МАТЕМАТИЧНЕ ТА КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ

MATHEMATICAL AND COMPUTER MODELING

UDC 51–74, 517.968.21

GENERALIZED FRACTIONAL GAUSSIAN NOISE PREDICTION BASED ON THE WALSH FUNCTIONS

Gorev V. N. – PhD, Associate Professor, Head of the Department of Physics, Dnipro University of Technology, Dnipro, Ukraine.

Gusev A. Yu. – PhD, Associate Professor, Professor of the Department of Information Security and Telecommunications, Dnipro University of Technology, Dnipro, Ukraine.

Korniienko V. I. – Dr. Sc., Professor, Head of the Department of Information Security and Telecommunications, Dnipro University of Technology, Dnipro, Ukraine.

Shedlovska Y. I. – PhD, Associate Professor of Department of Information Technology and Computer Engineering, Dnipro University of Technology, Dnipro, Ukraine.

ABSTRACT

Context. Some of the authors' recent papers were devoted to the Kolmogorov-Wiener filter for telecommunication traffic prediction in some stationary models, such as the fractional Gaussian noise model, the power-law structure function model, and the GFSD (Gaussian fractional sum-difference) model. Recently, the so-called generalized fractional Gaussian noise model was proposed for stationary telecommunication traffic description in some cases. So, in this paper the theoretical fundamentals of the continuous Kolmogorov-Wiener filter used for the prediction of the generalized fractional Gaussian noise are investigated.

Objective. The aim of the work is to obtain the filter weight function as an approximate solution of the corresponding Wiener–Hopf integral equation with the kernel equal to the generalized fractional Gaussian noise correlation function.

Method. A truncated Walsh function expansion is proposed in order to obtain the corresponding solution. This expansion is a special case of the Galerkin method, in the framework of which the unknown function is sought as a truncated series in orthogonal functions. The integral brackets and the results for the mean absolute percentage errors, which are a measure of discrepancy between the left-hand side and the right-hand side of the Wiener-Hopf integral equation, are calculated numerically on the basis of the Wolfram Mathematica package.

Results. The investigation is made for approximations up to sixty four Walsh functions. Different model parameters are investigated. It is shown that for different model parameters the proposed method is convergent and leads to small mean absolute percentage errors for approximations of rather large numbers of Walsh functions.

Conclusions. The paper is devoted to a theoretical construction of the continuous Kolmogorov-Wiener filter weight function for the prediction of a stationary random process described by the generalized fractional Gaussian noise model. As is known, this model may give a good description of some actual telecommunication traffic data in systems with packet data transfer. The corresponding weight function is sought on the basis of the truncated Walsh function expansion method. The corresponding discrepancy errors are small and the method is convergent.

KEYWORDS: continuous Kolmogorov-Wiener filter, weight function, Galerkin method, Walsh functions, generalized fractional Gaussian noise, telecommunication traffic.

ABBREVIATIONS

GFGN is a generalized fractional Gaussian noise;
FGN is a fractional Gaussian noise;
GFSD is Gaussian fractional sum-difference;
MAPE is a mean absolute percentage error.

NOMENCLATURE

T is a time interval on which the input process data are observed;

z is a time interval for which the forecast should be made;

$h(t)$ is the Kolmogorov-Wiener filter weight function;

H is the Hurst exponent;

$R(t)$ is a traffic correlation function in the GFGN model;

$a \in (0,1]$ is a GFGN model parameter;

σ is a traffic standard deviation;

n is a number of Walsh functions in the corresponding approximations;

g_s are coefficients multiplying the Walsh functions;

DOI 10.15588/1607-3274-2023-3-5

$\mathrm{wal}_s(t)$ are the Walsh functions in the Walsh numeration orthogonal on $t \in [0, T]$;

$\mathrm{Left}(t)$ is the left-hand side of the Wiener-Hopf integral equation;

$\mathrm{Right}(t)$ is the right-hand side of the Wiener-Hopf integral equation;

G_{ks} are integral brackets;

B_k are free terms in the linear system of algebraic equations in g_s;

$W_{ls}^{(n)}$ are values of the Walsh functions in corresponding points;

V_{ls}, Q_s are auxiliary integrals.

INTRODUCTION

The problem of traffic prediction is very important for telecommunications, see the corresponding description in [1, 2].

Our recent papers were devoted to such a simple approach as the Kolmogorov-Wiener filter for stationary traffic prediction. For example, in our recent paper [3] it is shown that both the continuous and the discrete Kolmogorov-Wiener filter may be applicable to the prediction of smoothed heavy-tail data similar to FGN which may describe telecommunication traffic in systems with data packet transfer.

Recently Ming Li proposed a GFGN model for stationary traffic description [4]. The theoretical fundamentals of the Kolmogorov-Winer filter construction for traffic in the GFGN model are still to be investigated, so this paper is devoted to the corresponding investigation.

The object of study is the Kolmogorov-Wiener filter for the prediction of continuous stationary telecommunication traffic in the GFGN model.

The subject of study is the weight function of the corresponding filter.

The aim of the work is to obtain the weight function on the basis of the truncated Walsh function expansion method.

1 PROBLEM STATEMENT

The weight function under consideration obeys the following Wiener-Hopf integral equation, see, for example, [5]:

$$\int_0^T d\tau h(\tau) R(t-\tau) = R(t+z) \tag{1}$$

where the traffic correlation function in the GFGN model is as follows [4]:

$$R(t) = \frac{\sigma^2}{2}\left(\left(|t|^a + 1\right)^{2H} + \left||t|^a - 1\right|^{2H} - 2|t|^{2aH}\right). \tag{2}$$

It should be stressed that in the case where $a = 1$ the GFGN model coincides with the FGN model.

The problem statement is as follows: to obtain the unknown filter weight function as an approximate solution of equation (1) with the correlation function (2) on the basis of the truncated Walsh function expansion method.

2 REVIEW OF THE LITERATURE

There are a variety of different and rather sophisticated approaches to traffic prediction, see, for example, [6–8]. Our recent papers were devoted to the Kolmogorov-Wiener filter approach. For example, the theoretical fundamentals of the weigh function construction were investigated for the power-law structure function model [9], for the FGN model [10], and for the GFSD model [11]. Telecommunication traffic in the systems with data packet transfer nowadays is treated as a heavy-tail random process, see, for example, [12–14] and references therein. In [3], the applicability of the Kolmogorov-Wiener filter to the prediction of smoothed heavy-tail data is shown.

In [4], the so-called GFGN model for traffic description is proposed. For example, in [4] it is stressed that such a model gives a good description of the experimental traffic data recorded by the Bellcore in 1989, recorded by the Digital Equipment Corporation in 1995, and recorded by the Measurement and Analysis on the Working Group Traffic Archive in 2019.

Any theoretical construction of the continuous Kolmogorov-Wiener filter for the prediction of a process described by the GFGN model is still to be done. So, the aim of this paper is to obtain the corresponding filter weight function.

The corresponding investigation is made in the framework of the Galerkin method [15] on the basis of a Walsh function expansion. The Walsh functions in the Walsh numeration are used, see [16].

3 MATERIALS AND METHODS

The unknown weight function is sought in the form

$$h(\tau) = \sum_{s=1}^{n} g_s \mathrm{wal}_s(\tau), \tag{3}$$

which on substitution into (1) followed by integration leads to the following matrix expression for the unknown coefficients g_s (see similar expressions, for example, in [11]):

$$\begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{pmatrix}^{-1} \cdot \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}, \tag{4}$$

where

DOI 10.15588/1607-3274-2023-3-5

$$G_{ks} = \int_0^T \int_0^T d\tau dt \mathrm{wal}_k(t) \mathrm{wal}_s(\tau) R(t-\tau),$$
$$B_k = \int_0^T dt \mathrm{wal}_k(t) R(t+z). \quad (5)$$

Similarly to [16], it may be shown that

$$G_{jk} = \sum_{l,s=1}^{n} W_{jl}^{(n)} W_{ks}^{(n)} V_{ls} \quad (6)$$

where

$$W_{jl}^{(n)} = \mathrm{wal}_j\left(\frac{(2l-1)T}{2n}\right),$$
$$V_{ls} = \int_{\frac{(l-1)T}{n}}^{\frac{lT}{n}} \int_{\frac{(s-1)T}{n}}^{\frac{sT}{n}} dt d\tau R(t-\tau). \quad (7)$$

Similarly to [16], it may be shown that

$$V_{ls} = V_{l+1,s+1},\ V_{ls} = V_{sl},\ G_{jk} = G_{kj}, \quad (8)$$

so only the quantities V_{1l}, $l = \overline{1,n}$ should be calculated on the basis of the integration (7), while all the other quantities V_{ls} may be calculated on the basis of (8) with account for calculated results for V_{1l}. The corresponding analytical calculation may meet difficulties, so the following approximate expressions were used:

$$V_{1l} = \int_0^{\frac{T}{n}} \int_{\frac{(l-1)T}{n}}^{\frac{lT}{n}} dt d\tau R(t-\tau) \approx \sum_{i,j} R(i-j) \cdot 10^{-3} \cdot 10^{-3},$$
$$i = 0.1 \cdot 10^{-3}, 2 \cdot 10^{-3}, \ldots, \text{ and so on while } i < T/n, \quad (9)$$
$$j = \frac{(l-1)T}{n}, \frac{(l-1)T}{n} + 1 \cdot 10^{-3}, \frac{(l-1)T}{n} + 2 \cdot 10^{-3}, \ldots,$$
$$\text{and so on while } j < lT/n.$$

The idea of (9) is that the square region $\tau \in [0, T/n]$, $t \in [(l-1)T/n, lT/n]$ is divided into small squares with a side equal to 10^{-3}. Of course, such a calculation is valid only if $T/n \gg 10^{-3}$, but this inequality holds in the framework of the numerical calculations made in this paper.

Similarly to [16], it may be shown that

$$B_k = \sum_{s=1}^{n} W_{ks}^{(n)} Q_s \quad (10)$$

where

$$Q_s = \int_{(s-1)T/n}^{sT/n} dt R(t+z). \quad (11)$$

The integrals Q_s are calculated in the Wolfram Mathematica package by direct integral calculation.

So, the quantities V_{1l} are calculated on the basis of (9), all the other quantities V_{ls} are calculated from V_{1l} on the basis of (8), and the corresponding integral brackets G_{jk} are calculated on the basis of (6) with account for (7) and (8). The coefficients B_k are calculated on the basis of (10) with account for (11). Then the unknown coefficients g_s are calculated on the basis of (4), and the unknown weight function is obtained on the basis of (3).

4 EXPERIMENTS

The following values of the parameters are investigated:

$$T = 100,\ z = 3,\ H = 0.75. \quad (12)$$

The following values of the model parameters are investigated: $a = 0.8$, $a = 0.4$, and $a = 0.08$. The quality of the obtained solution for the weight function is checked by the calculation of the corresponding MAPE error, which is a measure of discrepancy between the left-hand side and the right-hand side of equation (1):

$$\mathrm{MAPE} = \frac{1}{T} \int_0^T \left| \frac{\mathrm{Left}(t) - \mathrm{Right}(t)}{\mathrm{Right}(t)} \right| dt \cdot 100\% \quad (13)$$

where

$$\mathrm{Left}(t) = \int_0^T d\tau h(\tau) R(t-\tau),\ \mathrm{Right}(t) = R(t+z). \quad (14)$$

The method of trapezoids is used for an approximate numerical calculation of the function $\mathrm{Left}(t)$:

$$\mathrm{Left}(t) \approx \sum_j \left(\frac{T}{10^3} \cdot \frac{1}{2} \cdot \left(h(j) R(t-j) + \right.\right.$$
$$\left.\left. + h\left(j + \frac{T}{10^3}\right) R\left(t - j - \frac{T}{10^3}\right)\right)\right), \quad (15)$$
$$j = 0, 1 \cdot \frac{T}{10^3}, 2 \cdot \frac{T}{10^3}, \ldots, (10^3 - 1) \cdot \frac{T}{10^3}.$$

The MAPE is roughly estimated as

DOI 10.15588/1607-3274-2023-3-5

$$\mathrm{MAPE} = \frac{1}{10^2} \cdot \sum_j \left| \frac{\mathrm{Left}(j) - \mathrm{Right}(j)}{\mathrm{Right}(j)} \right| \cdot 100\%, \qquad (16)$$
$$j = 0, 1 \cdot \frac{T}{10^2}, 2 \cdot \frac{T}{10^2}, \ldots, \left(10^2 - 1\right) \cdot \frac{T}{10^2}.$$

Numerical results for the MAPE are given in the next section.

5 RESULTS

The obtained results for the MAPE are given in Table 1.

Table 1 – MAPE values (in %, rounded off to 2 significant digits) for the approximation of n Walsh functions for different values of the parameter a

	$a = 0.08$	$a = 0.4$	$a = 0.8$
$n = 2$	1.9	7.6	15
$n = 4$	0.93	3.4	6.7
$n = 8$	0.66	1.7	3.0
$n = 16$	0.44	0.92	1.5
$n = 32$	0.30	0.40	0.62
$n = 64$	0.23	0.37	0.50

Comparison graphs for the approximations of 64 Walsh functions are given in Fig. 1 – Fig. 3.

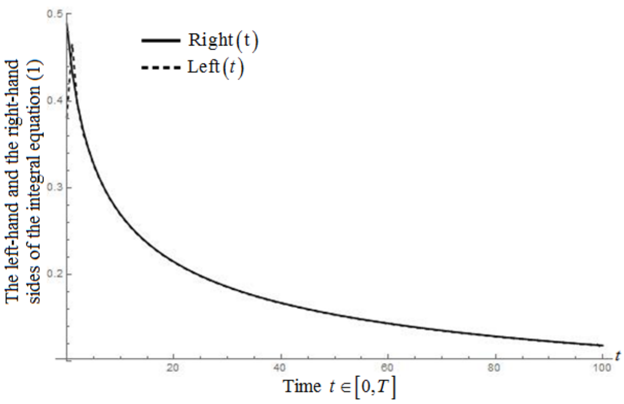

Figure 1 – Graphs of $\mathrm{Left}(t)$ and $\mathrm{Right}(t)$ for the approximation of 64 Walsh functions; $a = 0.8$

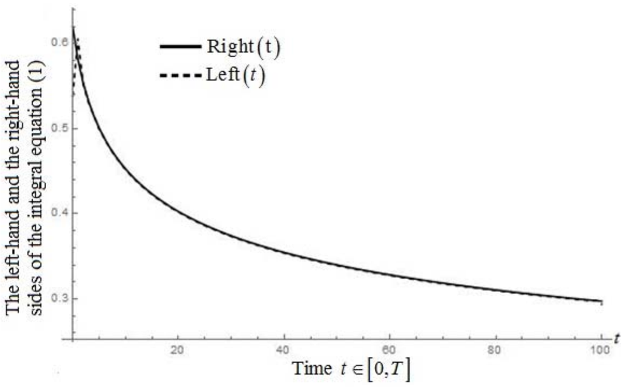

Figure 2 – Graphs of $\mathrm{Left}(t)$ and $\mathrm{Right}(t)$ for the approximation of 64 Walsh functions; $a = 0.4$

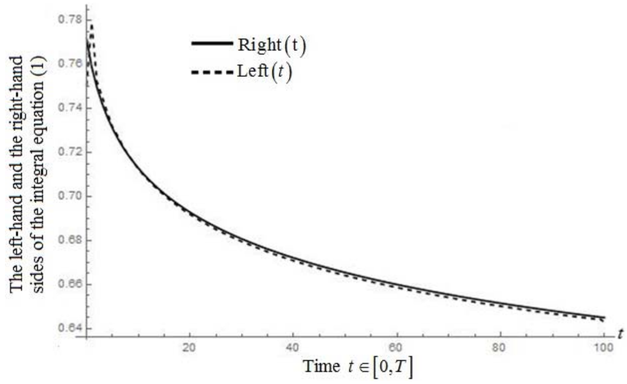

Figure 3 – Graphs of $\mathrm{Left}(t)$ and $\mathrm{Right}(t)$ for the approximation of 64 Walsh functions; $a = 0.08$

As can be seen, the left-hand side and the right-hand side of (1) are rather close.

6 DISCUSSION

As can be seen, the approximations of rather great numbers of the Walsh functions are accurate enough. The accuracy increases with the number of the Walsh functions. Thus, the proposed method of solving the integral equation (1) is convergent for the correlation function (2).

It should also be stressed that the corresponding approximations are rather accurate for all the three considered values of the parameter a, thus indicating that the proposed method works well. Of course, the estimation of the MAPE on the basis of (16) is rather rough, but it is adequate in order of magnitude. As can be seen from the obtained graphs, the left-hand side of the integral equation (1) is indeed rather close to the corresponding right-hand side for the approximation of 64 Walsh functions.

The paper is devoted to a theoretical construction of the continuous Kolmogorov-Wiener filter weight function for the prediction of a stationary random process described by the GFGN model. As indicated in [4], that model may give a good description of some actual telecommunication traffic data.

The corresponding weight function is the solution of the Wiener-Hopf integral equation (1). This equation is solved on the basis of the Galerkin method with the help of a truncated expansion in the Walsh functions. The Walsh functions in the Walsh numeration are used. The algorithm of the weight function derivation is described in detail. The quality of agreement between the left-hand side and the right-hand side of the integral equation is estimated by the MAPE parameter. It is shown that for a rather large number of Walsh functions the agreement is good enough, and the corresponding MAPE values are rather small. Graph comparisons are given to illustrate the fact that the corresponding left-hand side is indeed close to the corresponding right-hand side for the obtained solutions. It is shown that the MAPE decreases with the number of Walsh functions, which justifies the convergence of the method. The approximations of up to 64 Walsh functions are investigated. The calculations are made with the help of the Wolfram Mathematica package.

DOI 10.15588/1607-3274-2023-3-5

The use of a Walsh function expansion is known not only for integral equations, but also for variational calculus, see [17, 18]. As is known, variational calculus is widely used in optimal control problems, see, for example, [19, 20]. Paper [20] is devoted to a practical electrical engineering problem, so the mathematics of this paper may be applicable in part to, for example, electrical engineering problems.

CONCLUSIONS

The Kolmogorov-Wiener filter weight function for the prediction of continuous stationary telecommunication traffic in the GFGN model is calculated on the basis of the truncated Walsh function expansion method. Approximations up to 64 Walsh functions are investigated. The convergence of the method is illustrated.

The results of this paper may be useful for the practical prediction of stationary telecommunication traffic in systems with data packet transfer.

The scientific novelty of the paper is the fact that for the first time the Kolmogorov-Wiener filter weight function is calculated for the prediction of telecommunication traffic in the GFGN model.

The practical significance is that the obtained results may be applied to the practical prediction of telecommunication traffic in systems with data packet transfer.

Prospects for further research are to investigate the Galerkin method for other orthogonal systems of functions, for example, polynomial or trigonometric ones, and to compare the obtained MAPE results with the results based on the Walsh functions. Another plan for the future is to generate simulated data described by the GFGN model and to investigate the corresponding prediction for them.

ACKNOWLEDGEMENTS

The work was made in the framework of the Project "Research of methods of increase of efficiency of the automated control of thermal work of units of high power of industrial and household function" (State Registration No. 0122U002601) of the Dnipro University of Technology.

REFERENCES

1. Jacob S., Y. Qiao, Y. Ye et al.] Anomalous distributed traffic: Detecting cyber security attacks amongst micro-services using graph convolutional networks, *Computers & Security.* 2022, Vol. 118, 102728 (13 pages). DOI: 10.1016/j.cose.2022.102728.
2. Alqahtani H. Learning and prediction of cyber-attack based on correlation mapping in classical optical networks, *Optical and Quantum Electronics,* 2022, Vol. 54, 776. DOI: 10.1007/s11082-022-04176-8.
3. Gorev V., Gusev A., Korniienko V. et al. On the use of the Kolmogorov-Wiener filter for heavy-tail process prediction, *Journal of Cyber Security and Mobility*, 2023, Vol. 12, № 3, pp. 315–338. DOI: 10.13052/jcsm2245-1439.123.4.
4. Li M. Direct Generalized fractional Gaussian noise and its application to traffic modeling, *Physica A,* 2021, Vol. 579, 126138 (22 pages). DOI: 10.1016/j.physa.2021.126138
5. Miller S., Childers D. Probability and Random Processes With Applications to Signal Processing and Communications. Second edition. Amsterdam, Academic Press, 2012, 598 p. DOI: 10.1016/B978-0-12-386981-4.50001-1
6. Alghamdi VT., Mostafi S., Abdelkader G. et al. A Comparative Study on Traffic Modeling Techniques for Predicting and Simulating Traffic Behavior, *Future Internet,* 2022, Vol. 14, 294 (21 pages). DOI: 10.3390/fi14100294.
7. Dogra A. K., Kaur J. Oving towards smart transportation with machine learning and Internet of Things (IoT): a review, *Journal of Smart Environments and Green Computing,* 2022, Vol. 2, pp. 3–18. DOI: 10.20517/jsegc.2021.09.
8. Saha S., Haque A., Sidebottom G. An Empirical Study on Internet Traffic Prediction Using Statistical Rolling Model, *2022 International Wireless Communications and Mobile Computing (IWCMC),* Dubrovnik, 30 May – 03 June, 2022 : proceedings. Los Alamitos, IEEE, 2022, pp. 1058–1063. DOI: 10.1109/IWCMC55113.2022.9825059.
9. Gorev V. N., Gusev A. Yu., Korniienko V. I. Polynomial solutions for the Kolmogorov-Wiener filter weight function for fractal processes, *Radio Electronics, Computer Science, Control,* 2019, № 2, pp. 44–52. DOI: 10.15588/1607-3274-2019-2-5.
10. Gorev V. N., Gusev A. Yu., Korniienko V. I. Approximate solutions for the Kolmogorov-Wiener filter weight function for continuous fractional Gaussian noise, *Radio Electronics, Computer Science, Control*, 2021, № 1, pp. 29–35. DOI: 10.15588/1607-3274-2021-1-3.
11. Gorev V. N., Gusev A. Yu., Korniienko V. I. Kolmogorov-Wiener filter for continuous traffic prediction in the GFSD model, *Radio Electronics, Computer Science, Control,* 2022. № 3, pp. 31–37. DOI: 10.15588/1607-3274-2022-3-3.
12. Kawaminami I., Estrada A., Elsakkary Y. et al. Large Scale Enrichment and Statistical Cyber Characterization of Network Traffic, *2022 IEEE High Performance Extreme Computing Conference, Massachusetts, September 19–23, 2022 : proceedings*. Los Alamitos, IEEE, 2022, pp. 1–7. DOI: 10.1109/HPEC55821.2022.9926397.
13. Wu H., Liu Y., Cheng G. et al. Real-time Identification of VPN Traffic based on Counting Bloom Filter and Chained Hash Table from Sampled Data in High-speed Networks, The ICC 2022 – IEEE International Conference on Communications, Seoul, Republic of Korea, 16–20 May 2022 : proceedings. Los Alamitos, IEEE, 2022, pp. 5070–5075. DOI: 10.1109/ICC45855.2022.9839256.
14. Popoola J., Popoola O. J., Olaniran O. R. Modelling Internet Traffic Streams with Ga/M/1/K Queuing Systems under Self-similarity, *Tanzania Journal of Science,* 2022, Vol. 28, pp. 394–401. DOI: 10.4314/tjs.v48i2.14.
15. Polyanin A. D., Manzhirov A. V.. Handbook of integral equations. Second edition. New York, Chapman and Hall, 2008, 1144 p. DOI: 10.1201/9781420010558.
16. Gorev V., Gusev A., Korniienko V. Fractional Gaussian Noise Traffic Prediction Based on the Walsh Functions, *CEUR Workshop proceedings,* 2021, Vol. 2853, pp. 389–400.
17. Chen C. F., Hsiao C. H. A walsh series direct method for solving variational problems, *Journal of the Franklin Institute*, 1975, Vol. 300, № 4, pp. 265–280. DOI: 10.1016/0016-0032(75)90199-4

DOI 10.15588/1607-3274-2023-3-5

18. Ordokhani Y. Direct Walsh-Hybrid Method for Variational Problems, *International Journal of Nonlinear Science*, 2011, Vol. 11, № 1, pp. 114–120.
19. Kupenko O. P, Manzo R. Approximation of an optimal control problem in coefficient for variational inequality with anisotropic p-laplacian, *Nonlinear Differential Equations and Applications*, 2016, Vol. 23, № 3, 35 (18 pages). DOI: 10.1007/s00030-016-0387-9
20. Diachenko G., Aziukovskyi O., Rogoza M. et al. Optimal Field-Oriented Control of an Induction Motor for Loss Minimization in Dynamic Operation, *2019 IEEE International Conference on Modern Electrical and Energy Systems (MEES), Kremenchuk, 23–25 September 2019, proceedings*. Los Alamitos, IEEE, 2019, pp. 94–97. DOI: 10.1109/MEES.2019.8896455.

Received 05.06.2023.
Accepted 26.08.2023.

УДК 51–74, 517.968.21

ПРОГНОЗУВАННЯ УЗАГАЛЬНЕНОГО ФРАКТАЛЬНОГО ГАУСІВСЬКОГО ШУМУ НА ОСНОВІ ФУНКЦІЙ ВОЛША

Горєв В. М. – канд. фіз.-мат. наук, доцент, завідувач кафедри фізики, Національний технічний університет «Дніпровська Політехніка», Дніпро, Україна.

Гусєв О. Ю. – канд.фіз.-мат. наук, доцент, професор кафедри безпеки інформації та телекомунікацій, Національний технічний університет «Дніпровська Політехніка», Дніпро, Україна.

Корнієнко В. І. – д-р техн. наук, професор, завідувач кафедри безпеки інформації та телекомунікацій, Національний технічний університет «Дніпровська Політехніка», Дніпро, Україна.

Шедлоська Я. І. – канд. техн. наук, доцент кафедри інформаційних технологій та комп'ютерної інженерії, Національний технічний університет «Дніпровська Політехніка», Дніпро, Україна.

АНОТАЦІЯ

Актуальність. Деякі з нещодавніх статей авторів присвячені фільтру Колмогорова-Вінера для прогнозування телекомунікаційного трафіку в деяких стаціонарних моделях, таких як модель фрактального гаусівського шуму, модель степеневої структурної функції та GFSD (Gaussian fractional sum-difference) модель. Нещодавно так звана модель узагальненого фрактального гаусівського шуму була запропонована для опису стаціонарного телекомунікаційного трафіку в деяких випадках. Тож в цій статті досліджено теоретичні основи неперервного фільтра Колмогорова-Вінера, застосовного для прогнозування узагальненого фрактального гаусівського шуму.

Мета роботи. Метою роботи є отримати вагову функцію фільтра як наближений розв'язок відповідного інтегрального рівняння Вінера-Хопфа з ядром, що дорівнює кореляційній функції узагальненого фрактального гаусівського шуму.

Метод. Метод обірваних розвинень за функціями Волша запропоновано для отримання відповідного розв'язку. Таке розвинення є частинним випадком методу Галеркіна, в рамках якого невідома функція шукається у вигляді обірваного розвинення за ортогональними функціями. Інтегральні дужки та результати для середньої абсолютної відсоткової помилки відхилу лівої частини інтегрального рівняння Вінера-Хопфа від правої обчислені чисельно на основі пакету Wolfram Mathematica.

Результати. Дослідження зроблене для наближень включно до наближення шістдесяти чотирьох функцій Волша. Досліджено різні параметри моделі. Показано, що для різних параметрів моделі запропонований метод є збіжним та призводить до малих середніх абсолютних відсоткових помилок для наближень доволі великої кількості функцій Волша.

Висновки. Статтю присвячено теоретичній побудові вагової функції неперервного фільтра Колмогорова-Вінера для прогнозування стаціонарного випадкового процесу, що описується моделлю узагальненого фрактального гаусівського шуму. Як відомо, така модель може добре описувати певні експериментальні дані в системах з пакетною передачею даних. Відповідна вагова функція шукається на основі обірваного розвинення за функціями Волша. Відповідні помилки відхилу є малими та метод є збіжним.

КЛЮЧОВІ СЛОВА: неперервний фільтр Колмогорова-Вінера, вагова функція, метод Галеркіна, функції Волша, узагальнений фрактальний гаусівський шум, телекомунікаційний трафік.

ЛІТЕРАТУРА

1. Anomalous distributed traffic: Detecting cyber security attacks amongst micro-services using graph convolutional networks / [S. Jacob, Y. Qiao, Y. Ye et al.] // Computers & Security. – 2022. – Vol. 118. – 102728 (13 pages). DOI: 10.1016/j.cose.2022.102728.
2. Alqahtani H. Learning and prediction of cyber-attack based on correlation mapping in classical optical networks / H. Alqahtani // Optical and Quantum Electronics. – 2022. – Vol. 54. – 776. DOI: 10.1007/s11082-022-04176-8.
3. On the use of the Kolmogorov-Wiener filter for heavy-tail process prediction / [V. Gorev, A. Gusev, V. Korniienko et al.] // Journal of Cyber Security and Mobility. – 2023. – Vol. 12, № 3. – P. 315–338. DOI: 10.13052/jcsm2245-1439.123.4.
4. Li M. Direct Generalized fractional Gaussian noise and its application to traffic modeling / M. Li // Physica A. – 2021. – Vol. 579. – 126138 (22 pages). DOI: 10.1016/j.physa.2021.126138
5. Miller S. Probability and Random Processes With Applications to Signal Processing and Communications. Second edition / S. Miller, D. Childers. – Amsterdam: Academic Press, 2012. – 598 p. DOI: 10.1016/B978-0-12-386981-4.50001-1
6. A Comparative Study on Traffic Modeling Techniques for Predicting and Simulating Traffic Behavior / [VT. Alghamdi, S. Mostafi, G. Abdelkader et al.] // Future Internet. – 2022. – Vol. 14. – 294 (21 pages). DOI: 10.3390/fi14100294.
7. Dogra A. K. oving towards smart transportation with machine learning and Internet of Things (IoT): a review / A. K. Dogra, J. Kaur // Journal of Smart Environments and Green Computing. – 2022. – Vol. 2. – P. 3–18. DOI: 10.20517/jsegc.2021.09.
8. Saha S. An Empirical Study on Internet Traffic Prediction Using Statistical Rolling Model / S. Saha, A. Haque,

DOI 10.15588/1607-3274-2023-3-5

G. Sidebottom // 2022 International Wireless Communications and Mobile Computing (IWCMC), Dubrovnik, 30 May – 03 June, 2022 : proceedings. Los Alamitos: IEEE, 2022. – P. 1058–1063. DOI: 10.1109/IWCMC55113.2022.9825059.

9. Gorev V. N. Polynomial solutions for the Kolmogorov-Wiener filter weight function for fractal processes / V. N. Gorev, A. Yu. Gusev, V. I. Korniienko // Radio Electronics, Computer Science, Control. – 2019. – № 2. – P. 44–52. DOI: 10.15588/1607-3274-2019-2-5.
10. Gorev V. N. Approximate solutions for the Kolmogorov-Wiener filter weight function for continuous fractional Gaussian noise / V. N. Gorev, A. Yu. Gusev, V. I. Korniienko // Radio Electronics, Computer Science, Control. – 2021. – № 1. – P. 29–35. DOI: 10.15588/1607-3274-2021-1-3.
11. Gorev V. N. Kolmogorov-Wiener filter for continuous traffic prediction in the GFSD model / V. N. Gorev, A. Yu. Gusev, V. I. Korniienko // Radio Electronics, Computer Science, Control. – 2022. – № 3. – P. 31–37. DOI: 10.15588/1607-3274-2022-3-3.
12. Large Scale Enrichment and Statistical Cyber Characterization of Network Traffic / [I. Kawaminami, A. Estrada, Y. Elsakkary et al.] // 2022 IEEE High Performance Extreme Computing Conference, Massachusetts, September 19–23, 2022 : proceedings. Los Alamitos: IEEE, 2022. – P. 1–7. DOI: 10.1109/HPEC55821.2022.9926397.
13. Real-time Identification of VPN Traffic based on Counting Bloom Filter and Chained Hash Table from Sampled Data in High-speed Networks / [H. Wu, Y. Liu, G. Cheng et al.] // The ICC 2022 – IEEE International Conference on Communications, Seoul, Republic of Korea, 16–20 May 2022 : proceedings. Los Alamitos: IEEE, 2022. – P. 5070–5075. DOI: 10.1109/ICC45855.2022.9839256.
14. Popoola J. Modelling Internet Traffic Streams with Ga/M/1/K Queuing Systems under Self-similarity / J. Popoola, O. J. Popoola, O. R. Olaniran // Tanzania Journal of Science. – 2022. – Vol. 28. – P. 394–401. DOI: 10.4314/tjs.v48i2.14.
15. Polyanin A. D. Handbook of integral equations. Second edition / A. D. Polyanin, A. V. Manzhirov. – New York : Chapman and Hall, 2008. – 1144 p. DOI: 10.1201/9781420010558.
16. Gorev V. Fractional Gaussian Noise Traffic Prediction Based on the Walsh Functions / V. Gorev, A. Gusev, V. Korniienko // CEUR Workshop proceedings. – 2021. – Vol. 2853. – P. 389–400.
17. Chen C. F. A walsh series direct method for solving variational problems / C.F. Chen, C.H. Hsiao // Journal of the Franklin Institute. – 1975. – Vol. 300, № 4. – P. 265–280. DOI: 10.1016/0016-0032(75)90199-4
18. Ordokhani Y. Direct Walsh-Hybrid Method for Variational Problems / Y. Ordokhani // International Journal of Nonlinear Science. – 2011. – Vol. 11, №1. – P. 114–120.
19. Kupenko O. P. Approximation of an optimal control problem in coefficient for variational inequality with anisotropic p-laplacian / O. P. Kupenko, R. Manzo // Nonlinear Differential Equations and Applications. – 2016. – Vol. 23, № 3. – 35 (18 pages). DOI: 10.1007/s00030-016-0387-9
20. Optimal Field-Oriented Control of an Induction Motor for Loss Minimization in Dynamic Operation / [G. Diachenko, O. Aziukovskyi, M. Rogoza et al.] // 2019 IEEE International Conference on Modern Electrical and Energy Systems (MEES), Kremenchuk, 23–25 September 2019 : proceedings. Los Alamitos : IEEE, 2019. – P. 94–97. DOI: 10.1109/MEES.2019.8896455.

DOI 10.15588/1607-3274-2023-3-5