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METHOD OF MINIMIZATION SIDELOBES LEVEL AUTOCORRELATION FUNCTIONS OF SIGNALS WITH NON-LINEAR FREQUENCY MODULATION

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ABSTRACT

Context. At present, when creating new and upgrading existing radar systems, solid-state generator devices are widely used, which imposes certain restrictions on the peak power of probing signals. To overcome this limitation, longer duration signals with internal pulse modulation are used. The main efforts of the researchers are focused on reducing the maximum level of the side lobes of the autocorrelation function of such signals, which, without taking additional measures, has a significant level, which complicates the work of systems for detecting and stabilizing the level of false alarms. Attention is paid to signals with non-linear frequency modulation, which consist of two and three linearly frequency-modulated fragments. The maximum level of the side lobes of such signals depends significantly on the frequency-time parameters of the fragments, and therefore it is very difficult to obtain its stable value. Searching for signals with minimal side lobe level values by optimizing their time-frequency parameters is a difficult task, because changing the parameters of previous signal fragments leads to changes in the parameters of subsequent fragments

Objective. The aim of the work is to develop a method for simplifying the search for local minima of the level of side lobes of two- and three-fragment signals with nonlinear frequency modulation by using a modified mathematical model with a whole number of periods of radio oscillations of linear-frequency modulated fragments.

Method. The developed method is based on the proposed modification of the mathematical model, which corrects the frequency-time parameters of two- and three-fragment signals with non-linear frequency modulation by modifying the values of the frequency modulation speed while providing an integer number of complete periods of radio frequency oscillations for each of the fragments, which simplifies the process of finding local minima of the level of side lobes.

Results. Modification of the initial mathematical model leads to the expansion of the possible range of values of frequency-time parameters, ratios of durations and frequency deviations of linearly-frequency modulated fragments and ensures stability of the mathematical model with a decrease in the maximum level of side lobes of the autocorrelation function.

Conclusions. It has been experimentally confirmed that the use of the proposed method of modifying the input frequency-time parameters of signals with non-linear frequency modulation in the vast majority of cases reduces the maximum level of side lobes and simpli-fies the process of finding its local minima. The optimal ratios of durations and deviations of the frequency of the signal frag-ments are determined, subject to these, stable operation of the models is ensured and, in most cases, - less than the value of the maximum level of the side lobes.

KEYWORDS: mathematical model; a non-linear frequency modulation signal; autocorrelation function; maximum level of side lobes.

ABBREVIATIONS

ACF is a autocorrelation function;
SL is a side lobe;
Wt is a weighting;
LFM is a linear frequency modulation;
MM is a mathematical model;
MPSLL is a maximum side lobes level;
NLFM is a non-linear frequency modulation;
SLL is a side lobe level;
FM is a frequency modulation;
FMR is a frequency modulating rate.

NOMENCLATURE

n=1, 2, 3 is a fragment sequence number LFM signal;
f_0 is a initial signal frequency, Hz;
t is a time, sec;
T_n is a duration n-th fragment LFM signal, sec;
T_S is a duration NLFM signal, sec;
$\varphi_n(t)$ is a instantaneous phase n-th fragment LFM signal, rad;
$\beta_n = \Delta f_n / T_n$ is a FMR n-th fragment LFM signal, Hz/sec;
$\Delta\beta_{21}$ is a difference of FMR 2-nd and 1-st LFM fragments, Hz/sec;
$\Delta\beta_{31}$ is a difference of FMR 3-rd and 1-st LFM fragments, Hz/sec;

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$\Delta\beta_{32}$ is a difference of FMR 3-rd and 2-nd LFM fragments, Hz/sec;

$\tilde{\beta}_n$ is a modified FMR *n*-th fragment LFM signal, Hz/sec;

Δf_n is a frequency deviation *n*-th fragment LFM signal, Hz;

$\Delta\tilde{f}_n$ is a modified deviation *n*-th fragment LFM signal, Hz;

$\delta\varphi_{mn}$ is a phase jump during transition from $m=n-1$-st fragment LFM signal on *n*-th, rad;

INTRODUCTION

The modern stage of development of radar facilities is characterized by wide introduction of modular construction of receiving-transmitting devices using the technology of phased antenna arrays on solid-state generator devices and application of internal pulse modulation of frequency (phase) of probing signals [1–9]. These technologies and technical solutions are interrelated, since they aim to achieve the required radiated power under conditions of limiting the peak power of an individual transmitting module. Signals from LFM [1–10] are used as wide sounding signals, the methods of forming and processing of which are constantly being improved.

The main efforts of the researchers are focused on reducing the MPSLL of the autocorrelation function of LFM signals, which, without additional measures, is approximately –13 dB. As a rule, for this purpose, Wt is used in the time (spectral) region of the received radar signal [10–14] and signals with a rounded spectrum [10, 15–20] are used, which is equivalent to Wt in time.

Rounding the spectrum LFM signal brings its shape closer to the bell-shaped one and leads to a decrease in the effective spectrum width and, as a result, to the expansion of the main lobe of the ACF signal. That is, the deterioration of the range discriminating ability in the case of the use of Wt or the rounding of the signal spectrum is a fee for reducing the MPSLL. This is acceptable because from the point of view of detecting weak signals, minimizing MPSLL is a more important task. In [12, 19, 20] it is noted that the application of Wt to signals with a rounded spectrum gives a better final result than the separate use of these methods.

A common method of obtaining signals with a rounded spectrum is the use of intra-pulse NLFM, which, in comparison with LFM signals, provides significantly lower MPSLL [1, 2, 8–10, 12, 15–31]. NLFM signals are widely used, consisting of two or three fragments combined in time with linear or nonlinear FM, or their combinations [10, 12, 15, 17, 19–22, 27–31]. Unlike LFM signals, which have practically fixed MPSLL regardless of the values of the input variables of their MM [1–14], MPSLL of multifragment NLFM signals significantly depends on the ratio of the duration of fragments and deviations of their frequency and varies widely depending on their magnitude.

The studies carried out by the authors [30, 31] showed that the known MM NLFM signals, consisting of two and three LFM fragments [15, 25, 27–29, 32–34], have a significant drawback – they do not take into account frequency and phase jumps (or only phases, depending on the temporal representation of MM). These jumps are caused by an instantaneous change in the FM value, that is, the ratio of the frequency deviation of the FM fragment to its duration, at the moment of transition from one fragment to another. Several methods of their compensation are offered, which allow to take full advantage of such signals. It was also found that their MPSLL depends on what value the initial and final phase of each of the LMF fragments has.

Searching for signals with minimum MPSLL values by optimizing their frequency-time parameters is a complex problem that belongs to the class of dynamic programming problems, because changing the parameters of previous fragments of the NLFM signal leads to changes in the parameters of subsequent fragments. Therefore, the paper proposes a method that simplifies the search for local minima of the MPSLL by adjusting the input MM variables under the conditions of providing an integer number of radio oscillation periods in each of the fragments of the NLFM signal.

The object of study is the process of formation and processing of two- and three-fragment NLFM signals.

The subject of study are mathematical models of NLFM signals, which consist of two and three LFM fragments.

The purpose of the work is to develop a method for simplifying the search for local minima of two- and three-fragment NLFM signals by using a modified MM with a whole number of periods of radio oscillations of LFM fragments.

1 PROBLEM STATEMENT

The authors propose [30, 31] MM for calculating the instantaneous phase of two- and three-fragment NLFM signals in the current time, in which frequency and phase jumps are compensated during the transition from the previous to the next LFM fragment. Let's use MM [31] for the three-fragment NLFM signal:

$$\varphi(t)=2\pi\begin{cases} f_0 t+\dfrac{\beta_1 t^2}{2},\ 0\le t\le T_1; \\ \left(f_0-\Delta\beta_{21}T_1\right)t+\dfrac{\beta_2 t^2}{2}+\delta\varphi_{12},\ T_1\le t\le T_{12}; \\ \left(f_0-\Delta\beta_{31}T_1-\Delta\beta_{32}T_2\right)t+\dfrac{\beta_3 t^2}{2}-\delta\varphi_{23}, \\ T_{12}\le t\le T_S, \end{cases} \tag{1}$$

in which, to ensure compactness of mathematical records, the designation of the difference FMR between the second and first fragments is introduced $\Delta\beta_{21}=\beta_2-\beta_1$, the difference FMR between third and first fragments

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$\Delta\beta_{31} = \beta_3 - \beta_1$, the difference FMR between third and second fragments: $\Delta\beta_{32} = \beta_3 - \beta_2$.

Similarly, the total duration of the first and second LFM fragments $T_{12} = T_1 + T_2$ and the total duration of the NLFM signal are introduced $T_S = T_1 + T_2 + T_3$. Additional MM variables (1), which are determined using already specified parameters, are the phase jump during the transition from the first to the second LFM fragment:

$$\delta\varphi_{12} = \frac{1}{2}\Delta\beta_{21}T_1^2 \tag{2}$$

and phase jumps between the second and third LFM fragments:

$$\delta\varphi_{23} = \frac{1}{2}\left(\Delta\beta_{31}T_1^2 + \Delta\beta_{32}T_2^2\right) \tag{3}$$

Thus, in addition to the current time t and the initial frequency f_0, the input parameters of the model (1) are Δf_n and T_n that can be corrected.

In the presentation of the material, the designation of the operation of finding the nearest larger integer ceil was used. The two-step model is a separate case (1) when using the first two components. Model (1) can be adapted for the descending law of FM, if you use the negative sign of FMR in the calculated ratios.

The result of the research is to modify the model (1) so that the radio oscillations of each LFM fragment have an integer number of periods. The results of the assessment of MPSLL and the rate of decline of SLL are compared with those obtained earlier [15, 17, 19–22, 27–34].

2 REVIEW OF THE LITERATURE

The first publications on the use of NLFM signals, which have fragments with LFM and NLFM, appeared in the 60s of the last centuries [16, 17]. Subsequently, these developments received a more detailed theoretical basis, which emphasized the need to ensure the continuity of the instantaneous phase of such signals. For this, NLFM signals with symmetrical rounding of the spectrum were proposed, which in theory should have provided an MPSLL of –42.8 dB [10]. However, even today such an MPSLL is unattainable for known multi-fragment NLFM signals. Interest in using NLFM signals, which consist of LFM fragments, is associated with the results of developments in the field of their formation and processing [1–11, 13, 14].

A number of researchers [15–29, 32–34], along with tri-fragmental signals that can reduce MPSLL by an average of 6 dB, consider two-fragment NLFM signals, which, in comparison with LFM, provide a decrease in MPSLL by about 3dB. The peculiarity of the use of NLFM signals, which consist of two and three LFM fragments, is that the MPSLL of the resulting signal depends significantly on the frequency-time parameters of the fragments, and therefore it is very difficult to obtain a stable value of the MPSLL even with slight changes in the parameters of the signals. Therefore, the choice of parameters must be approached carefully, for example, in [27] an algorithm for selecting parameters is proposed in order to minimize the MPSLL of a two-fragment NLFM signal.

Frequency and phase jumps at the junctions of previous LFM fragments are included as components in the frequency and phase expressions of subsequent fragments (see the second and third fragments in expression (1)), which complicates the optimization of parameters. In the previously developed MM of both current and shifted time [10, 12, 15, 16, 17, 19–22, 25, 27–29, 32–34], such distortions were not taken into account. In works [30, 31] on the example of MM of the current time of two- and three-fragment NLFM signals it is shown that such frequency-phase distortions arise as a result of instantaneous change in the value of FMR and a method of their compensation is proposed. The range of change of initial parameters of NLFM signals (input variables of their MM) after compensation of frequency and phase jumps at the joints of LFM fragments is somewhat expanded, however, as practice shows, the possibilities of such expansion have not yet been exhausted. At the same time, the actual task remains to minimize the SLL of the correlation functions of NLFM signals.

3 MATERIALS AND METHODS

We apply the concept of the full phase Ψ_n -*nth* LFM fragment, which is equal to the phase of the radio frequency oscillation during its duration. For the first LFM fragment, based on (1), its total phase is:

$$\Psi_1 = 2\pi\left(f_0T_1 + \frac{\beta_1}{2}T_1^2\right). \tag{4}$$

We impose the condition that the complete phase of the LFM fragment (4) has an integer number of complete periods of radio frequency oscillations $2\pi N_1$. In this case:

$$N_1 = ceil\left(f_0T_1 + \frac{\beta_1}{2}T_1^2\right). \tag{5}$$

In order to satisfy (5) without applying the *ceil* operation, it is necessary to solve this equation with respect to β_1 and obtain a new modified FMR $\tilde{\beta}_1$ value, which for the first LFM fragment is:

$$\tilde{\beta}_1 = \frac{2(N_1 - f_0T_1)}{T_1^2}. \tag{6}$$

Since the frequency jump, and therefore the instantaneous phase at the junction of the fragments caused by the change in the FMR, remains, these jumps still need to be

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compensated. Taking into account the above and on the basis of (1) for the second LFM fragment, by analogy we have:

$$\Psi_2 = 2\pi\left([f_0 - (\beta_2 - \tilde{\beta}_1)T_1]T_2 + \frac{\beta_2}{2}T_2^2\right) = 2\pi N_2; \quad (7)$$

$$\tilde{\beta}_2 = \frac{2\left(N_2 - [f_0 - (\beta_2 - \tilde{\beta}_1)T_1]T_2\right)}{T_2^2}. \quad (8)$$

As you can see, in (7), (8), a modified value is already involved, that is, the calculation of the modified parameters $\tilde{\beta}_1$ of the next fragment is performed taking into account the modification of the parameters of the previous fragments of the NLFM signal. Similarly, for the third LFM fragment, we write:

$$\Psi_3 = 2\pi\left([f_0 - (\beta_3 - \tilde{\beta}_1)T_1 - (\beta_3 - \tilde{\beta}_2)T_2]T_3 + \frac{\beta_3}{2}T_3^2\right) = 2\pi N_3,$$

where:

$$\tilde{\beta}_3 = \frac{2\left\{N_3 - [f_0 - (\beta_3 - \tilde{\beta}_1)T_1 - (\beta_3 - \tilde{\beta}_2)T_2]T_3\right\}}{T_3^2}. \quad (9)$$

Thus, the essence of the proposed method for minimizing ACF SLL signals from NLFM is as follows. Due to modification of FMR values, an integer number of periods of radio frequency oscillations is formed at each of the signal sections, which eliminates the cause of phase jumps due to an arbitrary value of the final phase of the LFM fragment. However, the frequency jumps due to the change of the FMR with the transition to the next fragment are preserved and must be compensated. Taking into account the above, we have a modified MM in the current time for the instantaneous phase of the NLFM signal, which consists of three LFM fragments, with an integer number of radio oscillation periods and compensation for frequency and phase jumps at the joints of the fragments:

$$\varphi(t) = 2\pi \times \begin{cases} f_0 t + \dfrac{\tilde{\beta}_1 t^2}{2}, \ 0 \le t \le T_1; \\ [f_0 - (\tilde{\beta}_2 - \tilde{\beta}_1)T_1]t + \dfrac{\tilde{\beta}_2 t^2}{2} + \delta\tilde{\varphi}_{12}, \ T_1 \le t \le T_{12}; \\ [f_0 - (\tilde{\beta}_3 - \tilde{\beta}_1)T_1 - (\tilde{\beta}_3 - \tilde{\beta}_2)T_2]t + \dfrac{\tilde{\beta}_3 t^2}{2} - \delta\tilde{\varphi}_{23}, \\ T_{12} \le t \le T_S. \end{cases} \quad (10)$$

The compensating phase components (2), (3) in (10) should be calculated using expressions (11) and (12) already taking into account the modification of the FMR:

$$\delta\tilde{\varphi}_{12} = \frac{1}{2}(\tilde{\beta}_2 - \tilde{\beta}_1)T_1^2; \quad (11)$$

$$\delta\tilde{\varphi}_{23} = \frac{1}{2}\left((-\tilde{\beta}_3 + \tilde{\beta}_1)T_1^2 + (\tilde{\beta}_3 - \tilde{\beta}_2)T_2^2\right). \quad (12)$$

The modified FMR values for (10)–(12) are in accordance with (6), (8), (9). It should be noted that with the transition from the first fragment to the second, the FMR decreases, that is, it has a negative increase, and at the junction of the second and third fragments it increases – the increase is positive. This is taken into account by the opposite signs of the compensating phase components $\delta\tilde{\varphi}_{12}$, $\delta\tilde{\varphi}_{23}$, in (10) and by changing the signs of the FMR, $\tilde{\beta}_3$, $\tilde{\beta}_1$ in the first term (12).

The FMR rate is a time-frequency parameter because it is determined by frequency deviation Δf_n and duration T_n of the corresponding fragment. In case of modification of the FMR $\tilde{\beta}_n$ with constant duration of the LFM fragment, its frequency deviation changes:

$$\Delta\tilde{f}_n = \tilde{\beta}_n T_n, \quad (13)$$

Thus, MM (10) is obtained, which changes the frequency-time parameters of two- and three-fragment NLFM signals by modifying the values of the FMR fragments while providing an integer number of complete periods of radio frequency oscillations for each of the FMR fragments. A feature of the proposed MM is the correction of the initial values of the frequency deviations of the LFM fragments, which simplifies the process of finding local minima of the MPSLL. It should be noted that modification of the initial MM (1) leads to expansion of the possible range of values of frequency-time parameters, ratios of durations and deviations of frequency of LFM fragments, which ensure stability of MM operation.

4 EXPERIMENTS

Operability of the developed MM with adjustment of frequency-time parameters of two- and three-fragment NLFM signals was checked in the MATLAB application package. For verification, an experimental scheme was used, identical for both two- and three-fragment signals – groups of ten signals with the same input variables for each type of signals were studied. The obtained MPSLL for two-fragment NLFM signals is not higher than –18.0 dB, and for three-fragment signals – not higher than –22.0 dB. The ranges of possible changes in the ratios of the duration of LFM fragments, as well as their frequency deviations, in which the stable operation of the MM is observed, were also determined.

5 RESULTS

MPSLL was first evaluated for MM (1) with compensation for frequency and phase jumps at the joints of LFM fragments, and then for MM (10) with frequency parameters correction by modifying FMR values.

Table 1 shows the values of frequency-time parameters of two-fragment NFLM signals and the obtained MLSS values for the case of MM (1) with compensation of frequency and phase jumps. Corrected values of time-frequency parameters and corresponding MPSLL are

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given in parentheses using MM (10). The parameters in the table are arranged in the order of increasing the duration of NLFM signals, and in the case of the same duration – by increasing the total frequency deviation.

Table 1 – Frequency-time parameters and MPSLL of two-segment NLFM signals

No.	T_1, µs	T_2, µs	Δf_1, kHz	Δf_2, kHz	MPSLL, dB
1.	12	50	75 (100)	150 (280)	–18.25 (–20.85)
2.	25	140	100 (80)	180.0 (271.43)	–16.96 (–18.83)
3.	30	180	100.0 (66.66)	150.0 (233.33)	–15.61 (–18.91)
4.	30	180	150.0 (133.33)	300.0 (445.45)	–16.77 (–17.89)
5.	35	180	100.0 (114.29)	260.0 (388.89)	–17.20 (–18.40)
6.	40	180	100 (100)	220.0 (322.22)	–18.10 (–18.10)
7.	45	220	100.0 (88.88)	190 (280)	–16.84 (–18.12)
8.	45	220	150.0 (133.33)	300 (445.45)	–16.95 (–18.06)

Analysis of Table 1 indicates that the use of adjusted time-frequency parameters in 80% of cases for this data set provides a decrease in MPSLL. It should be noted that the use of the proposed method for selecting frequency-time parameters simplifies the process of finding local minima of the MPSLL. It should be noted that in some cases the modified values of frequency deviation of the LFM fragments differ quite significantly from the initial ones.

Frequency-time parameters of three-fragment NLFM signals and corresponding MPSLL values are given in Table 2.

Table 2 – Frequency-time parameters and MPSLL of three-fragment NLFM signals.

No.	T_1, µs	T_2, µs	T_3, µs	Δf_1, kHz	Δf_2, kHz	Δf_3, kHz	*MPSLL*, dB
1.	15	75	15	30 (133.33)	60 (77.3)	30 (72)	–13.80 (–24.62)
2.	25	100	25	80 (80)	200 (200)	100 (120)	–21.08 (–22.50)
3.	25	100	25	95 (80)	200 (200)	100 (120)	–22.68 (–22.50)
4.	30	150	30	95 (133.33)	200 (200)	90 (80)	–15.95 (–21.74)
5.	30	150	30	150 (200)	400 (400)	150 (133.33)	–18.96 (–22.61)
6.	30	150	30	260 (266.67)	590 (596)	260. (261.33)	–18.93 (–21.25)
7.	30	150	30	300 (333.33)	790 (796)	310 (328)	–21.13 (–21.58)
8.	40	200	40	100 (100)	200 (200)	90 (80)	21.13 (–21.96)

Comparison of the results of Table 2 indicates that the use of adjusted frequency-time parameters for three-fragment NLFM signals in the vast majority of cases (for the given data set – 90%) provides a decrease in MPSLL, which indicates the feasibility of using (10) in practical activities.

In the case of using modified frequency parameters of two- and three-fragment signals for MM (1) and (10), the obtained results completely coincide, which indicates the adequacy of the proposed method. So it should be, since model (10) is a separate case of model (1) under the condition of an integer number of periods of radio oscillations.

In case of FMR modification, the new values of fragment frequency deviations may differ from the given initial ones, which ensures their automatic selection and variability. Subsequently modified by (10) deviation frequency is advisable to use as input variables for MM (1).

The possibilities for obtaining potentially achievable MPSLL values and the rate of decline of the SLL in the case of adjusting the frequency-time parameters of three-fragment NLFM signals are shown in Fig. 1 – Fig. 4. Fig. 1 shows the results of application (10) for signal № 1 from Table 1, Fig. 1a shows the signal spectrum, and Fig. 1b shows its ACF on a logarithmic scale, the MPSLL is –20.85 dB. Note that such a MPSLL value is achieved with a relatively small value of the total frequency deviation when the spectrum type approaches the bell-shaped one more.

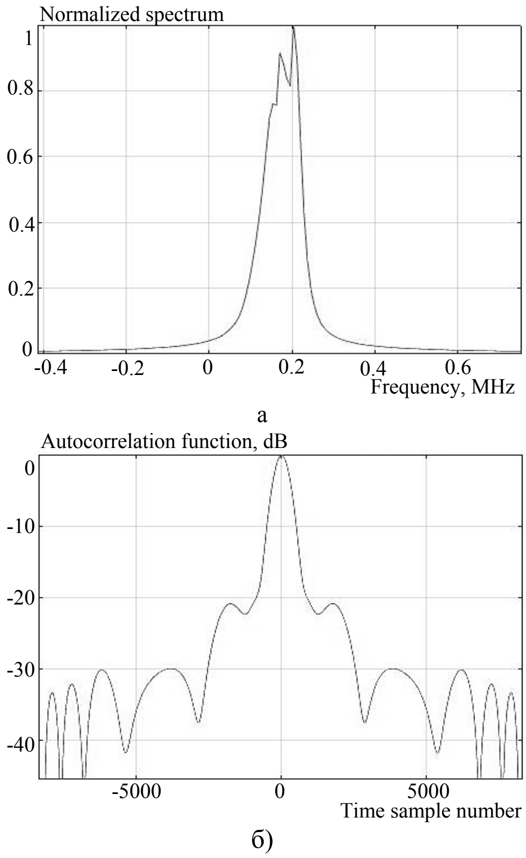

Figure 1– Spectrum (a), ACF (b) NLFM signal by model (10), parameter № 1 tab. 1

The simulation results shown in Fig. 2 are also obtained using (10), the set of parameters corresponds to signal № 1 of Table 2. The spectrum of Fig. 2a has a noticeable rounding, which led to a decrease in MPSLL to –24.62 dB (Fig. 2b).

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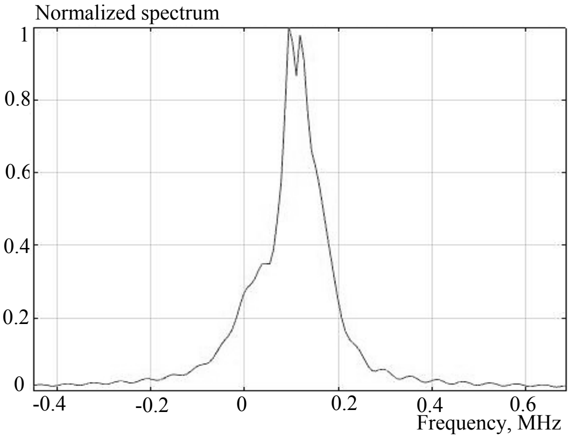

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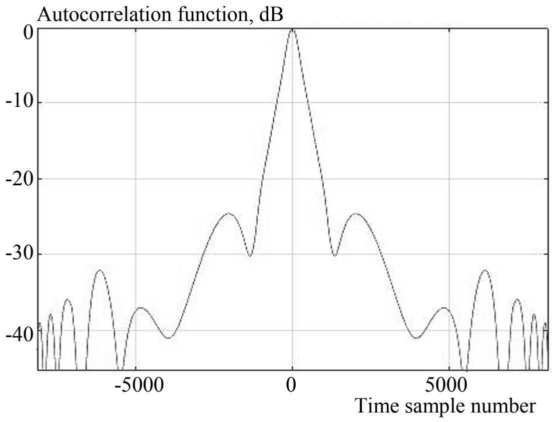
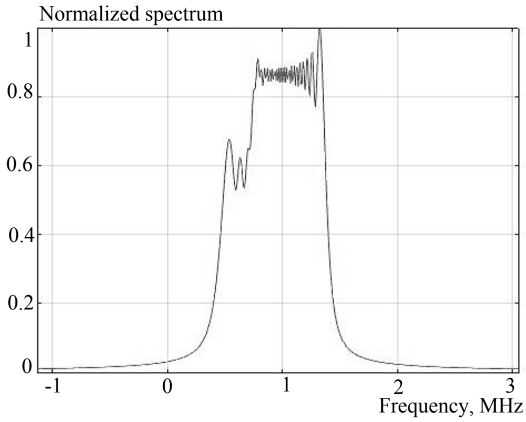


b

Figure 2– Spectrum (a), ACF (b) NLFM signal by model (10), parameters № 1 Tab. 2

To assess the rate of decline of the ACF SL, two- and three-fragment NLFM signals with larger values of the resulting duration and frequency deviation were used – parameter set № 8 from Table 1 and parameter set № 5 from Table 2. The results are shown in Fig. 3 and Fig. 4, respectively.

There is a clear decrease in the intensity of the signal spectrum in the low frequency region of Fig. 3a. Logarithmic scale along both coordinate axes is used for convenience of ACF signal analysis. For a two-segment NLFM signal, the SLL decay rate is estimated at about 22 dB/dec. (Fig. 3b). Analysis of Fig. 4a indicates a two-sided decrease in the intensity of the spectrum, as a result of which the ACF MPSLL decreased to the level of –22.61 dB. The rate of decline of the SLL is approximately 21.5 dB/dec.



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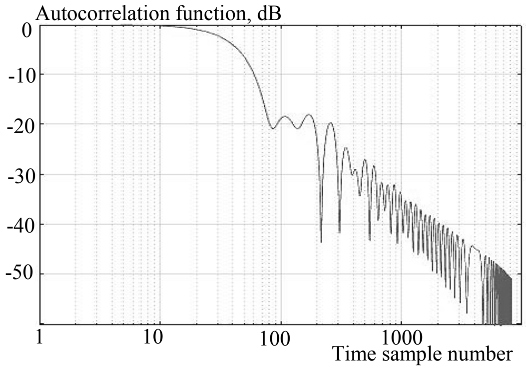

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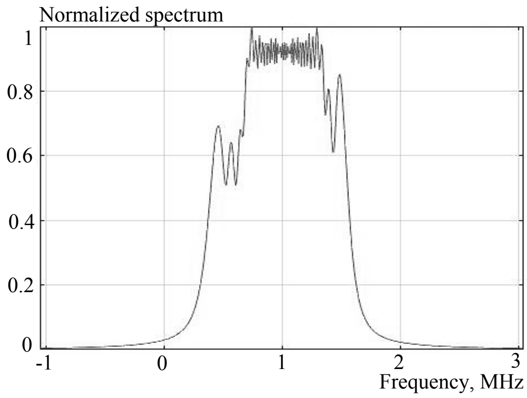
Figure 3 – Spectrum (a), ACF (b) NLFM signal by model (10), parameters № 10 Tab. 1



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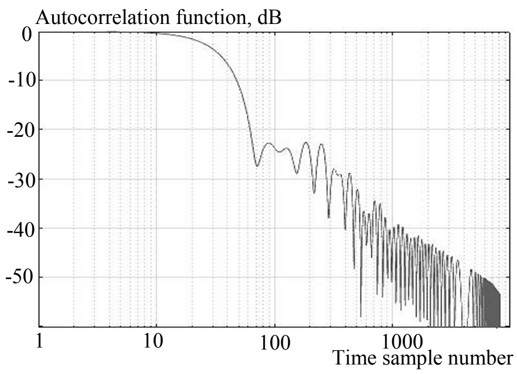

b

Figure 4 – Spectrum (a), ACF (b) NLFM signal by model (10), parameters № 7 tab. 2

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Analysis of Fig. 1 – Fig. 4 indicates that the spectra of Fig. 1a, 2a, 3a, 4a do not have significant additional distortions in the form of breaks and dips, and the corresponding ACF of Fig. 1b, 2b, 3b, 4b do not have a sharp change in the frequency of side lobe pulsations and SLL drops. This confirms the absence of frequency and phase jumps in the resulting NLFM signals.

For the studied groups of signals, the ratio of durations and deviations of the frequency of the LFM fragments is determined, which ensures the stability of the MM operation and minimization of the SLL in the case when only the frequency-phase jumps are compensated, as well as when the FMR is further modified to provide a whole number of radio oscillation periods. The obtained results for the ratios of LFM fragment durations are grouped in Table 3, and for frequency deviations – in Table 4. The ratio is relative to the value corresponding to the parameter of the first LFM fragment.

Table 3 – Ratio of LFM fragment durations

Compensation of frequency and phase jumps			
2 fragments		3 fragments	
from 1:4.1	to 1:5	from 1:5:0.5	to 1:6.7:1
Modifications FMR			
from 1:4.5	to 1:6	from 1:4:1	to 1:5:1

Table 4 – Frequency deviation ratio LFM fragments

Compensation of frequency and phase jumps			
2 fragments		3 fragments	
from 1:2	to 1:2.3	from 1:2:0.9	to 1:2.5:1
Modifications FMR			
from 1:2.8	to 1:3.5	from 1:2:0.67	to 1:2.5:1.5

A comparative analysis of the results of Table 3 and Table 4 indicates that the ranges of change in the input parameters for (1) and (10) differ, that is, as a result of using (10), the total variability of the input variables for determining the parameters of NLFM signals increased.

6 DISCUSSIONS

Comparison of results obtained using (1) and (10) suggests that the proposed method of finding new values of input variables for (1) simplifies their selection and increases variability. When changing the input data, models (1) and (10) can give excellent results both in favor of one and in favor of another model. It should be noted that they mutually complement each other. That is, in studies of NLFM signals, both MM should be used and already on the basis of the obtained results, it should be concluded that it is advisable to use unmodified input variables or the results of their modification.

In the case of obtaining a lower MPSLL by adjusting the initial time-frequency parameters, it is advisable to use them as input for (1). This approach does not always work in the opposite direction, since as a result of modification in (10), the input variables from (1) may receive new values, and the achieved result may even deteriorate. In the course of experimental studies, it was found that the proposed method allows for two-fragment NLFM signals to obtain MPSLL values below –20 dB, and for three-fragment signals – below –24 dB with a relatively small total frequency deviation, which can be useful in a number of practical applications, for example, sonar, ultrasound diagnostics, etc. At the same time, signal spectra do not have significant distortions, which provides a potential possibility of achieving a high degree of their coordination with the frequency characteristics of the receiving-transmitting paths.

CONCLUSIONS

The scientific novelty. The paper proposes a new method of simplifying the search for local minima of MPSLL by adjusting the input variables of MM under the conditions of providing a whole number of periods of radio oscillations in each of the fragments of the NLFM signal. Thus, each LFM fragment begins with a zero phase, and ends with a value of 2 π radians, which ensures the stability of the model and in the overwhelming number of cases – an additional decrease in MPSLL. Use of the proposed method of modification of input frequency-time parameters for two-fragment NLFM signals for the investigated group of signals ensured reduction of MPSLL by 3 dB, and for three-fragment signals – up to 3.5 dB. It should be noted that the use of the proposed method for selecting frequency-time parameters simplifies the process of finding local minima of the MPSLL.

It has been determined that two- and three-fragment NLFM signals generated with the help of the proposed MM have a higher rate of decline of MPSLL, which is estimated at 21–22 dB/dec, compared to the LFM signals. The optimal ratios of durations and deviations of the frequency of LFM fragments have been determined experimentally, subject to these, stable operation of models is ensured and, in most cases, – less than the MPSLL value.

The practical value of the obtained results lies in the possibility of using the proposed method for selecting parameters of NLFM signals, which consist of two and three LFM fragments. This can be used to develop devices for generating and processing radio signals of various applications, for example, radar of air targets, aviation and space systems for inspecting the earth's surface, weather location, sonar, ultrasonic diagnostics, etc., in which NLFM signals can be used to reduce MPSLL independently or in combination with Wt in the receiving device.

Prospects for further research. In the future, it is planned to improve MM [15, 25, 27–29, 32–34] in terms of compensating for phase jumps at the joints of fragments of NLFM signals and to explore the possibilities of optimizing input variables for such MM.

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СПОСІБ МІНІМІЗАЦІЇ РІВНЯ БІЧНИХ ПЕЛЮСТОК АВТОКОРЕЛЯЦІЙНИХ ФУНКЦІЙ СИГНАЛІВ З НЕЛІНІЙНОЮ ЧАСТОТНОЮ МОДУЛЯЦІЄЮ

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АНОТАЦІЯ

Актуальність. У теперішній час при створенні нових та модернізації існуючих радіолокаційних систем широко використовуються твердотільні генераторні прилади, що накладає певні обмеження на пікову потужність зондувальних сигналів. Для подолання цього обмеження застосовуються сигнали більшої тривалості з внутрішньо імпульсною модуляцією. Основні зусилля дослідників зосереджуються на зниженні максимального рівня бічних пелюсток автокореляційної функції таких сигналів, який без прийняття додаткових мір має суттєвий рівень, що утруднює роботу систем виявлення та стабілізації рівня хибних тривог. Увагою користуються сигнали з нелінійною частотною модуляцією, які складаються з двох та трьох лінійно-частотномодульованих фрагментів. Максимальний рівень бічних пелюсток таких сигналів суттєво залежить від частотно-часових параметрів фрагментів, а тому дуже складно отримати його стабільне значення. Пошук сигналів з мінімальними значеннями рівня бічних пелюсток шляхом оптимізації їх частотно-часових параметрів є складною задачею, бо зміна параметрів попередніх фрагментів сигналу призводить до змін параметрів наступних фрагментів.

Метою роботи є розробка способу для спрощення пошуку локальних мінімумів рівня бічних пелюсток дво- та трифрагментних сигналів з нелінійною частотною модуляцією за рахунок використання модифікованої математичної моделі з цілим числом періодів радіоколивань лінійно-частотномодульованих фрагментів.

Метод. Розроблений спосіб спирається на запропоновану модифікацію математичної моделі, яка здійснює коригування частотно-часових параметрів дво- та трифрагментних сигналів з нелінійною частотною модуляцією за рахунок модифікації значень швидкості частотної модуляції при забезпеченні цілого числа повних періодів радіочастотних коливань для кожного з фрагментів, що спрощує процес знаходження локальних мінімумів рівня бічних пелюсток.

Результати. Модифікація початкової математичної моделі призводить до розширення можливого діапазону значень частотно-часових параметрів, співвідношень тривалостей та девіацій частоти лінійно-частотномодульованих фрагментів та забезпечує стійкість роботи математичної моделі при зниженні значення максимального рівня бічних пелюсток автокреляційної функції.

Висновки. Експериментально підтверджено, що використання запропонованого способу модифікаціїї вхідних частотно-часових параметрів сигналів з нелінійною частотною модуляцією у переважній більшості випадків забезпечує зниження максимального рівня бічних пелюсток та спрощує процес знаходження його локальних мінімумів. Визначено оптимальні співвідношення тривалостей та девіацій частоти фрагментів сигналу, при дотриманні таких забезпечується стійка робота моделей та у більшості випадків – менше значення максимального рівня бічних пелюсток.

КЛЮЧОВІ СЛОВА: математична модель; сигнал з нелінійною частотною модуляцією; автокореляційна функція; максимальний рівень бічних пелюсток.

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