МАТЕМАТИЧНЕ ТА КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ

MATHEMATICAL AND COMPUTER MODELING

UDC 004.827:519.816

DEVELOPMENT OF TECHNIQUE FOR STRUCTURING OF GROUP EXPERT ASSESSMENTS UNDER UNCERTAINTY AND INCONCISTANCY

Davydenko Ye. O. – PhD, Associate professor, Head of Department of Software Engineering, Petro Mohyla Black Sea National University, Mykolayiv, Ukraine.

Shved A. V. – Dr. Sc., Professor, Professor of Department of Software Engineering, Petro Mohyla Black Sea National University, Mykolayiv, Ukraine.

Honcharova N. V. – Post-graduate student of Department of Software Engineering, Petro Mohyla Black Sea National University, Mykolayiv, Ukraine.

ABSTRACT

Context. The issues of structuring group expert assessments are considered in order to determine a generalized assessment under inconsistency between expert assessments. The object of the study is the process of synthesis of mathematical models of structuring (clustering, partitioning) of expert assessments that are formed within the framework of Shafer model under uncertainty, inconsistency (conflict).

Objective. The purpose of the article is to develop an approach based on the metrics of theory of evidence, which allows to identify a number of homogeneous subgroups from the initial heterogeneous set of expert judgments formed within the framework of the Shafer model, or to identify experts whose judgments differ significantly from the judgments of the rest of the group.

Method. The research methodology is based on the mathematical apparatus of theory of evidence and cluster analysis. The proposed approach uses the principles of hierarchical clustering to form a partition of a heterogeneous (inconsistent) set of expert evidence into a number of subgroups (clusters), within which expert assessments are close to each other. Metrics of the theory of evidence are considered as a criterion for determining the similarity and dissimilarity of clusters. Experts' evidence are considered consistent in the formed cluster if the average or maximum (depending on certain initial conditions) level of conflict between them does not exceed a given threshold level.

Results. The proposed approach for structuring expert information makes it possible to assess the degree of consistency of expert assessments within an expert group based on an analysis of the distance between expert evidence bodies. In case of a lack of consistency within the expert group, it is proposed to select from a heterogeneous set of assessments subgroups of experts whose assessments are close to each other for further aggregation in order to obtain a generalized assessment.

Conclusions. Models and methods for analyzing and structuring group expert assessments formed within the notation of the theory of evidence under uncertainty, inconsistency, and conflict were further developed. An approach to clustering group expert assessments formed under uncertainty and inconsistency (conflict) within the framework of the Shafer model is proposed in order to identify subgroups within which expert assessments are considered consistent. In contrast to existing clustering methods, the proposed approach allows processing expert evidence of a various structure and taking into account possible ways of their interaction (combination, intersection).

KEYWORDS: theory of evidence, distance metric, dissimilarity measure, clustering, expert evidence, uncertainty, inconsistency.

ABBREVIATIONS

bpa is a basic probability assignment;
CCT is a cophenetic correlation test;
DI is a Dunn index;
DST is a Dempster-Shafer theory;
SSE is a sum of the squared error.

NOMENCLATURE

A is a set of alternatives;
avg(·) is an arithmetic average of its argument;
B is a set of expert preference profiles;
B_j reflects the preferences (choice) of expert E_j;
b_k^j is a *k*-th evidence formed, within the given scale of preferences, by the expert E_j;
Conf(E_k, G_q) is a measure reflecting the degree of conflict between E_k and group G_q;
ConfLev is a given limit level of conflict;
$d(m_i, m_j)$ is a distance metric value;
$d_J(m_i, m_j)$ is a value of the *Jousselme's* distance measure between two groups of evidence;
Dst is a matrix of pairwise distances;
E is a group of experts;
E^* is a set of experts candidates for the subgroup with consistent estimates G_q;
E_o is an expert whose preferences are selected as a reference element;
E^{conf} is a group of experts whose assessments differ significantly from the assessments of the rest of the group;

DOI 10.15588/1607-3274-2023-4-3

f is a single group decision;

f_t is an individual expert preference;

G_q is a group of experts with consistent assessments;

G_i^j is a subgroup of expert evidence for which the level of conflict l_i is acceptable;

P is a preference relation of the type P={$\succ$} (strict ordering), or P={$\succ, \sim$} (non-strict ordering);

R_{rez} is a result ranking;

l_q is a predetermined threshold level of conflict responsible for expert E_j belonging to the subgroup G_q;

l_0 is considered equal to 0;

m_i is a 2^A-dimensional vector-column, the elements of which are the *bpa's* of focal elements formed over the *i*-th group of evidence;

$(m_i)^T$ is a transposed vector m_i (string vector);

m_j^q is a vector of *bpa's* formed by the expert E_j in group G_q;

(m1–m2) is a difference of the corresponding vectors;

n is a number of examination objects (alternatives);

p is a number of formed groups of experts G_q with consistent assessments;

r is a number of experts in G_q;

$S(B_i, B_j)$ is a *Jaccard* coefficient;

t is a number of experts in expert group E;

2^A is a set of all possible subsets formed on the set A;

$[\pi]$ is an operator for processing individual expert assessments (methods, rules, algorithms);

$|\cdot|$ is a cardinality of its argument.

INTRODUCTION

Group choice usually means the development of an agreed group decision on the order of preference of analyzed objects based on the individual judgments of experts. In other words, the problem of group choice is the problem of structuring individual preferences $f_1, f_2, \ldots, f_t$ into a single group decision f [1]:

$$\underset{[\pi]}{\overset{}{(f_1, f_2, \ldots, f_n) \Rightarrow f}}. \quad (1)$$

To select a method for obtaining a generalized assessment based on a set of group expert assessments, first need to test them for homogeneity (consistency). The results of such testing can lead to one of two possible cases:

1) the set of expert assessments is characterized by a high degree of consistency (which indicates their homogeneity);

2) the group of experts contains those whose assessments may differ in value from the assessments of the majority, the presence of such assessments in the total set of group expert assessments violates its homogeneity (consistency).

If the analysis reveals a high degree of consistency, a procedure of expert evidence aggregation is performed in order to obtain a final (group) ordering (ranking) of the analyzed objects (alternatives) in form of:

$$R_{rez} : A_j \mathrm{P} A_k \mathrm{P} \ldots \mathrm{P} A_z, \quad \forall (A_j, A_k, A_z) \in \mathrm{A}. \quad (2)$$

The lack of consistency (homogeneity) indicates the presence in the commission of such experts who have different (but similar (homogeneous, agreed upon) within the same subgroup) points of view on solving the problem under consideration. Such situation arises, for example, due to the presence among the group of experts of representatives of different scientific schools or even teams. In the worst case, as a result of an expert survey, a significant number of small subgroups of experts are formed, with consistent judgments.

As a result, two tasks arise:

1) identifying and excluding outlier observations;

2) division (clustering) of the initial set of experts' judgments into several subgroups (clusters) of experts with similar (agreed, homogeneous) assessments, for their further analysis and determination of the aggregated assessment.

The object of study is the process of synthesis of mathematical models of structuring (clustering, partitioning) of expert assessments that are formed within the framework of Shafer model under uncertainty, inconsistency (conflict).

The subject of study is the models and methods of the group expert assessment analysis and structuring in the context of multi-alternative, inconsistency, conflict, uncertainty and their combinations.

The purpose of the work is a development of an approach based on the metrics of theory of evidence, which allows to identify a number of homogeneous subgroups from the initial heterogeneous set of expert judgments formed within the framework of the Shafer model, or to identify experts whose judgments differ significantly from the judgments of the rest of the group.

1 PROBLEM STATEMENT

Let a group of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$, evaluating some initial set of objects of expertise (alternatives) $\mathrm{A} = \{A_i \mid i = \overline{1,n}\}$, forms profiles of expert preferences $\mathrm{B} = \{B_j \mid j = \overline{1,t}\}$, where B_j is a 2^A-dimensional vector. Profile $B_j = \{b_k^j \mid k = \overline{1,s}\}$, $s = 2^{|A|}$, reflects the preferences (choice) of expert E_j, each element of which is built on the basis of a system of rules:

$$\begin{aligned}
&1.\ b_k^j = \{\varnothing\};\\
&2.\ b_k^j = \{A_i\};\\
&3.\ b_k^j = \{A_i \mid i = \overline{1,v}\},\ v < n;\\
&4.\ b_k^j = \mathrm{A} = \{A_i \mid i = \overline{1,n}\}.
\end{aligned} \quad (3)$$

The task consists (in case of absence of agreement between the opinions of the members of the expert commission) to identify from the total set of expert judgments, subgroups of experts $\mathrm{E} \Rightarrow \{G_1\}, \{G_2\}, \ldots, \{G_q\}, \ldots, \{G_p\}$ ($G_q \subseteq \mathrm{E}$, $\{G_q\} = \{E_1, \ldots, E_r\}$, $t \geq r \geq 1$, $t \geq p \geq 1$), who have

DOI 10.15588/1607-3274-2023-4-3

a similar opinion and identify such experts E_l, who do not belong to any of these subgroups, that is, $E_l \subseteq G_q$, provided, that $|G_q| = 1$ (if any).

We will assume that:

1) judgments of $E_j \subseteq G_q$, $l \geq 2$ are considered consistent;

2) judgments of $E_j \subseteq G_q$, $|G_q| = 1$ are considered atypical, that is, significantly different (conflict) from other expert judgments.

Provided that $p = 1$ (and therefore $t = r$) the evidence of the entire group E are considered consistent.

If there is a trend $p \to t$ and $r \to 1$ (formation of a significant number of small groups G_q) the further analysis is inappropriate.

An example of the worst situation is the formation of the maximum possible number of subgroups, such that $\forall\, G_q : |G_q| = 1$ ($q = \overline{1, p}$, $p = t$); moreover, the best situation is considered to be in which $|G_q| = t$, q=1.

Thus, it is necessary to construct a decision rule that allows one to unambiguously determine whether the expert E_l belongs to the group Gr_q.

Further, additional procedures can be applied to bring together the opinions of different subgroups. Or, provided that the expert evidence are stable and final (formed taking into account the positions of all survey participants), the procedure for aggregating expert evidence is carried out for each of the resulting subgroups of experts Gr_q separately.

2 REVIEW OF THE LITERATURE

An analysis of methods that can be used to solve the problem of dividing group expert assessments into homogeneous, in a certain sense, subgroups has shown that their effective implementation is not always possible. For example, when analyzing expert assessments formed within the framework of numerical scales (absolute), the following methods have become widely used: cluster analysis methods based on the determination of distance functions, for example, Euclidean distance, Manhattan distance, Chebyshev distance, etc. [2–4]; clustering based on mathematical programming methods (dynamic programming, integer programming) [5, 6]; clustering based on estimation of probability density functions [7], etc.

To analyze expert judgments formed in ratio or order scales, non-numeric data clustering methods, for example, the Kemeny median method [8], can be used.

A justified choice and use of the considered methods for solving the problem of dividing group expert assessments in order to search for homogeneous subgroups can be carried out provided that various types of ignorance that arise in the process of obtaining and processing expert information are correctly taken into account. It is also necessary to take into account the possible structure of expert evidence (consonant, consistent, arbitrary, etc.), and take into account possible ways of their interaction (intersection, union, absorption) [9].

An effective mathematical apparatus that allows to correctly operate with such types of structures of expert evidence is the theory of evidence (Dempster-Shafer theory, DST) [10–12]. To solve the problem of assessing the distance between different types of structures of expert evidence in order to determine the degree of similarity of expert evidence, distance measures of evidence in Dempster-Shafer theory [13–16] can be applied.

3 MATERIALS AND METHODS

Let $\mathrm{A} = \{A_i \mid i = \overline{1, n}\}$ be a set of alternatives and a group of experts $\mathrm{E} = \{E_j \mid j = \overline{1, t}\}$ carrying out the examination. Within the notation of the DST, the set of initial data (alternatives, objects of examination) called the frame of discernment is a set of exhaustible and mutually exclusive elements [10–12]. Based on the analysis of A, according to the results of the expert survey, a subset system $\mathrm{B} = \{B_j \mid j = \overline{1, t}\}$ can be formed, where B_j is a 2^{A}-dimensional vector reflecting the preferences (choice) of the expert E_j, each element of which is built according to a system of rules (3).

So, for example, by assessing the initial set of alternatives A=$\{a, b, c\}$ by a group of experts E=$\{E_1, E_2\}$ the following profiles of expert preferences can be formed:

$$B_1=\{\{a\},\{b, c\}\}; \qquad B_2=\{\{a\},\{b\},\{c\}\}.$$

If the condition $\forall b_k^j \in B_j : (|\, b_k^j \,| = 1) \wedge (|\, B_j \,| = n)$ satisfied for $\forall\, B_j \subset \mathrm{B}$, then the results of expert survey in form of a set of group expert judgments (evidence) can be presented in the form of $n \times t$ dimension matrix:

$$\mathrm{B} = \begin{pmatrix} B_1 \\ B_2 \\ \dots \\ B_j \\ \dots \\ B_t \end{pmatrix} = \begin{pmatrix} b_1^1 & b_2^1 & \dots & b_n^1 \\ b_1^2 & b_2^2 & \dots & b_n^2 \\ \dots & \dots & \dots & \dots \\ b_1^j & b_2^j & \dots & b_n^j \\ \dots & \dots & \dots & \dots \\ b_1^t & b_2^t & \dots & b_n^t \end{pmatrix}. \tag{4}$$

In matrix (4), each row includes the judgments of an expert E_j, for all objects, and the column includes judgments of the entire group of experts for a given object A_i.

For each subset B_j, $j = \overline{1, t}$, a vector of *bpa's* $\mathrm{m}_j = \{m_i \mid i = \overline{1, s}\}$, $s = 2^{|\mathrm{A}|}$, will be constructed whose elements satisfy the condition [11, 12], m: $2^{\mathrm{A}} \to [0,1]$:

$$0 \leq m(b_k^j) \leq 1, \quad m(\varnothing) = 0, \quad \sum_{b_k^j \in B_j} m(b_k^j) = 1, \tag{5}$$

One of the metrics of the distance between expert evidence is taken as a measure of conflict [13–16]. Since expert evidence cannot be expressed in numerical terms, it is possible to establish that the original objects (experts) belong to any groups (classes) only on the basis of their similarity to each other.

The choice of metric is one of the main factors influencing the results of partitioning the initial set of expert

DOI 10.15588/1607-3274-2023-4-3

evidence and forming subgroups of experts with fairly close estimates. As a rule, the choice of metric is quite subjective and is determined by an analyst independently based on his / her own experience.

Let us consider the procedure for generation of G_q ($\forall\ G_q^{conf} \subset$ E) provided that the evidence of $E_j \subseteq G_q$ do not exceed the specified threshold *ConfLev* of the conflict coefficient.

1. Assessing the degree of similarity of expert evidence. For each pair <m_i, m_j>, $\forall (i,j)=\overline{1,t}$, $i \neq j$, estimates of the distance measure are determined, for example, the *Jousselme* distance [15]:

$$d_J(\mathrm{m}_1,\mathrm{m}_2)=\sqrt{\frac{1}{2}(\mathrm{m}_1-\mathrm{m}_2)^{\mathrm{T}}\underset{=}{D}(\mathrm{m}_1-\mathrm{m}_2)} \qquad (6)$$

where $\underset{=}{D}$ is a matrix of $2^{\mathrm{A}} \times 2^{\mathrm{A}}$ dimension, the elements of which are defined as

$$D(B,B)=\begin{cases}1, & \text{if } B=B;\\ S(B,B), & \forall B,B\in \mathrm{A}.\end{cases} \qquad (7)$$

The $S(B_i, B_j)$ function corresponds to the *Jaccard* coefficient $S(B_i,B_j)=|B_i \cap B_j|/|B_i \cup B_j|$.

The results are stored in the form of a matrix of pairwise distances, which is symmetrical about the main diagonal in form of:

$$Dst=\begin{pmatrix} - & d(\mathrm{m}_1,\mathrm{m}_2) & \dots & d(\mathrm{m}_1,\mathrm{m}_t)\\ d(\mathrm{m}_2,\mathrm{m}_1) & - & \dots & d(\mathrm{m}_2,\mathrm{m}_t)\\ \dots\dots & \dots\dots & - & \dots\dots\\ d(\mathrm{m}_t,\mathrm{m}_1) & d(\mathrm{m}_t,\mathrm{m}_2) & \dots & - \end{pmatrix} \qquad (8)$$

where $d(\mathrm{m}_i, \mathrm{m}_j) = d(\mathrm{m}_j, \mathrm{m}_i)$, $\forall (i,j)=\overline{1,t}$, $i \neq j$.

2. Formation of a set of candidates $\mathrm{E}^* = \mathrm{E} = \{E_j \mid j=\overline{1,t}\}$ in group G_q.

3. Determination of the acceptable conflict level *ConfLev*.

4. Formation of a subgroup of experts $G_q \subseteq$ E, $q=\overline{1,p}$.

4.1 In matrix (8), the minimum value of measure $d(\mathrm{m}_i, \mathrm{m}_j)$ is sought, which corresponds to the distance between the two closest evidence E_i and E_j.

If $d(m_i, m_j)$ does not exceed a given *ConfLev* level, then evidence E_i and E_j ($E_i, E_j \in \mathrm{E}^*$) are added to the cluster G_q and removed from the set $\mathrm{E}^* = \mathrm{E}^* \setminus (E_i \cup E_j)$.

If such a pair is not found, the algorithm stops. It is assumed that $|\mathrm{E}^*|$ single-element $E_k \in \mathrm{E}^*$ subgroups are formed from the elements of the set E^*.

4.2 For $\forall\ E_k \in \mathrm{E}^*$ in (8) the minimum value of the measure is sought, which reflects the degree of conflict between E_k and the group G_q [17]:

$$Conf(E_k,G_q)=\frac{1}{r}\sum_{j=1}^{r} d(\mathrm{m}_k,\mathrm{m}_j^q),\ r=|G_q|. \qquad (9)$$

If the value (9) does not exceed the specified *ConfLev* level (if necessary, an additional condition is imposed: $\forall\ E_j \in G_q$: $d(\mathrm{m}_k, \mathrm{m}_j) \le ConfLev$, $k \neq j$), then E_k evidence is added to the cluster G_q and removed from the set $\mathrm{E}^* = \mathrm{E}^* \setminus E_k$.

If all elements of E^* have been sorted out, then proceed to step 5.

5. Correction of the matrix *Dst* by removing elements belonging to a set $\mathrm{E} \setminus \mathrm{E}^*$.

6. Repeat steps 4–5 until $\mathrm{E}^* \neq \varnothing$.

Let us consider the procedure for generation of G_q provided that the estimates of $E_j \subseteq G_q$ do not exceed the specified threshold conflict level l_q.

1. Assessing the degree of similarity of expert evidence. Formation of matrix (8) elements.

2. Formation of a set of candidates $\mathrm{E}^* = \mathrm{E} = \{E_j \mid j=\overline{1,t}\}$.

3. Establishment of threshold values l_q, $q=\overline{1,p}$, responsible for certain levels of conflict (for example, low, medium, high conflict).

4. Selection a reference element $E_0 \in \mathrm{E}^*$.

*Algorithm*_1:

4.1*a* $\forall\ E_k \in \mathrm{E}^*$ it is defined estimates characterizing the degree of conflict between E_k and $\mathrm{E} \setminus E_k$ [17]:

$$Conf(E_k,\mathrm{E}^*)=\frac{1}{t^*-1}\sum_{j=1,\,j\neq k}^{t^*} d(\mathrm{m}_k,\mathrm{m}_j),\ t^*=|\mathrm{E}^*|. \qquad (10)$$

4.2*a* The reference element $E_0 \in \mathrm{E}^*$ is selected, which is provide $\min(Conf(E_0, \mathrm{E}^*))$. Element E_0 is a least conflicting in relation to the entire group of experts.

*Algorithm*_2**:**

4.1*b* $\forall\ E_k \in \mathrm{E}^*$ are determined estimates in accordance with (10).

4.2*b* A subgroup of elements $\mathrm{E}^{conf} \subset \mathrm{E}^*$ is formed such that for $E_j \in \mathrm{E}^{conf}$ the value of measure (10) is significantly different (sharply different) from the value of measure (10) for the rest of the group $\mathrm{E}^* \setminus \mathrm{E}^{conf}$.

4.3*b* The reference element $E_0 \in \mathrm{E}^*$ is selected, which is provide $\min(Conf(E_0, \mathrm{E}^* \setminus \mathrm{E}^{conf}))$. Element E_0 is a least conflicting in relation to a group of experts from which experts with conflicting evidence are excluded.

*Algorithm*_3:

4.1*c* Based on the values of matrix (8), a set G_1^j, $j=\overline{1,t}$ is formed. The subgroup G_1^j includes estimates of $E_k \in \mathrm{E}^*$ for which the following condition is satisfied:

$$\forall (E_j,E_k)\in G_1^j : d(\mathrm{m}_j, \mathrm{m}_k) \le l_1,\ k=\overline{1,t}, j\neq k. \qquad (11)$$

Thus, for a subgroup G_1^j, the $E_j \in \mathrm{E}^*$ is a reference element.

DOI 10.15588/1607-3274-2023-4-3

4.2*c* The reference element $E_o = E_j,\ E_j \in E^*$, is selected such that $\max_j(|G_1^j|)$. That is, a reference element E_0 ensures the formation of the largest group of consistent (with the lowest specified level of conflict) evidence.

5. Based on the values of matrix (8), $\forall\ l_q,\ q=\overline{1,p}$ according to the reference element E_o, the resulting subgroups are formed on the set of E^* in such a way that $\forall E_j \in G_q,\ q=\overline{1,p}$, $r \geq 1$ the following condition is satisfied:

$$j=\overline{1,r},\ \ l_{q-1} < d(m_o, m_j) \leq l_q. \qquad (12)$$

5.1 When forming a cluster $G_q,\ q=\overline{1,p}$, all elements of the set E^* are searched for compliance with condition (12).

The element E_j does not fall into the class G_q if the condition $\forall\ E_j, E_s \in G_q$: $d(m_j, m_s) \leq l_q,\ j \neq s,$ is not satisfied.

If E_j is added to the cluster G_q, then it is removed from the set $E^* = E^* \setminus E_j$.

5.2 The procedure provided for in clause 5.1 is repeated p–1 times or terminated early if $E^* = \varnothing$.

4 EXPERIMENTS

A comparative analysis of the proposed approaches to identification of homogeneous subgroups of expert assessments among an inconsistent initial set of expert evidence and agglomerative clustering methods have been carried out. The following classical methods have been considered: Ward's method (*Ward*), single-linkage (*Single*), complete-linkage (*Complete*), centroid (*Centroid*).

The class of agglomerative clustering methods was chosen due to the fact that, firstly, the proposed approach is based on the principles underlying agglomerative algorithms. Secondly, the goal of the proposed approach is to obtain such coverage (partitioning) of the initial set of expert evidence that ensures the formation of subgroups of experts with consistent assessments (consistent in the sense that the level of conflict between expert evidence belonging to the same group does not exceed a given threshold level of conflict) rather than determining the optimal number of classes. Accordingly, it is the principles and mechanisms underlying agglomerative algorithms that make it possible to terminate the agglomeration process at an iteration ahead of schedule, when the merging of clusters occurs at an unacceptable level of conflict. Thereby reducing the running time of the algorithm.

Case 1. For studying the effectiveness of the proposed approach (*Method*_1), which makes it possible to form a partition of a set of assessments into consistent (homogeneous) subgroups, provided that a certain threshold (acceptable) level of conflict *ConfLev* is specified, five test samples were formed, Table. 1.

The task was to form consistent subgroups of expert evidence with a *ConfLev* ≤ 0.3. Testing was carried out for samples of ten, 20 and 30 elements. The maximum sample size did not exceed 30 values, since usually a group of experts does not exceed 25–30 people.

Table 1 – Principles for test samples formation

Sample	Sample formation method
A	consistent estimates (max distance between evidence is equal to 0.2)
B	moderately conflicting expert evidence (30% of the sample is a group of expert evidence with an average distance equal to 0.3 in relation to the expert evidence of the main group)
C	conflicting expert evidence (30% of the sample is a group of expert evidence with an average distance equal to 0.3 in relation to the expert evidence of the main group; 17% of the sample is a group of expert evidence with an average distance equal to 0.4 in relation to the expert evidence of the main group)
D	highly conflicting expert evidence (17% of the sample is a group of expert evidence with an average distance equal to 0.5 in relation to the expert evidence of the main group)
E	highly conflicting expert evidence (17% of the sample is a single expert evidence with an average distance equal to 0.6 in relation to the expert evidence of the main group)

Case 2. To study the effectiveness of the approach (*Method*_2), which makes it possible to form a partition of a set of estimates into consistent (homogeneous) subgroups, provided that several different threshold levels of conflict l_q are specified, a method was chosen based on the search for a reference element using *Algorithm*_3.

Testing was carried out for samples of ten, 20 and 30 values.

Rule for generating a test sample:

– 50% of the sample is a group of expert evidence with max distance between evidence equal to: 0.170 (n = 10); 0.234 (n = 20); 0.220 (n = 30);

– 13% of the sample is a group of expert evidence with an average distance of 0.1 in relation to the expert evidence of the main group;

– 13% of the sample is a group of expert evidence with an average distance of 0.2 in relation to the expert evidence of the main group;

– 13% of the sample is a group of expert evidence with an average distance of 0.3 in relation to the expert evidence of the main group;

– 11% of the sample is a group of expert evidence with an average distance of 0.4 in relation to the expert evidence of the main group.

5 RESULTS

Let's analyze the results obtained.

Case 1. Table 2 shows the values of the obtained cophenetic correlation coefficient using the Mantel test for clustering results.

As can be seen from Table 2, in most cases the proposed method gives the maximum value of cophenetic correlation coefficient (p-value = 0.001).

For samples *C* and *E*, testing was carried out only for the samples of 20 and 30 elements.

Table 3 shows the results of a comparative analysis of the considered clustering methods (F_0 is an average dis-

DOI 10.15588/1607-3274-2023-4-3

tance in a cluster; F_1 is an average distance between clusters).

For samples *B* and *D*, all considered methods gave the same result. Both samples were formed according to the rule: one group of evidence with a moderate (sample *B*) and significant (sample *D*) level of conflict was added to the main consistent population.

For samples *C* and *D*, the proposed method provides the highest value of the silhouette index; for sample *D*, the proposed method provides the maximum silhouette index and modified Dunn index (*DI*); the lowest value of the ratio of the average intra-cluster distance (F_0) to the average inter-cluster distance (F_1), which indicates better separation of clusters and greater compactness of elements in the cluster compared to other methods.

Case 2. The results of the analysis are given in the Table 4.

For samples of sizes ten and 20, the proposed method provides the highest value of the cophenetic correlation coefficient according to the Mantel test (p-value = 0.001), and the formation of a cluster with the largest number of consistent expert evidence.

Table 2 – Analysis of the quality of clustering results

Sample	Method	*n* = 10		*n* = 20		*n* = 30		Sample	*n* = 20		*n* = 30	
		max $d(m_i,m_j)$	***CCT***	**max** $d(m_i,m_j)$	***CCT***	**max** $d(m_i,m_j)$	***CCT***		**max** $d(m_i,m_j)$	***CCT***	**max** $d(m_i,m_j)$	***CCT***
A	Method_1	0.092	**0.700**	0.169	**0.784**	0.169	**0.696**	*C*	0.411	**0.930**	0.441	**0.926**
	Ward		0.684		0.622		0.612			0.923		0.920
	Single		0.669		0.760		0.600			0.912		0.878
	Complete		0.698		0.769		0.613			0.906		0.915
	Centroid		0.626		0.719		0.641			0.929		0.920
B	Method_1	0.334	**0.976**	0.341	**0.960**	0.354	**0.932**	*E*	0.657	**0.973**	0.654	**0.956**
	Ward		0.975		0.957		0.926			0.947		0.937
	Single		0.974		0.957		0.914			0.957		0.783
	Complete		0.975		0.925		0.907			0.965		0.201
	Centroid		0.975		**0.960**		0.923			0.965		0.951
D	Method_1	0.576	**0.981**	0.530	**0.939**	0.646	**0.978**					
	Ward		0.979		0.932		0.976					
	Single		**0.981**		0.935		0.975					
	Complete		**0.981**		0.841		0.952					
	Centroid		0.980		0.938		**0.978**					

Table 3 – Comparative analysis of clustering methods (*n* = 30)

Sample	Distance		Method	Clusters						Silhouette score		*SSE*	*DI*	F_0 / F_1
				Generated		Detected								
	max	**min**		**№**	**Size**	**№**	**Size**	**Diameter**	**avg** $d(m_i,m_j)$	S_i	**avg(S)**			
B	0.354	0.004	All methods	1	20	1	20	0.188	0.082	0.695	0.728	0.115	3.425	0.263
				2	10	2	10	0.111	0.057	0.796				
C	0.441	0.004	Method_1, Ward, Single, Centroid	1	16	1	16	0.188	0.078	0.712	**0.674**	0.098	1.997	0.240
				2	9	2	9	0.111	0.060	0.603				
				3	5	3	5	0.080	0.050	0.681				
			Complete	1	16	1	14	0.117	0.060	0.464	0.562	**0.050**	**2.32**	**0.207**
				2	9	2	9	0.111	0.059	0.603				
				3	5	3	5	0.080	0.050	0.681				
						4	2	0.023	0.023	0.771				
D	0.646	0.004	All methods	1	25	1	25	0.200	0.071	0.863	0.850	0.116	4.65	0.150
				2	5	2	5	0.180	0.112	0.783				
E	0.654	0.004	Method_1, Centroid	1	25	1	25	0.192	0.074	0.777	**0.738**	0.106	**2.590**	**0.158**
						2	2	0.100	0.100	0.612				
				2	5	3	1	–	–	–				
						4	2	0.087	0.087	0.745				
			Ward	1	25	1	25	0.192	0.074	0.752	0.734	0.149	2.127	0.180
				2	5	2	3	0.278	0.206	0.536				
						3	2	0.087	0.087	0.800				
			Single	1	25	1	22	0.228	0.126	0.408	0.375	0.284	0.960	0.300
				2	5	2	4	0.078	0.045	0.569				
						3–6	1	–	–	–				
			Complete	1	25	1	21	0.169	0.068	0.189	0.274	**0.067**	0.926	0.201
						2	4	0.048	0.038	0.389				
				2	5	3	2	0.100	0.100	0.612				
						4	1	–	–	–				
						5	2	0.087	0.087	0.745				

DOI 10.15588/1607-3274-2023-4-3

Table 4 – Comparative analysis of clustering methods when forming the largest group of consistent evidence ($l_1 = 0.2$)

Sample	**Distance**		**Method**	***CCT***	**Clusters**			**Elements of the largest cluster with $l_1 = 0.2$**	
	max	**min**			**Count**	**max size**	**Diameter**	**Detected**	**Generated**
10	0.421	0.030	Method_2	**0.934**	**3**	**8**	**0.183**	$\{E_1,\mathbf{E}_{2},E_3,\mathbf{E}_{4},\mathbf{E}_{6},E_7,\mathbf{E}_{8},\mathbf{E}_{10}\}$	$\{E_2,E_4,E_6,E_8,E_{10}\}$
			Single	0.925					
			Centroid	0.929					
			Ward	0.828	2	8	0.183	$\{E_1,\mathbf{E}_{2},E_3,\mathbf{E}_{4},\mathbf{E}_{6},E_7,\mathbf{E}_{8},\mathbf{E}_{10}\}$	
			Complete	0.836					
20	0.399	0.021	Method_2	0.869	4	**10**	**0.134**	$\{E_1,\mathbf{E}_{3},E_4,\mathbf{E}_{5},E_8,\mathbf{E}_{10},\mathbf{E}_{11},E_{12},\mathbf{E}_{18},\mathbf{E}_{20}\}$	$\{\mathbf{E}_{2},E_3,E_5,E_7,E_{10},E_{11},\mathbf{E}_{14},\mathbf{E}_{16},E_{18},E_{20}\}$
			Centroid	0.861					
			Single	0.870	4	8	0.124	$\{\mathbf{E}_{3},\mathbf{E}_{5},\mathbf{E}_{7},\mathbf{E}_{10},\mathbf{E}_{11},E_{15},\mathbf{E}_{18},\mathbf{E}_{20}\}$	
			Ward	**0.872**					
			Complete	0.766	10	4	0.145	$\{E_3,\mathbf{E}_{8},\mathbf{E}_{18},\mathbf{E}_{20}\}$	
30	0.394	0.030	Method_2	**0.881**	5	**16**	**0.189**	$\{E_2,E_3,\mathbf{E}_{4},E_5,E_{10},\mathbf{E}_{12},E_{15},\mathbf{E}_{18},E_{19},E_{20},\mathbf{E}_{21},\mathbf{E}_{22},E_{25},\mathbf{E}_{26},E_{29},\mathbf{E}_{30}\}$	$\{E_1,E_4,\mathbf{E}_{7},E_8,E_{12},E_{14},E_{17},E_{18},E_{21},E_{22},E_{24},E_{26},E_{28},E_{30}\}$
			Ward	0.827	4	12	0.144	$\{E_3,\mathbf{E}_{4},E_{10},\mathbf{E}_{12},\mathbf{E}_{18},E_{19},E_{20},\mathbf{E}_{21},\mathbf{E}_{22},E_{25},\mathbf{E}_{26},\mathbf{E}_{30}\}$	
			Single	0.849	11	9	0.132	$\{E_3,E_{10},\mathbf{E}_{18},E_{19},E_{20},\mathbf{E}_{21},\mathbf{E}_{22},E_{25},\mathbf{E}_{30}\}$	
			Complete	0.752	5	12	0.144	$\{E_3,\mathbf{E}_{4},E_{10},\mathbf{E}_{12},\mathbf{E}_{18},E_{19},E_{20},\mathbf{E}_{21},\mathbf{E}_{22},E_{25},\mathbf{E}_{26},\mathbf{E}_{30}\}$	
			Centroid	0.864	8	14	0.155	$\{\mathbf{E}_{1},E_3,\mathbf{E}_{4},\mathbf{E}_{8},E_{10},\mathbf{E}_{12},\mathbf{E}_{14},\mathbf{E}_{17},E_{19},\mathbf{E}_{21},\mathbf{E}_{24},\mathbf{E}_{26},\mathbf{E}_{28},\mathbf{E}_{30}\}$	

As can be seen from Table 4, for a sample size of 20, none of the methods under consideration identified the evidence of experts E_2, E_{14} and E_{16} (for a sample size of 30, this is the evidence of expert E_7) as belonging to the initially formed consensus subgroup. But this is explained by the fact that when forming the initial group of consistent expert evidence (which is 50% of the test sample) for $n = 20$, the maximum distance between expert evidence was 0.234 (with $n = 30$, the maximum distance between expert evidence was 0.220), and the splitting of the totality of expert evidence into clusters occurred at the level of conflict (distance) $l_1 = 0.200$.

6 DISCUSSION

The analysis of tasks and methods for processing group expert assessments allows to conclude that solving the problem of finding generalized (aggregated) assessments, on the basis of which recommendations are formed for the decision maker, largely depends on the effective solution of clustering and ranking problems.

The problem of clustering (partitioning) expert assessments arises in situations where the results of the examination are characterized by a lack of consistency, which creates certain difficulties in determination of generalized assessments.

To solve the problem, two approaches are proposed. The first is to form subgroups of experts that have agreed upon assessments, provided that a certain threshold (acceptable) level of conflict *ConfLev* is specified. The evidence of experts included in subgroup G_q does not exceed a certain conflict level *ConfLev*. In this case, p subgroups of experts can be formed, within which the expert opinions can be considered consistent, but formed subgroups can be in conflict with each other.

The second approach allows to identify subgroups of experts within which expert opinions can be considered consistent, but with different threshold levels of conflict l_q. Thus, for example, a group of experts G_1 will be obtained with a low level of conflict between the expert evidence belonging to it; group of experts G_2 – with a moderate level of conflict; a group of experts G_3 – with a significant level of conflict, etc.

CONCLUSIONS

The paper proposes a technique for structuring group expert assessments, which is based on the mathematical apparatus of the theory of evidence. The proposed approach allows, in the absence of an acceptable level of consistency (consensus, homogeneous) between expert evidence, to identify from the original set of experts subgroups with similar (in a certain sense) assessments (preferences). Various distance measures of evidence (in the framework of DST) were used as a degree of similarity.

The scientific novelty of obtained results is that the models and methods of group expert assessment analysis and structuring under inconsistency, conflict, uncertainty and their combinations are received the further development.

Unlike existing methods for clustering expert assessments, the proposed approach allows processing expert evidence of a various structure: consonant, consistent, arbitrary, etc.; take into account possible combinations and overlaps of expert evidence.

The proposed approach is based on the mathematical apparatus of distances in evidence theory, which allows to assess the degree of dissimilarity (conflict) between selected groups of expert evidence, taking into account their structure. Expert evidence is considered consistent (homogeneous) if the value of the selected metric for all evidence of the selected subgroup does not exceed a specified threshold level.

The practical significance of the obtained results is that the proposed approach can be used as an additional tool for identifying experts (one or more) whose assessments are based on the results of several examinations, largely from the assessments of the main group. Next, it

DOI 10.15588/1607-3274-2023-4-3

can be studied the reason for such behavior of the expert: is it his / her creative opinion, reflecting a non-standard approach to solving the current problem; an attempt to manipulate the results of an expert survey or lack of sufficient knowledge of the subject area.

The prospects for further research are to study of the influence of the choice of distance measure on the results of partitioning under different structures of expert judgments (consonant, consistent, arbitrary).

ACKNOWLEDGEMENTS

The work is partially supported by the state research project of Petro Mohyla Black Sea National University "Development of modern information and communication technologies for the management of intellectual resources to decision-making support of operational management" (research project no. 0121U107831, financed by the Government of Ukraine).

REFERENCES

1. Blokdyk G. Group Decision Making A Complete Guide. Toronto, 5STARCooks, 2021, 208 p.
2. Cichosz P. Data Mining Algorithms: Explained Using R. Chichester, Wiley, 2015, 720 p.
3. Hair J. F. et al. Multivariate Data Analysis (6th ed.). New Jersy, Pearson Prentice Hall, 2006, 899 p.
4. Scitovski R., Sabo K., Martínez-Álvarez F., Ungar S. Cluster Analysis and Applications. Switzerland. Springer Cham, 2021, 271 p. DOI: 10.1007/978-3-030-74552-3
5. Alhameli F., Elkamel A., Alkatheri M., Betancourt-Torcat A., Almansoori A. New Class of Simple and Efficient Clustering Algorithms for Multiscale Mathematical Programming with Demand Data Applications, *Industrial Engineering and Operations Management*, Fourth North American International Conference, Toronto, 23–25 October 2019: proceedings. Canada, IEOM Society International, 2019, pp. 497–505.
6. Hubert L. J., Arabie P., Meulman J. Dynamic Programming in Clustering. In: Floudas C., Pardalos P. (eds) Encyclopedia of Optimization. Boston, Springer, 2008, pp. 837–844. DOI: 10.1007/978-0-387-74759-0_145
7. Nguyentrang T., Vovan T. Fuzzy clustering of probability density functions, *Journal of Applied Statistics*, 2017, Vol. 44(4), pp. 583–601. DOI: 10.1080/02664763.2016.1177502
8. Kemeny J. G., Snell J. L. Mathematical models in the social sciences. Introduction to higher mathematics. New York, Toronto, London, Blaisdell Publishing Company, A Division of Ginn and Company, 1963, 145 p.
9. Sentz K., Ferson S. Combination of evidence in Dempster-Shafer theory. Technical report SAND 2002-0835. Albuquerque, Sandia National Laboratories, 2002, 94 p.
10. Dempster A. P. Upper and lower probabilities induced by a multi-valued mapping, *Annals of Mathematical Statistics*, 1967, Vol. 38(2), pp. 325–339. DOI: 10.1214/aoms/1177698950
11. Shafer G. A mathematical theory of evidence. Princeton, Princeton University Press, 1976, 297 p.
12. Smarandache F., Dezert J. Advances and applications of DSmT for information fusion. Rehoboth, American Research Press, 2004, Vol. 1, 760 p.
13. Bhattacharyya A. On a measure of divergence between two statistical populations defined by their probability distribution, *Bulletin of the Calcutta Mathematical Society*, 1943, Vol. 35, pp. 99–110.
14. Cuzzolin F. A geometric approach to the theory of evidence, *Transactions on Systems, Man, and Cybernetics*, Part C: Applications and Reviews, 2007, Vol. 38(4), pp. 522–534. DOI: 10.1109/TSMCC.2008.919174
15. Jousselme A. L., Grenier D., Boss´e E. A new distance between two bodies of evidence, *Information Fusion,* 2001, Vol. 2, pp. 91–101. DOI: 10.1016/S1566-2535(01)00026-4
16. Tessem B. Approximations for efficient computation in the theory of evidence, *Artificial Intelligence*, 1993, Vol. 61, pp. 315–329. DOI: 10.1016/0004-3702(93)90072-J
17. Martin A., Jousselme A. L., Osswald C. Conflict measure for the discounting operation on belief functions, *Information Fusion* (FUSION 2008): the 11th International Conference, Cologne. Germany, 30 June–3 July 2008: proceedings. Cologne, IEEE, 2008, pp. 1–8.

Received 28.09.2023.
Accepted 20.11.2023.

УДК 004.827:519.816

РОЗРОБКА МЕТОДИКИ СТРУКТУРИЗАЦІЇ ГРУПОВИХ ЕКСПЕРТНИХ ОЦІНОК В УМОВАХ НЕВИЗНАЧЕНОСТІ ТА НЕУЗГОДЖЕНОСТІ

Давиденко Є. О. – канд. техн. наук, доцент, завідувач кафедри інженерії програмного забезпечення Чорноморського національного університету імені Петра Могили, Миколаїв, Україна.

Швед А. В. – д-р наук, професор, професор кафедри інженерії програмного забезпечення Чорноморського національного університету імені Петра Могили, Миколаїв, Україна.

Гончарова Н. В. – аспірантка кафедри інженерії програмного забезпечення Чорноморського національного університету імені Петра Могили, Миколаїв, Україна.

АНОТАЦІЯ

Актуальність. Розглянуті питання структуризації групових експертних оцінок з метою визначення узагальненої оцінки у випадку відсутності узгодженості експертних оцінок. Об'єктом дослідження є процеси синтезу математичних моделей структуризації (кластеризації, розбиття) експертних оцінок, що формуються в рамках моделі Шейфера в умовах невизначеності, неузгодженості (конфлікту). Мета роботи – розробка підходу на основі метрик теорії свідоцтв, що дозволяє із вихідної неоднорідної сукупності експертних оцінок, сформованих в рамках моделі Шейфера, виділяти ряд однорідних підгруп, або ідентифікувати експертів чиї оцінки в значній мірі відрізняються від оцінок решти групи.

Метод. Методика дослідження ґрунтується на математичному апараті теорії свідоцтв, кластерному аналізі. Запропонований підхід використовує принципи ієрархічної кластеризації при формуванні розбиття неоднорідної (неузгодженої) суку-

DOI 10.15588/1607-3274-2023-4-3

пності експертних свідоцтв на ряд підгруп (кластерів), всередині яких оцінки експертів близькі між собою. В якості критерію визначення схожості та відмінності кластерів розглянуті метрики теорії свідоцтв. Оцінки експертів вважаються узгодженими у сформованому кластері, якщо середній або максимальний (в залежності від визначених початкових умов) рівень конфлікту між ними не перевищує заданий пороговий рівень.

Результати. Запропонована методика структуризації експертної інформації дозволяє оцінювати рівень узгодженості експертних оцінок усередині експертної групи на основі аналізу відстані між експертними свідоцтвами. У разі відсутності узгодженості всередині експертної групи запропоновано виділяти з неоднорідної сукупності оцінок підгрупи експертів, оцінки яких близькі для подальшого їх агрегування з метою отримання узагальненої оцінки. Наявність у комісії небагатьох груп експертів із узгодженими оцінками може свідчити про наявність експертів, що мають різний погляд на аналізовану проблему.

Висновки. Дістали подальшого розвитку моделі та методи аналізу та структуризації групових експертних оцінок, сформованих в рамках нотації теорії свідоцтв в умовах невизначеності, неузгодженості, конфлікту. Запропоновано метод кластеризації групових експертних оцінок, що формуються в умовах невизначеності та неузгодженості (конфлікту) в рамках моделі Шейфера, з метою виділення підгруп, всередині яких оцінки експертів вважаються узгодженими. На відміну від існуючих методів кластеризації, запропонований підхід дозволяє обробляти експертні свідоцтва довільної структури, враховувати можливі способи їх взаємодії (об'єднання, перетин).

КЛЮЧОВІ СЛОВА: теорія свідоцтв, метрики теорії свідоцтв, кластеризація, міри відстані, експертні свідоцтва, невизначеність, неузгодженість.

ЛІТЕРАТУРА

1. Blokdyk G. Group Decision Making A Complete Guide / G. Blokdyk. – Toronto: 5STARCooks, 2021. – 208 p.
2. Cichosz P. Data Mining Algorithms: Explained Using R / P. Cichosz – Chichester : Wiley, 2015. – 720 p.
3. Multivariate Data Analysis (6th ed.) / J. F. Hair et al. – New Jersy: Pearson Prentice Hall, 2006. – 899 p.
4. Cluster Analysis and Applications / [R. Scitovski, K. Sabo, F. Martínez-Álvarez, S. Ungar]. – Switzerland : Springer Cham, 2021. – 271 p. DOI: 10.1007/978-3-030-74552-3
5. New Class of Simple and Efficient Clustering Algorithms for Multiscale Mathematical Programming with Demand Data Applications / [F. Alhameli, A. Elkamel, M. Alkatheri et al.] // Industrial Engineering and Operations Management: Fourth North American International Conference, Toronto, 23–25 October 2019: proceedings. – Canada: IEOM Society International, 2019. – P. 497 – 505.
6. Hubert L. J. Dynamic Programming in Clustering / L. J. Hubert, P. Arabie, J. Meulman. In: Floudas C., Pardalos P. (eds) // Encyclopedia of Optimization. – Boston: Springer, 2008. – P. 837–844. DOI: 10.1007/978-0-387-74759-0_145
7. Nguyentrang T. Fuzzy clustering of probability density functions / T. Nguyentrang, T. Vovan // Journal of Applied Statistics. – 2017. – Vol. 44(4). – P. 583–601. DOI: 10.1080/02664763.2016.1177502
8. Kemeny J. G. Mathematical models in the social sciences. Introduction to higher mathematics / J. G. Kemeny, J. L. Snell. – New York, Toronto, London: Blaisdell Publishing Company, A Division of Ginn and Company, 1963. – 145 p.
9. Sentz K. Combination of evidence in Dempster-Shafer theory. Technical report SAND 2002-0835 / K. Sentz, S. Ferson. – Albuquerque: Sandia National Laboratories, 2002. – 94 p.
10. Dempster A. P. Upper and lower probabilities induced by a multi-valued mapping / A. P. Dempster // Annals of Mathematical Statistics. – 1967. – Vol. 38(2). – P. 325–339. DOI: 10.1214/aoms/1177698950
11. Shafer G. A mathematical theory of evidence / G. Shafer. – Princeton: Princeton University Press, 1976. – 297 p.
12. Smarandache F. Advances and applications of DSmT for information fusion / F. Smarandache, J. Dezert. – Vol. 1. – Rehoboth: American Research Press, 2004. – 760 p.
13. Bhattacharyya A. On a measure of divergence between two statistical populations defined by their probability distribution / A. Bhattacharyya // Bulletin of the Calcutta Mathematical Society. – 1943. – Vol. 35. – P. 99–110.
14. Cuzzolin F. A geometric approach to the theory of evidence / F. Cuzzolin // Transactions on Systems, Man, and Cybernetics (Part C: Applications and Reviews). – 2007. – Vol. 38(4). – P. 522–534. DOI: 10.1109/TSMCC.2008.919174
15. Jousselme A. L. A new distance between two bodies of evidence / A. L. Jousselme, D. Grenier, E. Boss´e // Information Fusion. – 2001. – Vol. 2. – P. 91–101. DOI: 10.1016/S1566-2535(01)00026-4
16. Tessem B. Approximations for efficient computation in the theory of evidence / B. Tessem // Artificial Intelligence. – 1993. – Vol. 61. – P. 315–329. DOI: 10.1016/0004-3702(93)90072-J
17. Martin A. Conflict measure for the discounting operation on belief functions / A. Martin, A. L. Jousselme, C. Osswald // Information Fusion (FUSION 2008): the 11th International Conference, Cologne, Germany, 30 June–3 July 2008: proceedings. – Cologne: IEEE, 2008. – P. 1–8.

DOI 10.15588/1607-3274-2023-4-3