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FACE RECOGNITION USING THE TEN-VARIATE PREDICTION ELLIPSOID FOR NORMALIZED DATA BASED ON THE BOX-COX TRANSFORMATION

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ABSTRACT

Context. Face recognition, which is one of the tasks of pattern recognition, plays an important role in the modern information world and is widely used in various fields, including security systems, access control, etc. This makes it an important tool for security and personalization. However, the low probability of identifying a person by face can have negative consequences, so there is a need for the development and improvement of face recognition methods. The object of research is the face recognition process. The subject of the research is a mathematical model for face recognition.

One of the frequently used methods of pattern recognition is the construction of decision rules based on the prediction ellipsoid. An important limitation of its application is the need to fulfill the assumption of a multivariate normal distribution of data. However, in many cases, the multivariate distribution of real data may deviate from normal, which leads to a decrease in the probability of recognition. Therefore, there is a need to improve mathematical models that would take into account the specified deviation.

The objective of the work is to increase the probability of face recognition by constructing a ten-variate prediction ellipsoid for data normalized by the Box-Cox transformation.

Method. Application of the Mardia test to test the deviation of a multivariate distribution of data from normality. Building decision rules for face recognition using a ten-variate prediction ellipsoid for data normalized based on the Box-Cox transformation. Obtaining estimates of the parameters of the univariate and ten-variate Box-Cox transformations using the maximum likelihood method.

Results. A comparison of the results of face recognition using decision rules, which were built using a ten-variate ellipsoid of prediction for data normalized by various transformations, was carried out. In comparison with the use of univariate normalizing transformations (decimal logarithm and Box-Cox) and the absence of normalization, the use of the ten-variate Box-Cox transformation leads to an increase in the probability of face recognition.

Conclusions. For face recognition, a mathematical model in the form of a ten-variate prediction ellipsoid for data normalized using the multivariate Box-Cox transformation has been improved, which allows to increase in the probability of recognition in comparison with the use of corresponding models that are built either without normalization or with the use of univariate normalizing transformations. It was found that a mathematical model built for normalized data using a multivariate Box-Cox transformation has a higher probability of recognition since univariate transformations neglect the correlation between geometric features of the face.

KEYWORDS: face recognition, prediction ellipsoid, multivariate Box-Cox transformation, normalizing transformation.

ABBREVIATIONS

BCT is the Box-Cox transformation;

SMD is the squared Mahalanobis distance;

PRFP is the probability of recognizing the first person.

NOMENCLATURE

k is a number of variables (geometrical facial features);

m is a number of degrees of freedom;

N is a number of data points;

$\mathbf{S_Z}$ is a sample covariance matrix for normalized data;

$\mathbf{X}$ is a non-Gaussian random vector;

$\overline{\mathbf{X}}$ is a vector of sample means of the X_j variables;

X_j is a j-th non-Gaussian variable;

$\overline{X}_j$ is a sample mean of the X_j values;

$\mathbf{Z}$ is a Gaussian random vector;

$\overline{\mathbf{Z}}$ is a vector of sample means of the Z_j variables;

Z_j is a j-th Gaussian variable that is obtained by transforming the variable;

$\overline{Z}_j$ is a sample mean of the Z_j values;

α is a significance level;

β_1 is a multivariate skewness;

β_2 is a multivariate kurtosis;

$\chi^2_{m,\alpha}$ is the Chi-Square distribution quantile with m degrees of freedom and significance level α;

$\boldsymbol{\psi}$ is a vector of multivariate normalizing transformation;

$\boldsymbol{\Theta}$ is a k-variate vector of normalizing transformation parameters.

INTRODUCTION

Facial recognition is becoming increasingly popular due to its wide range of applications in fields like computer vision, security systems, and others. The process of facial recognition involves automatically identifying individuals based on distinct facial features, including the shape of the eyes, nose, mouth, and other characteristics. The technology behind facial recognition is continually advancing leading to higher accuracy and opening up a multitude of possibilities for its use in various aspects of life.

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The accuracy and effectiveness of facial recognition systems are heavily reliant on the specific decision rule chosen for the identification process. This decision rule essentially dictates how an individual's facial features are classified into predefined categories or classes within the system.

In modern methods used for face recognition, an important limitation is the assumption of a multivariate normal distribution of the data [1]. However, real data often have a non-Gaussian distribution. As a result, such deviations can be the cause of errors in the face recognition process. Therefore, there is a need to improve mathematical models that can take into account deviations from the normal distribution of data.

The object of study is the process of face recognition.

The facial recognition process involves a series of key steps, it initiates with image preprocessing, involving face detection, and alignment for optimal analysis. Feature extraction follows, identifying key facial elements such as the position of the eyes, nose, mouth, and other distinctive attributes. These extracted features serve as the foundation for generating a feature vector, a mathematical representation encapsulating the unique facial characteristics, which is pivotal in the recognition process. Pattern recognition, through the application of mathematical models, determines which individual the feature vector corresponds to. This process involves comparing the feature vector to a database of known individuals [2].

The subject of study is a mathematical model for face recognition. One of the frequently employed methods in pattern recognition involves building decision rules based on prediction ellipsoids.

The purpose of the work is to increase the probability of face recognition by constructing a ten-variate prediction ellipsoid for normalized data using Box-Cox transformation.

1 PROBLEM STATEMENT

Suppose given the original data sample set of the ten geometrical facial features the multivariate distribution for which is not Gaussian. Suppose that there are bijective ten-variate normalizing transformation $\boldsymbol{\psi}=\{\psi_Y,\psi_1,\psi_2,\dots,\psi_{10}\}^T$ of non-Gaussian random vector $\mathbf{X}=\{X_1,X_2,\dots,X_{10}\}^T$ to Gaussian random vector $\mathbf{Z}=\{Z_1,Z_2,\dots,Z_{10}\}^T$ is given by:

$$\mathbf{Z}=\boldsymbol{\psi}(\mathbf{X}) \tag{1}$$

and the inverse transformation for (1)

$$\mathbf{X}=\boldsymbol{\psi}^{-1}(\mathbf{Z}). \tag{2}$$

It is required to build the prediction ellipsoid for normalized data in the form:

$$\left(\mathbf{Z}-\overline{\mathbf{Z}}\right)^T \mathbf{S}_{\mathbf{Z}}^{-1}\left(\mathbf{Z}-\overline{\mathbf{Z}}\right)=\chi^2_{m,\alpha}, \tag{3}$$

where

$$\mathbf{S}_{\mathbf{Z}}=\frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{Z}_i-\overline{\mathbf{Z}}\right)\left(\mathbf{Z}_i-\overline{\mathbf{Z}}\right)^T .$$

Also, it is required to develop the decision rule for face recognition based on equation (3) and the transformations (1) and (2).

2 REVIEW OF THE LITERATURE

Mahalanobis distance is a way of measuring how far a point is from a distribution of points, taking into account the shape and orientation of the distribution. It is based on the idea that the distance between two points should be scaled by the variance and covariance of the variables involved. The squared Mahalanobis distance (SMD) is widely used in statistics and multivariate data analysis for pattern recognition [3], classification [4, 5], and outlier detection [6–8]. This metric provides a valuable measure for assessing the relationships between data points in various applications.

The value of the SMD is approximately equal to the value of the Chi-squared distribution with k degrees of freedom, which is equal to the number of characteristics [9, 10]. Recognition takes place with the help of a prediction ellipsoid, which defines the space of allowed values such that all elements within the same class are inside the ellipsoid, while others are outside it.

Many statistical procedures assume that the variables are normally distributed, and an assumption of homoscedasticity or homogeneity of variance. Significant violations of either assumption can increase the chances of committing either a type I or II error. Rectifying these issues through data transformations can significantly improve analysis accuracy [11].

The construction of the ellipsoid relies on the assumption that the data follows a multivariate normal distribution [12]. However, real data may have a non-normal multivariate distribution, which leads to a lower recognition probability. To solve certain practical problems, which are based on the use of the Mahalanobis distance, in the case of non-Gaussian data, normalization is used [13, 14]. Its application allows solving the corresponding problems for data whose multivariate distribution deviates from normal.

In [15], a decision rule for pattern recognition was improved based on the application of the SMD for normalized data from 10 characteristics using a decimal logarithm transformation. This allowed to increase in the probability of recognition, but when using the decimal logarithm, the probability of recognition is not always satisfactory, so it is necessary to apply other normalizing transformations, such as the Box-Cox transformation (BCT).

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3 MATERIALS AND METHODS

To create feature vectors, a special program was developed in Python using the Dlib computer vision library. After detecting a face in the input image, the program performs several image processing steps, including cropping and aligning the face so that the eyes are at the same level. Such processing helps to remove some of the distortions caused by the position of the face in the input image [16]. In the last step, the program obtains a set of characteristics from the aligned image. Each feature is the pixel distance between facial landmarks defined by the Dlib library.

After analyzing the studies [17–19], 17 key landmarks of the face were identified. Using the pixel distances between these landmarks, a vector consisting of 10 features was constructed. The symmetrical distances were averaged, resulting in the following features: X_1 – the average distance from the eyes to the middle of the nose, X_2 – the average distance from the eyes to the center of the mouth, X_3 – the average distance from the eyes to the center of the eyebrows, X_4 – the average distance from the eyebrows to the top of the nose, X_5 – the average distance from the corners of the eyes to the top of the nose, X_6 – the distance between the eyebrows, X_7 – the distance between the nose and the middle of the mouth, X_8 – the distance between the corners of the mouth, X_9 – the distance between the edges of the nose, X_{10} – the distance from the mouth to the chin.

To account for variations in the position of the face in the image and different distances to the camera, a normalization process is used by dividing each feature by the distance between the eyes [20].

In the final version, the vector takes the form: $\mathbf{X}$={(v1+v2)/2d, (v3+v4)/2d, (v5+v6)/2d, (v7+v8)/2d, (v9+v10)/2d, v11/d, v12/d, v13/d, v14/d, v15/d}.

A dataset from work [15] containing 200 photos of two people was chosen, where 100 photos are used to build a prediction ellipsoid for recognizing the first person, and 300 are used for testing.

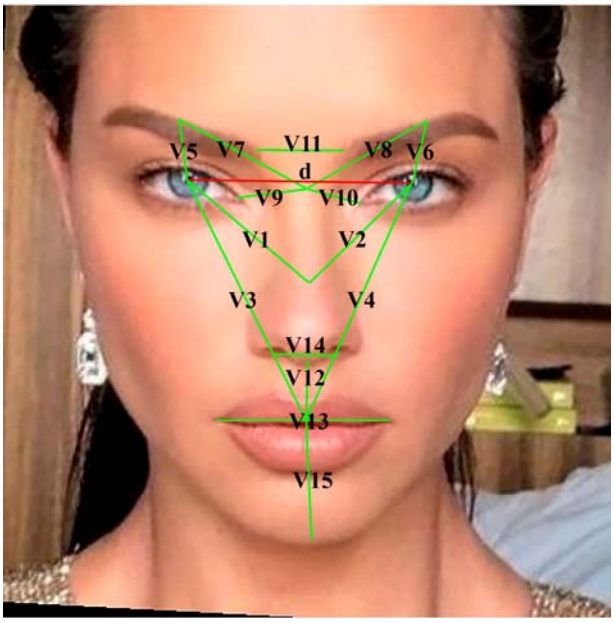

Figure 1 – Distances between face key points used for recognition

As a result, 400 feature vectors were obtained, one per photo, consisting of 10 elements.

A vector of first-person sample means $\overline{\mathbf{X}} = \left\{ \overline{X}_1, \overline{X}_2, \ldots, \overline{X}_{10} \right\}^T$ to construct a prediction ellipsoid: $\overline{\mathbf{X}}$ = {0.6330; 1.1629; 0.3010; 0.6259; 0.2994; 0.3670; 0.3015; 0.8011; 0.3584; 0.6290}, the covariance matrix is shown in Table 1, and the characteristic ranges in Table 2.

Table 1 – Covariance matrix of the initial sample

0.00101	0.00089	–0.00060	–0.00006	–0.00012	–0.00011	–0.00052	0.00001	0.00034	–0.00113
0.00089	0.00200	0.00000	0.00026	–0.00016	0.00006	0.00052	–0.00004	0.00046	0.00024
–0.00060	0.00000	0.00111	0.00047	0.00001	0.00019	0.00071	–0.00009	–0.00009	0.00121
–0.00006	0.00026	0.00047	0.00039	–0.00003	0.00017	0.00031	–0.00027	–0.00006	0.00034
–0.00012	–0.00016	0.00001	–0.00003	0.00008	0.00002	0.00003	0.00000	–0.00010	0.00008
–0.00011	0.00006	0.00019	0.00017	0.00002	0.00073	0.00027	0.00003	–0.00002	0.00002
–0.00052	0.00052	0.00071	0.00031	0.00003	0.00027	0.00119	–0.00015	–0.00007	0.00149
0.00001	–0.00004	–0.00009	–0.00027	0.00000	0.00003	–0.00015	0.00276	0.00100	0.00129
0.00034	0.00046	–0.00009	–0.00006	–0.00010	–0.00002	–0.00007	0.00100	0.00104	0.00016
–0.00113	0.00024	0.00121	0.00034	0.00008	0.00002	0.00149	0.00129	0.00016	0.00416

Table 2 – Ranges of characteristics of the initial sample

	1	2	3	4	5	6	7	8	9	10
Min	0.55734	1.07032	0.23998	0.57780	0.27292	0.31088	0.22610	0.69870	0.29332	0.46811
Max	0.70846	1.28726	0.39580	0.66983	0.32167	0.43156	0.39956	1.00075	0.46041	0.80951

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The Mardia test was used to assess the deviation of the multivariate data distribution from normality. It is based on the analysis of the multivariate skewness β_1 and kurtosis β_2 of the data, which are indicators of how much the data deviate from the normal distribution and are calculated according to the following formulas:

$$\beta_1 = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{\left(\mathbf{X}_i - \overline{\mathbf{X}}\right)^T \mathbf{S}_{\mathbf{X}}^{-1}\left(\mathbf{X}_j - \overline{\mathbf{X}}\right)\right\}^3, \quad (4)$$

$$\beta_2 = \frac{1}{N}\sum_{j=1}^{N}\left\{\left(\mathbf{X}_j - \overline{\mathbf{X}}\right)^T \mathbf{S}_{\mathbf{X}}^{-1}\left(\mathbf{X}_j - \overline{\mathbf{X}}\right)\right\}^2, \quad (5)$$

where

$$\mathbf{S}_{\mathbf{X}} = \frac{1}{N}\sum_{i=1}^{N}\left(\mathbf{X}_i - \overline{\mathbf{X}}\right)\left(\mathbf{X}_i - \overline{\mathbf{X}}\right)^T .$$

According to the Mardia test, the multivariate distribution of the received sample is not Gaussian since the test statistic for multivariate skewness $N\beta_1/6$ of the data, which equals 289.20, is greater than the quantile of the Chi-Square distribution, which is 277.77 for 220 degrees of freedom and 0.005 significance level. In contrast, the test statistic for multivariate kurtosis β_2, which equals 122.35, does not exceed the value of the Gaussian distribution quantile, which is 127.97 for the mean of 120, the variance of 9.6, and a significance level of 0.005. That is why, there is a need to apply a normalizing transformation (1).

The original BCT is a univariate transformation with one parameter λ and is applied element-wise to a vector. For multivariate data, it is usually applied k times as univariate mapping to each column with different values for λ. Therefore, the overall transformation is specified by a k-variate vector $\mathbf{\Theta} = \{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ [21].

As in [22], normalization by the BCT is given by:

$$Z_j = x(\lambda_j) = \begin{cases} \left(X_j^{\lambda_j} - 1\right)\big/\lambda_j, & \lambda_j \neq 0; \\ \ln\left(X_j\right), & \lambda_j = 0. \end{cases} \quad (6)$$

The main task when using the method is to find the optimal value of the input parameter in such a way that, as a result of the transformation, the distribution of the output value is as close as possible to the normal one. The most popular method of finding the optimal value of the lambda parameter is the maximum likelihood estimation:

$$l(\lambda) = C - \frac{N}{2}\ln\sum_{i=1}^{N}\frac{\left(x(\lambda)_i - \overline{x(\lambda)}\right)^2}{N} + (\lambda - 1)\sum_{i=1}^{N}\ln(x_i). \quad (7)$$

The multivariate Box-Cox method uses a separate transformation parameter for each variable. When variables are transformed to joint normality, they become approximately linearly related, constant in conditional variance, and marginally normal in distribution. In the case of using the ten-dimensional BCT, the components of the vector $\mathbf{T}$ are defined as (6).

For the ten-variate BCT, the log-likelihood function can be written as:

$$l(X,\theta) = \sum_{j=1}^{k}\left(\lambda_j - 1\right)\sum_{i=1}^{N}\ln(x_{ji}) - \frac{N}{2}\ln\left[\det(\mathbf{S}_{\mathbf{Z}})\right]. \quad (8)$$

After applying normalizing transformations, a ten-variate prediction ellipsoid is built based on (3):

$$\left(\mathbf{Z} - \overline{\mathbf{Z}}\right)^T \mathbf{S}_{\mathbf{Z}}^{-1}\left(\mathbf{Z} - \overline{\mathbf{Z}}\right) = \chi^2_{10,\,0.005}. \quad (9)$$

The value of the quantile of the Chi-square distribution is 25.19 for 10 degrees of freedom and a significance level of 0.005. The decision rule is based on a prediction ellipsoid (9), which describes the space of admissible values for each class such that all objects of one class must lie within the bounds of this ellipsoid, and of another class – outside the bounds.

4 EXPERIMENTS

For comparison, two prediction ellipsoids are built based on the data from [14] and two normalization transformations: the univariate BCT and the ten-variate BCT.

A univariate BCT is applied to the initial sample. As a result of solving the task using the maximum likelihood method of the logarithmic function (7), the following parameter estimates were obtained: $\hat{\lambda}_1 = 1.7451$, $\hat{\lambda}_2 = -4.5493$, $\hat{\lambda}_3 = -0.7145$, $\hat{\lambda}_4 = 0.3643$, $\hat{\lambda}_5 = 5.1055$, $\hat{\lambda}_6 = -0.8785$, $\hat{\lambda}_7 = -0.4222$, $\hat{\lambda}_8 = -2.7221$, $\hat{\lambda}_9 = -1.8611$, $\hat{\lambda}_{10} = 1.0102$.

As a result of the application of the univariate BCT with components (6), where each element of the vector $\mathbf{T}$ is calculated independently of the others, a sample with the following vector of means $\overline{\mathbf{Z}} = \left\{\overline{Z}_1, \overline{Z}_2, \ldots, \overline{Z}_{10}\right\}^T$ was obtained: $\overline{\mathbf{Z}}$ = {–0.31461; 0.10718; –1.92555; –0.43103; –0.19545; –1.62004; –1.57614; –0.31836; –3.16345; –0.37015}. The covariance matrix of the sample in Table 3, ranges of characteristics in Table 4.

The normalized sample obtained as a result of applying the univariate BCT does not deviate from the multivariate normal distribution, because the test statistic for multivariate skewness $N\beta_1/6$, which equals 276.51, does not exceed the critical value 277.77; the test statistic for multivariate kurtosis β_2, which equals 120.4, is less than the critical value of 127.97.

Applying the ten-variate BCT to normalize the initial sample. As a result of solving the task using the maximum likelihood method of the logarithmic function (8), the following parameter estimates were obtained:

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$\hat{\lambda}_1 = -0.5799$, $\hat{\lambda}_2 = -0.9732$, $\hat{\lambda}_3 = 0.4871$, $\hat{\lambda}_4 = 2.888$, $\hat{\lambda}_5 = 4.0714$, $\hat{\lambda}_6 = -0.231$, $\hat{\lambda}_7 = 0.087$, $\hat{\lambda}_8 = -1.2973$, $\hat{\lambda}_9 = -1.1866$, $\hat{\lambda}_{10} = 1.3321$.

As a result of the application of the ten-variate BCT with components (6), a sample with the following vector of means $\overline{\mathbf{Z}} = \{\overline{Z}_1, \overline{Z}_2, \ldots, \overline{Z}_{10}\}^T$ was obtained: $\overline{\mathbf{Z}}$ = {–0.52627; 0.13911; –0.91083; –0.25654; –0.24379; –1.13224; –1.14383; –0.26329; –2.03304; –0.34494}.

The covariance matrix of the sample in Table 5, ranges of characteristics in Table 6.

The normalized sample by using the ten-variate BCT does not deviate from the multivariate normal distribution, because the test statistic for multivariate skewness $N\beta_1/6$, which equals 265.59, does not exceed the critical value 277.77; the test statistic for multivariate kurtosis β_2, which equals 121.52, is less than the critical value of 127.97.

Table 3 – Covariance matrix of the sample normalized by univariate Box-Cox

0.00051	0.00027	–0.00329	–0.00006	0.00000	–0.00054	–0.00197	0.00006	0.00437	–0.00080
0.00027	0.00035	–0.00016	0.00014	0.00000	0.00022	0.00115	–0.00006	0.00294	0.00005
–0.00329	–0.00016	0.06670	0.00489	0.00000	0.00923	0.02936	–0.00234	–0.01845	0.00937
–0.00006	0.00014	0.00489	0.00070	0.00000	0.00149	0.00234	–0.00085	–0.00193	0.00046
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	–0.00001	0.00000
–0.00054	0.00022	0.00923	0.00149	0.00000	0.03103	0.00950	0.00025	–0.00533	0.00025
–0.00197	0.00115	0.02936	0.00234	0.00000	0.00950	0.03558	–0.00250	–0.01137	0.00810
0.00006	–0.00006	–0.00234	–0.00085	0.00000	0.00025	–0.00250	0.01315	0.03704	0.00273
0.00437	0.00294	–0.01845	–0.00193	–0.00001	–0.00533	–0.01137	0.03704	0.34189	0.00232
–0.00080	0.00005	0.00937	0.00046	0.00000	0.00025	0.00810	0.00273	0.00232	0.00412

Table 4 – Ranges of characteristics of the sample normalized by univariate Box-Cox

	1	2	3	4	5	6	7	8	9	10
Min	–0.36642	0.05845	–2.48076	–0.49720	–0.19561	–2.03872	–2.06852	–0.60754	–4.72990	–0.53009
Max	–0.25900	0.15013	–1.31438	–0.37285	–0.19527	–1.24335	–1.12041	0.00075	–1.73859	–0.19029

Table 5 – Covariance matrix of the sample normalized by ten-variate Box-Cox

0.00436	0.00134	–0.00233	–0.00006	–0.00001	–0.00080	–0.00323	0.00007	0.00616	–0.00204
0.00134	0.00106	–0.00002	0.00008	0.00000	0.00017	0.00112	–0.00004	0.00281	0.00013
–0.00233	–0.00002	0.00374	0.00036	0.00000	0.00115	0.00381	–0.00035	–0.00198	0.00192
–0.00006	0.00008	0.00036	0.00007	0.00000	0.00023	0.00038	–0.00018	–0.00027	0.00012
–0.00001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	–0.00002	0.00000
–0.00080	0.00017	0.00115	0.00023	0.00000	0.00846	0.00270	0.00012	–0.00122	0.00009
–0.00323	0.00112	0.00381	0.00038	0.00000	0.00270	0.01047	–0.00090	–0.00277	0.00378
0.00007	–0.00004	–0.00035	–0.00018	0.00000	0.00012	–0.00090	0.00705	0.01390	0.00175
0.00616	0.00281	–0.00198	–0.00027	–0.00002	–0.00122	–0.00277	0.01390	0.08567	0.00116
–0.00204	0.00013	0.00192	0.00012	0.00000	0.00009	0.00378	0.00175	0.00116	0.00305

Table 6 – Ranges of characteristics of the sample normalized by ten-variate Box-Cox

	1	2	3	4	5	6	7	8	9	10
Min	–0.69589	0.06576	–1.02852	–0.27523	–0.24437	–1.34124	–1.39454	–0.45651	–2.76948	–0.47758
Max	–0.38152	0.22388	–0.74583	–0.23742	–0.24319	–0.92748	–0.88171	0.00075	–1.27277	–0.18419

After data normalization by univariate and multivariate BCTs, ten-variate ellipsoids were constructed based on (9). The computer program implementing the constructed models was developed to conduct experiments. The program was written in the Python language.

5 RESULTS

The recognition check is based on two criteria, such as the probability of recognizing the first person (PRFP), and the probability of type II errors, which occur when the decision rule mistakenly identifies another person as person 1.

Application of (9) for normalized data with a univariate BCT to recognize 300 test photos allowed obtaining the following results: the PRFP is 94%, with a probability of type II errors being 5.5%. The application of (9) for data normalized using the ten-variate BCT allowed obtaining the PRFP of 97% with type II errors of 2.5%.

Table 7 shows a comparison of the results of using models for non-normalized data (Source); data normalized by the transformation of the decimal logarithm (Lg) [15]; normalized data using univariate BCT; normalized data using the ten-variate BCT.

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Table 7 – Comparison of the results

	Source	Lg	Univariate BCT	Ten-variate BCT
PRFP	92%	95%	94%	97%
Type II errors	4.5%	5.5%	5.5%	2.5%

The use of ten-variate prediction ellipsoid (9) for data normalized using a multivariate BCT resulted in the highest probability of recognition.

6 DISCUSSION

As is evident from Table 7, the decision rule built for the initial data resulted in the lowest recognition probability. Application of decision rules for normalized data by decimal logarithm and univariate BCT has increased the PRFP and reduced the possibility of type II errors. The most notable enhancement in recognition accuracy was achieved with the ten-variate BCT.

Taking into account that the use of univariate transformations had a lesser impact, it is important to note that this can be explained by the fact that univariate transformations do not account for data correlation. Thus, their limitation lies in their inability to consider the interrelationships between different variables, which affect the analysis results. Unlike univariate transformations, the ten-variate BCT preserves inter-variable relationships crucial in capturing complex facial features.

CONCLUSIONS

The important problem of increasing the probability of face recognition by constructing a ten-variate prediction ellipsoid for data normalized by the BCT is solved.

The scientific novelty of the obtained results is that the ten-variate prediction ellipsoid for normalized data for face recognition is firstly constructed based on the BCTs. The application of univariate BCT resulted in a slight improvement, which is explained by the fact that the method does not take into account the correlation between features. The construction prediction ellipsoid for normalized data based on ten-variate BCT allowed increased PRFP and reduced type II errors.

The practical significance of the obtained results is that the software realizing the constructed model is developed in the Python language. The experimental results allow us to recommend the constructed model for use in practice.

Prospects for further research may include the use of other multivariate normalizing transformations to construct prediction ellipsoid for face recognition.

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РОЗПІЗНАВАННЯ ОБЛИЧЧЯ ЗА ДОПОМОГОЮ ДЕСЯТИВИМІРНОГО ЕЛІПСОЇДА ПРОГНОЗУВАННЯ ДЛЯ НОРМАЛІЗОВАНИХ НА ОСНОВІ ПЕРЕТВОРЕННЯ БОКСА-КОКСА ДАНИХ

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АНОТАЦІЯ

Актуальність. Розпізнавання обличчя, яке є одним із завдань розпізнавання образів, відіграє важливу роль у сучасному інформаційному світі та знаходить широке застосування в різних галузях, включаючи системи безпеки, управління доступом та ін. Це робить його важливим інструментом для забезпечення безпеки та персоналізації. Однак низька ймовірність ідентифікації особи за обличчям може мати негативні наслідки, тому існує потреба в розробці та вдосконаленні методів розпізнавання обличчя. Об'єктом дослідження є процес розпізнавання обличчя. Предметом дослідження є математична модель для розпізнавання обличчя.

Один з часто використовуваних методів розпізнавання образів полягає в побудові правил прийняття рішень на основі еліпсоїда прогнозування. Важливим обмеженням його застосування є необхідність виконання припущення про багатовимірний нормальний розподіл даних. Однак у багатьох випадках багатовимірний розподіл реальних даних може відхилятися від нормального, що призводить до зниження ймовірності розпізнавання. Тому виникає необхідність удосконалення математичних моделей, які враховували б зазначене відхилення.

Мета роботи полягає у підвищенні ймовірності розпізнавання обличчя шляхом побудови десятивимірного еліпсоїду прогнозування для нормалізованих за допомогою перетворення Бокса-Кокса даних.

Метод. Застосування тесту Мардіа для перевірки відхилення багатовимірного розподілу даних від нормального. Побудова правил прийняття рішень для розпізнавання обличчя за допомогою десятивимірного еліпсоїду прогнозування для нормалізованих на основі перетворення Бокса-Кокса даних. Отримання оцінок параметрів одновимірного та десятививимірного перетворень Бокса-Кокса за допомогою методу максимальної правдоподібності.

Результати. Здійснено порівняння результатів розпізнавання облич за допомогою правил прийняття рішень, які побудовані за допомогою десятивимірного еліпсоїду прогнозування для нормалізованих за різними перетвореннями даних. У порівнянні із застосуванням одновимірних нормалізуючих перетворень (десяткового логарифму та Бокса-Кокса) та у випадку відсутності нормалізації використання десятивимірного перетворення Бокса-Кокса призводить до збільшення ймовірності розпізнавання облич.

Висновки. Для розпізнавання обличчя удосконалено математичну модель у вигляді десятивимірного еліпсоїду прогнозування для нормалізованих за допомогою багатовимірного перетворення Бокса-Кокса даних, що дозволяє підвищити ймовірність розпізнавання у порівнянні із застосуванням відповідних моделей, які побудовані або без нормалізації, або із використанням одновимірних нормалізуючих перетворень. Досліджено, що математична модель, побудована для нормалізованих даних за допомогою багатовимірного перетворення Бокса-Кокса, має більшу ймовірність розпізнавання за рахунок того, що одновимірні перетворення нехтують кореляцією між геометричними ознаками обличчя.

КЛЮЧОВІ СЛОВА: розпізнавання обличчя, еліпсоїд прогнозування, нормалізуюче перетворення, багатовимірне перетворення Бокса-Кокса, правило прийняття рішення.

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