}s_r^{(z)}\varphi_r\right)\right]dt =$$
$$= \int_{t_0}^{T_f}\left[\left(\sum_{r=0}^{R-1}\dot{s}_r^{(i)}\varphi_r\right)\left(\varphi_p\right)\left(\sum_{r=0}^{R-1}s_r^{(z)}\varphi_r\right)\right]dt(z=\overline{1,n}),(p=\overline{0,R-1});$$
$$\sum_{j=1}^{n}\int_{t_0}^{T_f}\left[\left(\sum_{r=0}^{R-1}\hat{a}_r^{(ij)}\varphi_r\right)\left(\varphi_p\right)\left(\sum_{r=0}^{R-1}s_r^{(z)}\varphi_r\right)\left(\sum_{r=0}^{R-1}u_r^{(t)}\varphi_r\right)\right]dt + \quad (9)$$
$$+\sum_{k=1}^{m}\int_{t_0}^{T_f}\left[\left(\sum_{r=0}^{R-1}\hat{b}_r^{(ik)}\varphi_r\right)\left(\varphi_p\right)\left(\sum_{r=0}^{R-1}u_r^{(k)}\varphi_r\right)\left(\sum_{r=0}^{R-1}u_r^{(t)}\varphi_r\right)\right]dt =$$
$$= \int_{t_0}^{T_f}\left[\left(\sum_{r=0}^{R-1}\dot{s}_r^{(i)}\varphi\right)\left(\varphi_p\right)\left(\sum_{r=0}^{R-1}u_r^{(t)}\varphi_r\right)\right]dt(t=1,m),(p=\overline{0,R-1}),(i=\overline{1,n}).$$

Taking into account the multiplicativity property of the Walsh function system, after a series of transformations, we obtain at a given interval, equations (9) in the form

$$\sum_{j=1}^{n}\int_{t_0}^{T_f}\left[\left(\sum_{r=0}^{R-1}\hat{a}_r^{(ij)}\varphi_{r\oplus p}\right)\left(\sum_{r_1=0}^{R-1}f_{r_1}^{(jz)}\varphi_{r_1}\right)\right]dt + \sum_{k=1}^{m}\int_{t_0}^{T_f}\left[\left(\sum_{r=0}^{R-1}\hat{b}_r^{(ik)}\varphi_{r\oplus p}\right)\right]\times$$
$$\times\sum_{r_1=0}^{R-1}h_{r_1}^{(kz)}\varphi_{r_1})]dt = \int_{t_0}^{T_f}[(\sum_{r=0}^{R-1\bullet(i)}s_r\varphi_{r\oplus p})(\sum_{r=0}^{R-1}s_r^{(z)}\varphi_r)]dt(z=\overline{1,n}),(p=\overline{0,R-1});$$
$$\sum_{j=1}^{n}\int_{t_0}^{T_f}[(\sum_{r=0}^{R-1}\hat{a}_r^{(ij)}\varphi_{r\oplus p})(\sum_{r_1=0}^{R-1}w_{r_1}^{(jt)}\varphi_{r_1})]dt + \sum_{k=1}^{m}\int_{t_0}^{T_f}[(\sum_{r=0}^{R-1}\hat{b}_r^{(ik)}\varphi_{r\oplus p})\times$$
$$\times\sum_{r_1=0}^{R-1}v_{r_1}^{(kt)}\varphi_{r_1})]dt = \int_{t_0}^{T_f}[(\sum_{r=0}^{R-1\bullet(i)}s_r\varphi_{r\oplus p})(\sum_{r=0}^{R-1}u_r^{(t)}\varphi_r)]dt(t=\overline{1,m}),(p=\overline{0,R-1}),(i=\overline{1,n}),$$

where $d \oplus c$ – bitwise addition modulo 2;

$$\overline{f}^{(jz)T} = \{f_0^{(jz)},...,f_{r_1}^{(jz)},...,f_{R-1}^{(jz)}\}, \overline{h}^{(kz)T} = \{h_0^{(kz)},...,h_{r_1}^{(kz)},...,h_{R-1}^{(kz)}\},$$
$$\overline{w}^{(jt)T} = \{w_0^{(jt)},...,w_{r_1}^{(jt)},...,w_{R-1}^{(jt)}\}, v^{(kt)T} = \{v_0^{(kt)},...,v_{r_1}^{(kt)},...,v_{R-1}^{(kt)}\} -$$
,

R-dimensional vectors whose elements are composed of the sum of products of known Fourier coefficients $S_r^{(j)}$ and $S_r^{(z)}$, $u_r^{(k)}$ and $S_r^{(z)}, S_r^{(j)}$ and $u_r^{(t)}, u_r^{(k)}$ and $u_r^{(t)}$ $(r=\overline{0,R-1})$ of the orthogonal expansion into the Walsh series of splines and control functions $S_j(t)$ and $S_z(t)$, $u_k(t)$ and $S_z(t)$, $S_j(t)$ and $u_t(t)$, $u_k(t)$ and $u_t(t)$, respectively, determined from the following relations:

$$f_{r_1}^{(jz)} = \sum_{r=0}^{R-1}s_r^{(j)}s_{r\oplus r_1}^{(z)}, \quad (10)$$

$$h_{r_1}^{(kz)} = \sum_{r=0}^{R-1}u_r^{(k)}s_{r\oplus r_1}^{(z)}, \quad (11)$$

$$w_{r_1}^{(jt)} = \sum_{r=0}^{R-1}s_r^{(j)}u_{r\oplus r_1}^{(t)}, \quad (12)$$

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$$v_{r_1}^{(kt)} = \sum_{r=0}^{R-1} u_r^{(k)} u_{r\oplus r_1}^{(t)}, (r_1 = \overline{0, R-1}). \quad (13)$$

or

$$\sum_j (\sum_r (\sum_{r_1} (\hat{a}_r^{(ij)} f_{r_1}^{(jz)} \int_{t_0}^{T_f} \varphi_{r\oplus p} \varphi_{r_1} dt))) +$$

$$+\sum_k (\sum_r (\sum_{r_1} (\hat{b}_r^{(ik)} h_{r_1}^{(kz)} \int_{t_0}^{T_f} \varphi_{r\oplus p} \varphi_{r_1} dt))) =$$

$$= \sum_r (\sum_{r_1} (\overset{\bullet}{s}{}_r^{(i)} s_{r_1}^{(z)} \int_{t_0}^{T_f} \varphi_{r\oplus p} \varphi_{r_1} dt)) \ (z = \overline{1, n}), (p = \overline{0, R-1});$$

$$\sum_j (\sum_r (\sum_{r_1} (\hat{a}_r^{(ij)} w_{r_1}^{(jt)} \int_{t_0}^{T_f} \varphi_{r\oplus p} \varphi_{r_1} dt))) +$$

$$+\sum_k (\sum_r (\sum_{r_1} (\hat{b}_r^{(ik)} v_{r_1}^{(kt)} \int_{t_0}^{T_f} \varphi_{r\oplus p} \varphi_{r_1} dt))) =$$

$$= \sum_r (\sum_{r_1} (\overset{\bullet}{s}{}_r^{(i)} u_r^{(t)} \int_{t_0}^{T_f} \varphi_{r\oplus p} \varphi_{r_1} dt)) \ (t = \overline{1, m}), (p = \overline{0, R-1}), (i = \overline{1, n}).$$

Due to the orthogonality property of the system of Walsh functions on a given interval, the latter system of equations is transformed into $(n+m)\times R$ a system of linear algebraic equations to obtain an estimate of the vector of parameters $\overline{g}^{(i)} (i = \overline{1, n})$ (7) of the form

$$\begin{aligned} &\sum_j (\sum_r \hat{a}_r^{(ij)} f_{r\oplus p}^{(jz)}) + \sum_k (\sum_r \hat{b}_r^{(ik)} h_{r\oplus p}^{(kz)}) = \\ &= \sum_r \overset{\bullet}{s}{}_r^{(i)} s_{r\oplus p}^{(z)} (z = \overline{1, n}), (p = \overline{0, R-1}); \\ &\sum_j (\sum_r \hat{a}_r^{(ij)} w_{r\oplus p}^{(jt)}) + \sum_k (\sum_r \hat{b}_r^{(ik)} v_{r\oplus p}^{(kt)}) = \\ &= \sum_r \overset{\bullet}{s}{}_r^{(i)} u_{r\oplus p}^{(t)} (t = \overline{1, m}), (p = \overline{0, R-1}). \end{aligned} \quad (14)$$

Equations (14) can be written as

$$C^{(i)} \overline{g}^{(i)} = \overline{d}^{(i)}. \quad (15)$$

Here $C^{(i)}$ is a size $(n+m)R \times (n+m)R$ matrix having a block structure

$$C^{(i)} = \begin{bmatrix} F_{nR\times nR}^{(i)} & \vdots & H_{nR\times mR}^{(i)} \\ \cdots & \cdots & \cdots \\ W_{mR\times nR}^{(i)} & \vdots & V_{mR\times mR}^{(i)} \end{bmatrix},$$

the elements of which are defined as follows:

$$F^{(i)} = \begin{bmatrix} \overline{f}_0^{(1)} \\ \vdots \\ \overline{f}_{R-1}^{(1)} \\ \vdots \\ \overline{f}_0^{(z)} \\ \vdots \\ \overline{f}_p^{(z)} \\ \vdots \\ \overline{f}_{R-1}^{(z)} \\ \vdots \\ \overline{f}_0^{(n)} \\ \vdots \\ \overline{f}_{R-1}^{(n)} \end{bmatrix}, H^{(i)} = \begin{bmatrix} \overline{h}_0^{(1)} \\ \vdots \\ \overline{h}_{R-1}^{(1)} \\ \vdots \\ \overline{h}_0^{(z)} \\ \vdots \\ \overline{h}_p^{(z)} \\ \vdots \\ \overline{h}_{R-1}^{(z)} \\ \vdots \\ \overline{h}_0^{(n)} \\ \vdots \\ \overline{h}_{R-1}^{(n)} \end{bmatrix}, W^{(i)} = \begin{bmatrix} \overline{w}_0^{(1)} \\ \vdots \\ \overline{w}_{R-1}^{(1)} \\ \vdots \\ \overline{w}_0^{(t)} \\ \vdots \\ \overline{w}_p^{(t)} \\ \vdots \\ \overline{w}_{R-1}^{(t)} \\ \vdots \\ \overline{w}_0^{(m)} \\ \vdots \\ \overline{w}_{R-1}^{(m)} \end{bmatrix}, V^{(i)} = \begin{bmatrix} \overline{v}_0^{(1)} \\ \vdots \\ \overline{v}_{R-1}^{(1)} \\ \vdots \\ \overline{v}_0^{(t)} \\ \vdots \\ \overline{v}_p^{(t)} \\ \vdots \\ \overline{v}_{R-1}^{(t)} \\ \vdots \\ \overline{v}_0^{(m)} \\ \vdots \\ \overline{v}_{R-1}^{(m)} \end{bmatrix}$$

where

– $\overline{f}_p^{(z)} = \{f_{r\oplus p}^{(iz)}\} (j = \overline{1, n}), (r = \overline{0, R-1}) - 1\times nR$ – dimensional vector whose elements are determined from the relations (10);

– $\overline{h}_p^{(z)} = \{h_{r\oplus p}^{(kz)}\} (k = \overline{1, n}), (r = \overline{0, R-1}) - 1\times nR$ – dimensional vector whose elements are determined from the relations (11);

– $\overline{w}_p^{(z)} = \{w_{r\oplus p}^{(it)}\} (j = \overline{1, n}), (r = \overline{0, R-1}) - 1\times nR$ – dimensional vector whose elements are determined from the relations (12);

– $\overline{v}_p^{(t)} = \{v_{r\oplus p}^{(kt)}\} (k = \overline{1, m}), (r = \overline{0, R-1}) - 1\times mR$ – dimensional vector whose elements are determined from the relations (13);

$$\overline{d}^{(i)T} = \{q_0^{(i1)},...,q_{R-1}^{(i1)},...,q_0^{(iz)},...,q_p^{(iz)},...q_{R-1}^{(iz)},...,q_0^{(in)},...,q_{R-1}^{(in)},$$
$$e_0^{(i1)},...,e_{R-1}^{(i1)},...,e_0^{(it)},...e_p^{(it)},...,e_{R-1}^{(it)},...,e_{R-1}^{(im)}\} -$$

– $\overline{d}^{(i)T} = \{d_1^{(i)},...,d_n^{(i)}, d_{n+1}^{(i)}...,d_{n+m}^{(i)}\} - (n+m)R -$ dimensional vector of free terms of equation (15), whose elements are determined from the relations

$$q_p^{(iz)} = \sum_{r=0}^{R-1} \overset{\bullet}{s}{}_r^{(i)} \ s_{r\oplus p}^{(z)} (p = \overline{0, R-1}), (z = \overline{1, n}), \quad (16)$$

$$q_p^{(it)} = \sum_{r=0}^{R-1} \overset{\bullet}{s}{}_r^{(i)} \ u_{r\oplus p}^{(t)} (p = \overline{0, R-1}), (t = \overline{1, m}) \quad (17)$$

To ensure the required accuracy of parameter estimation, it is necessary to solve the problem of optimal choice of the number of terms of the Walsh series expansion. To solve this problem, an algorithm for adaptive partitioning of the observation interval based on piecewise constant approximation is proposed below.

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Thus, the desired unknown coefficients of equation (15) are determined from relations (10)–(14), (16), (17).

The algorithm is constructed as follows:

Step 1. We accept l=1. An estimate $\overline{g}_l^{(i)}$ is obtained in the interval $[t_l^{(i)}, t_{l+1}^{(i)H}]$, where $t_{l+1}^{(i)H} = t_l^{(i)} + \delta_l, \delta_l > 0$ according to the generalized parametric identification algorithm given above.

Step 2. In the interval $[t_{l+1}^{(i)H}, t_{l+1}^{(i)\prime}]$, where $t_{l+1}^{(i)\prime} = t_l^{(i)H} + \eta_0 \delta_l$, according to the equations of state of the model (1) with estimates $\overline{g}_l^{(i)}$, the state of the model $x_i^M(t)$ is calculated, while we assume $x_i^M(t_{l+1}^{(i)H}) = x_i(t_{l+1}^{(i)H})$.

Step 3. In the neighborhood $[t_{l+1}^{(i)\prime} - \delta', t_{l+1}^{(i)\prime}](\delta' < t_{l+1}^{(i)\prime} - t_{l+1}^{(i)H})$ of the interval $[t_{l+1}^{(i)H}, t_{l+1}^{(i)\prime}]$, the functional $I = 1/P \sum_{p=1}^{P} (x_i(t_p) - x_i^M(t_p))^2$ is considered.

If $I > \varepsilon$ then the parameter η_0 is reduced, the parameter $\eta_1 = \mu\eta_0 (0 < \mu < 1, \mu = const)$ is entered and the transition to step 2 occurs. If after m steps $\eta_m = \mu^m \eta_0 \approx 0$, then the transition to step 4.

Otherwise, when $I \le \varepsilon$. the interval $[t_l^{(i)}, t_{l+1}^{(i)\prime}]$ is formed, where the estimate $\overline{g}_l^{(i)}$ $[t_{l+1}^{(i)H}, t_{l+1}^{(i)\prime}]$ is taken and the transition to step 2. Otherwise, go to step 4.

Step 4. In the interval $[t_l^{(i)}, t_{l+1}^{(i)}]$ the estimate $\overline{g}_l^{(i)} (i = \overline{1, n})$ is taken.

Step 5. We assume $l = l + 1$ and make the transition to step 1. The algorithm provides for viewing the entire observation interval.

The algorithm continues until the entire observation interval has been viewed. The smallest segment of the piecewise constant approximation determines the number of terms in the expansion in the Walsh series for a given accuracy ε. The block diagram of the adaptive algorithm for sampling the observation interval is shown in Fig. 1.

The smallest segment of the piecewise constant approximation determines the number of terms in the expansion in the Walsh series for a given accuracy ε.

As noted earlier, the system of equations (15) is a system with approximately given initial data, the error of which depends on the error of approximation of the state of the system (1) by splines, the choice of the number of terms of the expansion of functions into a Walsh series, computational errors. To solve the system (15), the regularization method of A. N. Tikhonov is used [24–26]. Thus, the algorithm for estimating the parameters of a linear non-stationary system (1) is reduced to solving n systems of linear algebraic equations of the form (15).

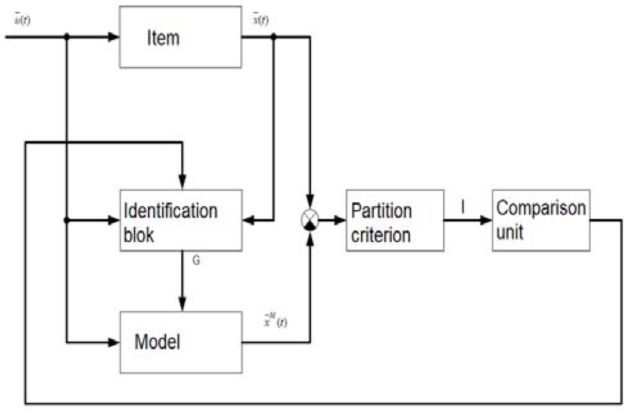

Figure 1 – Block-diagram of the algorithm for adaptive partitioning of the observation interval

4 EXPERIMENTS

As an example of the practical implementation of a generalized parametric identification algorithm, consider the problem of parametric identification of a non-stationary object of the second order

$$\dot{x}_1(t) = a_{12}(t)x_2(t),$$
$$\dot{x}_2(t) = b_2(t)u(t), t \in [0,1],$$
$$x_1(0) = 5, x_2(0) = 3, u(t) = -1.$$

The exact values of the estimated parameters are described by the following functions:

$$a_{12}(t) = \exp(-0{,}5t), b_2(t) = 1{,}5\exp(-0{,}3t).$$

As an example of the practical implementation of the algorithm for adaptive partitioning of the observation interval, consider the problem of identifying a non-stationary object of the second order

$$\dot{x}_1(t) = a_{12}(t)x_2(t),$$
$$\dot{x}_2(t) = b_2(t)u(t), t \in [0,100].$$

We accept the following values of variable $x_1(0) = 30, x_2(0) = 50, b_2(t) = 1, u(t) = -1$.

The exact value of the estimated parameter $a_{12}(t) = 0{,}000012t^3 - 0{,}0014t^2 + 0{,}033t + 2$.

The parameters of the time interval partitioning algorithms were set as follows:

$$L = 10; P = 5 : \delta' = 1; \delta_l = 1 \, for\ all \quad l; \eta_0 = 10; \varepsilon = 0{,}2; \mu = 0{,}5.$$

5 RESULTS

The results of parameter estimation $a_{12}(t), b_2(t)$ are shown in Fig. 2 and Fig. 3, respectively, where the following designations are accepted: solid curve – exact value $a_{12}(t), b_2(t)$, respectively; black dots indicate the values of the estimates $\hat{a}_{12}(t), \hat{b}_2(t)$, respectively, using 8 Walsh functions; white – 4 Walsh functions.

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The estimation results for fixed and adaptive partitioning of the observation interval are shown in Fig. 4, where curve 1 is the exact value of $a_{12}(t)$; curve 2 is the estimate $\hat{a}_{12}(t)$ for a fixed partition; curve 3 is the estimate $\hat{a}_{12}(t)$ for adaptive partitioning. The parameter estimation accuracy is characterized by the value

$$\delta^2 = \sum_{m=0}^{M} [\delta\hat{a}_{12}(t_m)]^2 / \sum_{m=0}^{M} [a_{12}(t_m)]^2 .$$

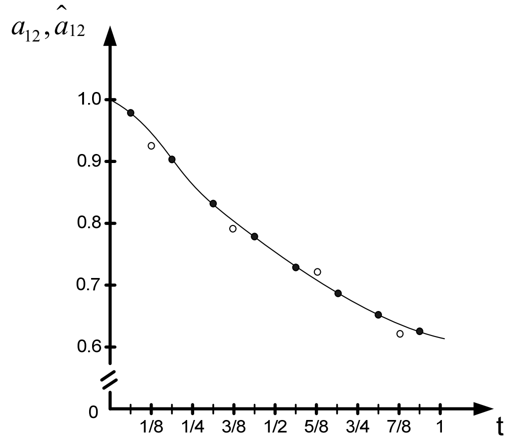

Figure 2 – Parameter Estimation $a_{12}(t)$

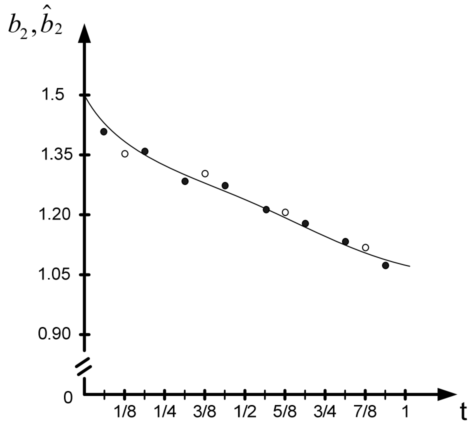

Figure 3 – Parameter Estimation $b_2(t)$

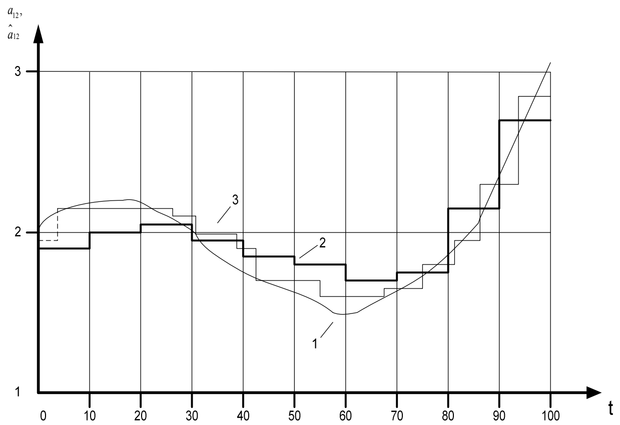

Figure 4 – Plots of piecewise constant approximation

6 DISCUSSION

From the above graphs Fig. 2 and Fig. 3, it follows that with an increase in the number of Walsh functions, the accuracy of parameter estimation increases, to ensure which the required number of expansions in a Walsh series which is determined based on the algorithm for adaptive partitioning of the observation interval.

Comparison of the estimates obtained with a fixed and adaptive partitioning (Fig.4) of the time interval for the considered example allows us to conclude that the accuracy of the parameter estimate can be significantly improved when using an algorithm with an adaptive choice of the interval by choosing the number of expansion terms in the Walsh series, based on the smallest segment of the piecewise constant approximation of a non-stationary parameter. Determination of non-stationary parameters of linear dynamic systems in analytical form in the form of Walsh series allows solving problems of analysis and synthesis of optimal control systems [27]. When optimizing systems with a priori unknown non-stationary parameters, it is advisable to use this adaptive algorithm for partitioning the observation interval in combination with the well-known method MPC (model predictive control).

7 CONCLUSION

A generalized procedure for determining a wide class of linear systems is proposed. Exact estimation of the parameters controlled by the collection of a spline with a given frequency and the choice of the number of expansion terms in the Walsh series, when referring to the system linear algebraic regularity rules A. N. Tikhonov. In connection with the need for a regularity in the division of the observation interval. The splitting criterion is the weighted square of the residual between object control state variables and model state variables. The choice of the number of expansion terms in the Walsh series was carried out on the basis of a responsible approximation of non-stationary parameters in the observation interval, based on the given value of their estimates. The results of modeling models of linear non-stationary systems for the efficiency of using the proposed approaches to estimating non-stationary parameters are presented. The joint use of spline functions and Walsh functions allows, on the basis of the proposed generalized procedure, to obtain the parameter of determining the estimated parameters in an analytical form, which is very convenient for use in the synthesis of optimal controls for real technical objects. This procedure is applicable to the distributed class of linear distributions of systems with lumped and distributed parameters. For systems with distributed parameters, spline interpolation is carried out by a two-dimensional spline and, accordingly, a double Walsh series is used for orthogonal decomposition of the population and unknown functions.

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ЗАСТОСУВАННЯ СПЛАЙН-ФУНКЦІЙ ТА ФУНКЦІЙ УОЛША В ЗАДАЧАХ ПАРАМЕТРИЧНОЇ ІДЕНТИФІКАЦІЇ ЛІНІЙНИХ НЕСТАЦІОНАРНИХ СИСТЕМ

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АНОТАЦІЯ

Актуальність. У статті запропоновано узагальнену процедуру параметричної ідентифікації лінійних нестаціонарних систем, яка використовує сплайн-функції та ортогональне розкладання в ряд за системою функцій Уолша, що дозволяє знаходити оцінки шуканих параметрів шляхом мінімізації інтегрального квадратичного критерію незбіжності на основі розв'язання системи лінійних алгебраїчних рівнянь для широкого класу лінійних динамічних систем. Точність оцінювання параметрів забезпечується побудовою сплайна із заданою точністю та вибором кількості членів розкладу в ряд Уолша при розв'язанні систем лінійних алгебраїчних рівнянь методом регуляризації А. Н. Тихонова. Для підвищення точності оцінки запропоновано алгоритм адаптивного розбиття інтервалу спостереження. Критерієм розбиття є зважений квадрат розбіжності між змінними стану об'єкта керування та змінними стану моделі. Вибір кількості членів розкладу в ряд Уолша здійснюється на основі адаптивної апроксимації нестаціонарних параметрів на інтервалі спостереження, виходячи із заданої точності їх оцінок. Якість управління об'єктами зі змінними параметрами значною мірою визначається точністю оцінювання їх параметрів. Тому отримання достовірної інформації про дійсний характер зміни параметрів є, безперечно, актуальною задачею.

Мета. Підвищення точності оцінювання параметрів широкого класу лінійних динамічних систем шляхом спільного використання сплайн-функцій та функцій Уолша.

Метод. Запропоновано узагальнену процедуру параметричної ідентифікації широкого класу лінійних динамічних систем. Вибір кількості членів розкладу в ряд Уолша здійснюється на основі запропонованого алгоритму адаптивного розбиття інтервалу спостереження.

Результати. Результати моделювання конкретних лінійних нестаціонарних систем підтверджують ефективність використання запропонованих підходів до оцінювання нестаціонарних параметрів.

Висновки. Спільне використання сплайн-функцій та функцій Уолша дозволяє на основі запропонованої узагальненої процедури параметричної ідентифікації отримати оцінку параметрів в аналітичному вигляді, що є дуже зручним для подальшого використання при синтезі оптимальних систем управління реальними об'єктами. Дана процедура застосовна до широкого класу лінійних динамічних систем з зосередженими та розподіленими параметрами.

КЛЮЧОВІ СЛОВА: лінійні нестаціонарні системи, сплайн-функції, функції Уолша, операційна матриця, метод регуляризації Тихонова, кусково-стала апроксимація.

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