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METHOD FOR SIGNAL PROCESSING BASED ON KOLMOGOROV-WIENER PREDICTION OF MFSD PROCESS

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ABSTRACT

Context. We investigate a method to signal processing based on the Kolmogorov-Wiener filter weight function calculation for the prediction of a continuous stationary heavy-tail process in the MFSD (multifractal fractional sum-difference) model. Such a process may describe telecommunication traffic in some systems with data packet transfer, the consideration of the continuous filter may be reliable in the case of the large amount of data.

Objective. The aim of the work is to obtain an approximate solution for the Kolmogorov-Wiener filter weight function and to show the applicability of the method to signal processing used in the paper.

Method. The Galerkin method based on the orthogonal Chebyshev polynomials of the first kind is used for the calculation of the weight function under consideration. The approximations up to the thirteen-polynomial one are investigated. The corresponding integrals are calculated numerically on the basis of the Wolfram Mathematica package. The higher is the packet rate, the higher accuracy of the integral calculation is needed.

Results. It is shown that for rather large number of polynomials the misalignment between the left-hand side and the right-hand side of the Wiener-Hopf integral equation under consideration is rather small for the obtained solutions. The corresponding mean absolute percentage errors of misalignment for different packet rates are calculated. The method to signal processing used in the paper leads to reliable results for the Kolmogorov-Wiener filter weight function for the prediction of a process in the MFSD model.

Conclusions. The theoretical fundamentals of the continuous Kolmogorov-Wiener filter construction for the prediction of a random process in the MFSD model are investigated. The filter weight function is obtained as an approximate solution of the Wiener-Hopf integral equation with the help of the Galerkin method based on the Chebyshev polynomials of the first kind. It is shown that the obtained results for the filter weight function are reliable. The obtained results may be useful for the practical telecommunication traffic prediction. The paper results may also be applied to the treatment of heavy-tail random processes in different fields of knowledge, for example, in agriculture.

KEYWORDS: Kolmogorov-Wiener filter weight function, telecommunication traffic, Galerkin method, MFSD model, Chebyshev polynomials of the first kind, stationary random heavy-tail process.

ABBREVIATIONS

GFSD is the Gaussian fractional sum-difference;

ARIMA is an autoregressive integrated moving average;

MFSD is a multifractal fractional sum-difference;

MAPE is a mean absolute percentage error.

NOMENCLATURE

T is a time interval on which the traffic data are observed;

p/s packets per second;

z is a time interval for which the traffic prediction should be made;

$h(t)$ is the Kolmogorov-Wiener filter weight function;

α is a packet rate;

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θ, λ, ξ are auxiliary quantities which depend on the packet rate;

$R(t)$ is a traffic correlation function in the MFSD model;

$\rho(t)$ is a traffic correlation function in the GFSD model;

$\Gamma(x)$ is a gamma function;

d is a fractional differencing parameter of the model;

a, b are auxiliary constants;

n is a number of polynomials in the corresponding approximations;

g_s are coefficients multiplying the polynomials;

$S_s(t)$ are Chebyshev polynomials of the first kind orthogonal on $t \in [0, T]$;

$T_s(x)$ are Chebyshev polynomials of the first kind orthogonal on $x \in [-1, 1]$;

$\mathrm{Left}(t)$ is a left-hand side of the Wiener-Hopf integral equation;

$\mathrm{Right}(t)$ is a right-hand side of the Wiener-Hopf integral equation;

G_{ks} are integral brackets;

B_k are free terms in the linear system of algebraic equations in g_s;

K, N, L are numbers of points in the numerical integration.

INTRODUCTION

Telecommunication traffic nowadays is usually treated as a heavy-tail random process; see, for example, [1]. The traffic prediction is an important problem for telecommunications. Improving the accuracy of traffic prediction can help companies to develop adequate business planning and improve the economic benefits. Moreover, accurate prediction results can also be urgent for optimal resource management, sophisticated network design, and so on, see [2–4]. Recently several models of the heavy-tail stationary heavy-tail processes which may describe telecommunication traffic were proposed, for example, the generalized fractional Gaussian noise model [5, 6] and the GFSD and the MFSD models [7].

There are a plenty of different methods for traffic prediction, for example, such as ARIMA, neural networks, etc., see, for example, [2, 4, 8]. Recently in our papers we investigated such a simple prediction method as the one based on the Kolmogorov-Wiener filter.

It was shown that both continuous and discrete Kolmogorov-Wiener filter may be applied to the prediction of heavy-tail data if the data are smooth enough [9]. As is known, see, for example, [9], in the continuous case the Kolmogorov-Wiener filter weight function obeys the Wiener-Hopf integral equation. This equation can be solved on the basis of the Galekrin method, see the method description, for example, in [10] and references therein. In particular, it was shown that the Galerkin method may be applied to the solution of the corresponding Wiener-Hopf equation in the case of the GFSD and the generalized fractional Gaussian noise models, see [11, 12]. But the investigation of the Wiener-Hopf equation solution for the process in the framework of the MFSD model is not yet done. So, this paper is devoted to the search of an approximate solution of the Wiener-Hopf integral equation in the case where the corresponding continuous Kolmogorov-Wiener filter is used to the prediction of the heavy-tail process in the MFSD model.

The object of study is the Kolmogorov-Wiener filter for the prediction of continuous heavy-tail process in the MFSD model.

The subject of study is the weight function of the corresponding filter.

The aim of the work is to obtain an approximate solution for the weight function on the basis of Galerkin method.

1 PROBLEM STATEMENT

The Kolmogorov-Wiener filter weight function in the continuous case obeys the following Wiener-Hopf equation (see, for example, [9]):

$$\int_0^T d\tau h(\tau) R(t-\tau) = R(t+z). \qquad (1)$$

The problem statement is as follows: to obtain the unknown filter weight function as an approximate solution of the integral equation (1) on the basis of the Galerkin method.

2 REVIEW OF THE LITERATURE

The MFSD model of a stationary random heavy-tail process which may describe telecommunication traffic was proposed in [7]. In some sense, the MFSD model is a modification of the GFSD model which was also proposed in [7].

Our recent papers were devoted to the development of the theoretical fundamentals of the continuous Kolmogorov-Wiener filter construction for the prediction of stationery processes in different models; see [11–13]. In particular, paper [11] was devoted to the corresponding investigation for the GFSD model. However, the MFSD model was not investigated in our previous papers.

In this paper we investigate the continuous Kolmogorov-Wiener filter applied to the prediction of the telecommunication traffic described by the MFSD model. The Wiener-Hopf integral equation (1) is solved on the basis of the Galerkin method [10], the Chebyshev polynomials of the first kind are chosen as the required orthogonal function system. The numerical investigation of the misalignment between the left-hand side and the right-hand side for the obtained solutions is made. It is shown that the proposed method leads to reliable results.

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3 MATERIALS AND METHODS

According to [7], the traffic correlation function for the discrete MFSD model is as follows:

$$R(t)=\frac{e^{\xi(\alpha)\rho(t)}-1}{e^{\xi(\alpha)}-1} \tag{2}$$

where

$$\rho(t)=(1-\theta(\alpha))\frac{2(1-d)t^2-(1-d)^2}{t^2-(1-d)^2}\times \\ \times\frac{\Gamma(1-d)}{\Gamma(d)}\frac{\Gamma(t+d)}{\Gamma(t-d+1)}, \\ \theta(\alpha)=\frac{2^{-7.21}\alpha^{0.75}}{2^{-7.21}\alpha^{0.75}+1}, \tag{3}$$

and

$$\xi(\alpha)=\ln\left(\Gamma\left(1+\frac{2}{\lambda(\alpha)}\right)\right)-2\ln\left(\Gamma\left(1+\frac{1}{\lambda(\alpha)}\right)\right), \\ \lambda(\alpha)=\frac{2^{-5.36}\alpha^{0.63}}{2^{-5.36}\alpha^{0.63}+1}, \tag{4}$$

the packet rate $\alpha\in\left[2^{10.22}\text{ p/s},2^{17.5}\text{ p/s}\right]$, $d=0.31$, see [7]. The definition of the Gamma function $\Gamma(x)$ is given, for example, in [14]. The asymptotic behavior of $R(t)$ at $t\to+\infty$ is $R(t)\sim\text{const}\cdot t^{2d-1}$ [7], so the MFSD model describes a heavy-tail process. In [7] it is indicated that the results (2) and (3) are valid for $t\geq 1$. In [11] it is proposed to redefine the function $\rho(t)$ for the continuous case as follows:

$$\rho(t)=\begin{cases} a|t|^b+1, |t|\leq 1 \\ (1-\theta(\alpha))\dfrac{2(1-d)t^2-(1-d)^2}{t^2-(1-d)^2}\times \\ \times\dfrac{\Gamma(1-d)}{\Gamma(d)}\dfrac{\Gamma(|t|+d)}{\Gamma(|t|-d+1)}, |t|\geq 1\end{cases}, \tag{5}$$

where the values a, b are chosen in such a way that

$$\lim_{t\to 1-0}\rho(t)=\lim_{t\to 1+0}\rho(t), \\ \lim_{t\to 1-0}\frac{d\rho(t)}{dt}=\lim_{t\to 1+0}\frac{d\rho(t)}{dt}. \tag{6}$$

So, in what follows we use the correlation function $R(t)$ in the form (2) where the function $\rho(t)$ is taken according to (5) and (6).

Similarly to the calculations [11], the weight function is sough in the form

$$h(\tau)=\sum_{s=0}^{n-1}g_sS_s(\tau),\ S_s(\tau)=T_s\left(\frac{2\tau}{T}-1\right) \tag{7}$$

where the coefficients g_s obey the following system of linear algebraic equations:

$$\sum_{s=0}^{n-1}g_sG_{ks}=B_k,\ k=\overline{0,n-1}, \\ G_{ks}=\int_0^T\int_0^T d\tau dt S_k(t)S_s(\tau)R(t-\tau), \\ B_k=\int_0^T dtS_k(t)R(t+z). \tag{8}$$

The free terms B_k are calculate with the help of the Wolfram Mathematica package on the basis of the NIntegrate procedure built in the package. As for the calculation of the integral brackets G_{ks}, the NIntegrate procedure requires too much computation time, so the integral brackets are approximately calculated as follows:

$$G_{ks}\approx\Delta^2\sum_{i,j=0}^{K-1}S_k\left(i\Delta+\frac{1}{2}\Delta\right)S_s\left(j\Delta+\frac{1}{2}\Delta\right)R(i\Delta-j\Delta), \\ \Delta=T/K, \tag{9}$$

it should be stressed that only the calculation of G_{ks} where k and s are of the same parities is needed; $G_{ks}=0$ if k and s are of different parities, see [11] and references therein. Expression (9) on the basis of the fact that the function $R(t)$ is an even one may be rewritten as

$$G_{ks}\approx\Delta^2\sum_{i=1}^{K-1}\sum_{j=0}^{i-1}\left(S_k\left(i\Delta+\frac{1}{2}\Delta\right)S_s\left(j\Delta+\frac{1}{2}\Delta\right)+\right. \\ \left.+S_k\left(j\Delta+\frac{1}{2}\Delta\right)S_s\left(i\Delta+\frac{1}{2}\Delta\right)\right)R(i\Delta-j\Delta)+ \\ +\Delta^2\sum_{i=0}^{K-1}S_k\left(i\Delta+\frac{1}{2}\Delta\right)S_s\left(i\Delta+\frac{1}{2}\Delta\right)R(0). \tag{10}$$

So, first of all the quantities G_{ks} and B_k should be calculated on the basis of (8) and (10). Then the coefficients g_s are calculated as the solution of the system (8), the corresponding calculations are made in the Wolfram Mathematica package. The obtained weight function $h(t)$ is given by (7).

The left-hand side and the right-hand side of the integral equation (1) for the obtained solutions are as follows:

$$\text{Left}(t)=\int_0^T d\tau h(\tau)R(t-\tau), \\ \text{Right}(t)=R(t+z), \tag{11}$$

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the calculation of the weight function $h(t)$ is described above in detail. The corresponding misalignment is described by the MAPE:

$$\text{MAPE} = \frac{1}{T}\int_0^T \left|\frac{\text{Left}(t) - \text{Right}(t)}{\text{Right}(t)}\right| dt \cdot 100\% . \quad (12)$$

The corresponding integrals can hardly be calculated, so they are estimated as follows, see [11, 12]:

$$\text{MAPE} \approx \frac{1}{N}\sum_{j=0}^{N-1}\left|\frac{\text{Left}\left(\frac{jT}{N}\right) - \text{Right}\left(\frac{jT}{N}\right)}{\text{Right}\left(\frac{jT}{N}\right)}\right| \cdot 100\% ;$$

$$\text{Left}(t) \approx \frac{\delta}{2}\sum_{j=0}^{L-1}\Big(h(j\delta)R(t - j\delta) + \quad (13)$$

$$+ h\big((j+1)\delta\big)R\big(t - (j+1)\delta\big)\Big), \ \delta = T/L .$$

The ccorresponding numerical results are given in the next section.

4 EXPERIMENTS

The following values of the parameters are chosen: $T = 100$, $z = 3$, a similar choice is made in [11].

The numerical results for the coefficients a, b for different packet rates are given in Table 1, the results in Table 1 are written rounded off to 3 significant digits.

Table 1 – Results for the coefficients in (5) for different packet rates

α, p/s	a, b
2^{11}	$a = -0.746$, $b = 0.381$
2^{13}	$a = -0.886$, $b = 0.144$
2^{15}	$a = -0.956$, $b = 0.0522$
2^{17}	$a = -0.984$, $b = 0.0186$

The obtained MAPE results for different packet rates are given in Tables 2 and 3. The results in Table 2 and Table 3 are rounded off to two decimal places.

Table 2 – MAPE for the approximations of n polynomials for rather low packet rates

$\alpha = 2^{11}$ p/s		$\alpha = 2^{13}$ p/s	
n	MAPE, %	n	MAPE, %
1	27.39	1	26.49
2	18.58	2	17.78
3	11.77	3	11.09
4	8.67	4	8.09
5	6.01	5	5.53
6	4.71	6	4.31
7	3.49	7	3.15
8	2.92	8	2.62
9	2.28	9	2.05
10	2.03	10	1.79
11	1.69	11	1.50
12	1.57	12	1.40
13	1.34	13	1.23

The values $K = 3 \cdot 10^3$, $N = 10^2$, $L = 10^4$ are chosen during the calculation of the results shown in Table 2. The values $K = 6 \cdot 10^3$, $N = 10^2$, $L = 10^4$ are chosen during the calculation of the results shown in Table 3. The value $K = 3 \cdot 10^3$ does not provide the satisfactory accuracy of the calculations of the integrals used in the obtaining of the results given in Tabe 3, so a higher value of K is needed. The increase of K leads to a rather significant increase in the computation time for the integral brackets, so we restricted ourselves to the value $K = 6 \cdot 10^3$.

Table 3 – MAPE for the approximations of n polynomials for rather high packet rates

$\alpha = 2^{15}$ p/s		$\alpha = 2^{17}$ p/s	
n	MAPE, %	n	MAPE, %
1	26.30	1	25.46
2	17.57	2	16.75
3	10.94	3	10.18
4	7.97	4	7.32
5	5.46	5	4.96
6	4.25	6	3.86
7	3.14	7	2.91
8	2.61	8	2.47
9	2.07	9	2.10
10	1.83	10	1.95
11	1.56	11	1.81
12	1.45	12	1.78
13	1.29	13	1.74

As can be seen, the left-hand side and the right-hand side of the integral equation (1) are really close for the obtained solutions.

5 RESULTS

The obtained results are graphically illustrated on Fig. 1 – Fig. 4.

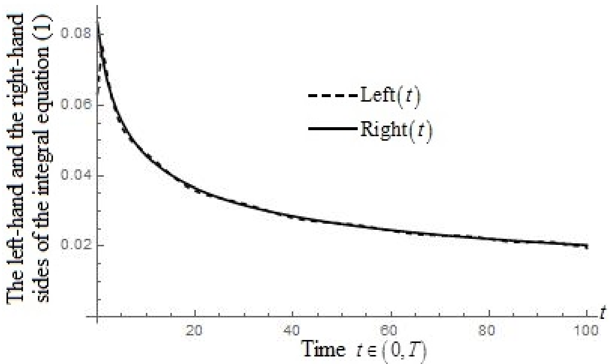

Figure 1 – Comparison of the left-hand and right-hand sides of eq. (1) for the packet rate $\alpha = 2^{11}$ p/s for the thirteen-polynomial approximation

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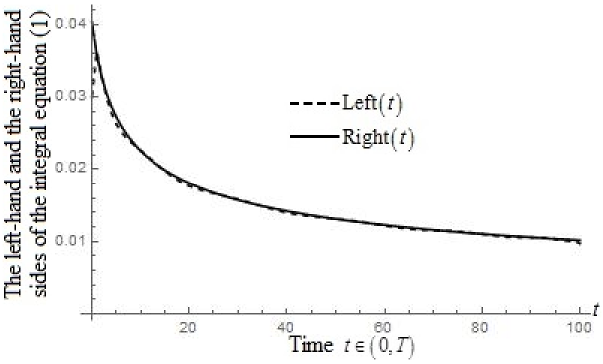

Figure 2 – Comparison of the left-hand and right-hand sides of eq. (1) for the packet rate $\alpha = 2^{13}$ p/s for the thirteen-polynomial approximation

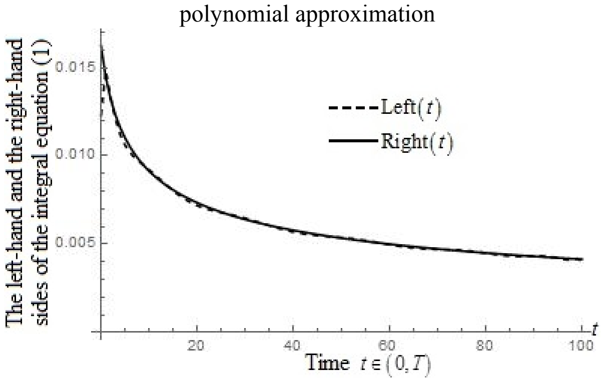

Figure 3 – Comparison of the left-hand and right-hand sides of eq. (1) for the packet rate $\alpha = 2^{15}$ p/s for the thirteen-polynomial approximation

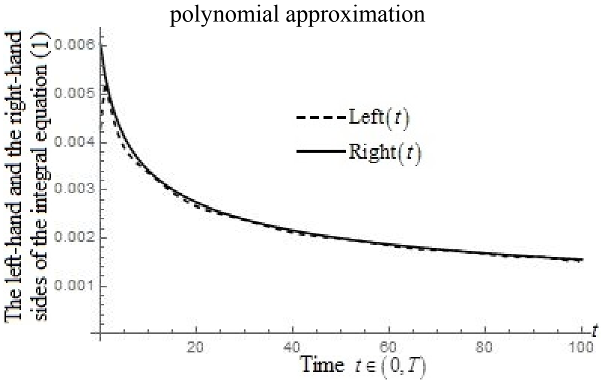

Figure 4 – Comparison of the left-hand and right-hand sides of eq. (1) for the packet rate $\alpha = 2^{17}$ p/s for the thirteen-polynomial approximation

As can be seen from the corresponding graphs, the left-hand side and the right-hand side of the integral equation (1) are really close for the obtained solutions. According to Table 2 and Table 3, the corresponding misalignment MAPE is less than 2%.

6 DISCUSSION

The paper is devoted to the development of the theoretical fundamentals of the continuous Kolmogorov-Wiener filter construction for the prediction of a stationary process in the MFSD model. Such a process may describe telecommunication traffic in the systems with data packet transfer, see [7]. Of course, the traffic measurements in fact lead to a discrete time series, but if we deal with large amount of data, the consideration of the continuous limit may be reliable, so the results may be also used to digital signal processing of telecommunication traffic.

An approximate solution for the filter weight function is obtained on the basis of the Galerkin method based on the Chebyshev polynomials of the first kind. The main difficulty of the calculation is the fact that in the framework of the considered problem the integral brackets can hardly be calculated either analytically or on the basis of the numerical integration procedure built in the Wolfram Mathematica package. So, the formula (10) is used for the integral brackets obtaining. The higher the number of points K is, the more precise the result is; however, the increase of the value of K leads to the significant increase of the computation time. The value $K = 3 \cdot 10^3$ is enough for rather low packet rates, but higher values are required for rather high packet rates, the corresponding value $K = 6 \cdot 10^3$ is chosen. The misalignment of the left-hand side and the right-hand side of the integral equation (1) is rather low for rather large number of polynomials (corresponding MAPE is less than 2%), so we can conclude that the solutions for the weight function obtained in this paper are rather accurate.

The plans for the future are as follows. The practical applications of the constructed theoretical fundamentals are of interest. Moreover, another orthogonal system may be used in order to avoid such computational difficulties, for example, the Walsh function system, see [13, 15]. The method used in the paper may be applied to the self-similar processes in other models, for example, for the recently developed rational Gaussian noise model [16]. The heavy-tail processes play important role not only for the traffic prediction, but also in other fields of knowledge, for example, in agriculture [17, 18]. So the proposed method may be useful for data treatment in agriculture, for time series in mining (see [19, 20]) and so on. These plans may be realized in other papers.

CONCLUSIONS

The Kolmogorov-Wiener filter weight function for the prediction of continuous stationary self-similar heavy-tail process in the MFSD) model is obtained on the basis of the Galerkin method based on the Chebyshev polynomials of the first kind. The obtained results may be used to digital signal processing of telecommunication traffic. Approximations up to the 13-polynomial one are investigated. It is shown that if the number of polynomials is rather large, the coincidence between the left-hand side and the right-hand side of the Wiener-Hopf integral equation is rather good, so rather accurate solutions for the weight function under consideration are obtained.

The scientific novelty of the paper is the fact that for the first time the weight function of the continuous Kolmogorov-Wiener filter is obtained for the heavy-tail process prediction in the MFSD model, which may be important both for analog and for digital signal processing.

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The practical significance, in particular, is that the obtained results may be applied to telecommunication traffic prediction in systems with data packet transfer.

Prospects for further research, in particular, are to apply the obtained theoretical results to practical prediction and to investigate the Galerkin method in the framework of the problem under consideration on the basis of another orthogonal function systems, for example on the basis of the Walsh functions. The method used in the paper may be also applied to treatment of time series which may be measured by wireless sensor networks, including agrotechnical purposes.

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МЕТОД ОБРОБКИ СИГНАЛІВ НА ОСНОВІ ПРОГНОЗУВАННЯ КОЛМОГОРОВА-ВІНЕРА ПРОЦЕСУ В MFSD МОДЕЛІ

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АНОТАЦІЯ

Актуальність. Досліджено метод обробки сигналів, що базується на обчисленні вагової функції фільтра Колмогорова-Вінера для прогнозування неперервного стаціонарного процесу з важким хвостом в MFSD (multifractal fractional sum-difference) моделі. Такий процес може описувати телекомунікаційний трафік у деяких системах з пакетною передачею даних, розгляд неперервного фільтра може бути доцільним у випадку великої кількості даних.

Мета роботи. Метою роботи є отримання наближеного розв'язку для вагової функції фільтра Колмогорова-Вінера та показати застосовність методу обробки сигналів, використаного в роботі.

Метод. Для розрахунку вагової функції, яку розглянуто в роботі, використовується метод Галеркіна, що базується на ортогональних поліномах Чебишова першого роду. Досліджено різні наближення включно до наближення тринадцяти поліномів. Відповідні інтеграли обчислені числовими методами на основі математичного пакету Wolfram Mathematica. Чим більша швидкість передачі пакетів, тим вищою має бути точність обчислення інтегралів.

Результати. Показано, що для досить великої кількості поліномів нев'язка між лівою та правою частинами інтегрального рівняння Вінера-Хопфа, що розглядається в роботі, є досить малою для отриманих розв'язків. Розраховано відповідні середні абсолютні відсоткові похибки нев'язки для різних швидкостей передачі пакетів. Метод обробки сигналів, використаний у роботі, дає адекватні результати для вагової функції фільтра Колмогорова-Вінера для прогнозування процесу в MFSD моделі.

Висновки. Досліджено теоретичні основи побудови неперервного фільтра Колмогорова-Вінера для прогнозування випадкового процесу в MFSD моделі. Вагову функцію фільтра отримано як наближений розв'язок інтегрального рівняння Вінера-Хопфа за допомогою методу Галеркіна, що базується на поліномах Чебишова першого роду. Показано, що отримані результати для вагової функції фільтра є адекватними. Отримані результати можуть бути корисними для практичного прогнозування телекомунікаційного трафіку. Результати роботи також можуть бути застосовані для розгляду випадкових процесів з важким хвостом у різних галузях знань, наприклад, у сільському господарстві.

КЛЮЧОВІ СЛОВА: вагова функція фільтра Колмогорова-Вінера, телекомунікаційний трафік, метод Галеркіна, MFSD модель, поліноми Чебишева першого роду, стаціонарний випадковий процес з важким хвостом.

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