# УПРАВЛІННЯ У ТЕХНІЧНИХ СИСТЕМАХ

# CONTROL IN TECHNICAL SYSTEMS

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# MARGIN OF STABILITY OF THE TIME-VARYING CONTROL SYSTEM FOR ROTATIONAL MOTION OF THE ROCKET

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## ABSTRACT

**Context.** The rocket motion control system is time-varying, since its parameters during flight depend on the point of the trajectory and fuel consumption. Stability margin indicators are determined in a limited area of individual points of the trajectory using algorithms that are developed only for linear stationary systems, which leads to the need to enter stock factor in hardware. In the available sources, due attention is not paid to the development of methods for determining the quantitative assessment of the stability margin of the time-varying control system.

**Objective** is to develop a methodological support for the construction of an algorithm for calculating the stability margin indicators of the time-varying system for controlling the rocket rotational motion in the plane of yawing using the equivalent stationary approximation at a selected trajectory section**.**

**Method.** The mathematical model of the control system for the rocket rotational movement in one plane is adopted in the form of a linear differential equation without considering the inertia of the executive device and other disturbing factors. The effect of deviation of parameters from their average values for a certain trajectory section is considered as a disturbance, which makes it possible to transition from a non-stationary model to an equivalent approximate stationary one. The Nyquist criterion is used to estimate the stability margin indicators, which is based on the analysis of the frequency characteristic of an open system, for the determination of which the Laplace transform mathematical apparatus is used. To simplify the transition from functions of time in the differential equation of perturbed motion to functions of a complex variable in the Laplace transform, time-varying model parameters are presented in the form of a sum of exponential functions.

**Result.** Methodological support was developed for building an algorithm for determining the stability margin of the rocket's rotary motion control system at a given trajectory section with time-inconstant parameters.

**Conclusions.** Using the example of the time-varying system for controlling the rocket rotational movement, the possibility of using the Laplace transformation to determine the stability margin indicators is shown.

The obtained results can be used at the initial stage of project work.

The next stage of the research is an assessment of the level of algorithm complexity, considering the inertia of the executive device and the disturbed movement of the mass center.

**KEYWORDS:** rocket motion control, time-varying system, Laplace transform.

## ABBREVIATIONS

APFC is an amplitude-phase frequency characteristic;

LC is law of control;

LTV is a linear time-varying system;

MLS is the method of least squares;

CO is the control object;

TF is a transfer function;

CS is the control system for rotational motion of the rocket in the yaw plane;

LF is a Lyapunov function;

CP is a characteristic polynomial.

## NOMENCLATURE

$\bar{a}_{\psi\psi}, \bar{a}_{\psi\delta}$ are average values of parameters of the CS model at the trajectory section;

$\tilde{a}_{\psi\psi}(t), \tilde{a}_{\psi\delta}(t)$ are variable components of the model parameters at the trajectory section depending on the time from the beginning of the section;

$C_{\psi i}, C_{\delta i}$ are coefficients in the $i$-th term of the approximation of variable component of the model parameters by the sum of exponential functions;

$d \cdot 1(t)$ is a signal at the input of the CS;

$f_1$ is a specified value of the frequency of the missile body oscillations in the transient process of disturbance compensation;

$f_{2k}$ , $f_2$ are frequency of the rocket body oscillations in the transient process of disturbance compensation at the $k$-th step of the iterations and one after their end;

$j$ is an imaginary unit;


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$k$ is a current number of iteration;

$k_{\psi}, k'_{\psi}$ are coefficients of LC;

$l$ is the number of rows in the array $N$;

$L\{f(t)\}$ is the Laplace transform operator of the time function;

$m$ is a disruptive acceleration;

$N$ is a array of polynomial $Q_k(s)$ depending on the $l$ argument values;

$P(s), Q(s)$ are numerator and denominator of the TF $w_z(s)$;

$Q_a(s)$ is a denominator of the TF $w(s)$;

$Q_0$ is a first approximation of the denominator of the TF $w_z(s)$;

$Q_k(s)$ is a denominator of TF $w_z(s)$ at the $k$-th iterations step;

$Qd(s,\psi_{cur})$ is a component CP of CS caused by the instability of the model parameters depending on the complex variable $s$ and the image $\psi_{cur}$ of the signal at the output of the CS;

$q_{2k}, q_{1k}, q_{0k}$, $q_2, q_1, q_0$ are coefficients of CP $Q(s)$ at the $k$-th iterations step and after their completion;

$r_{\psi i}$, $r_{\delta i}$ are exponents of the exponential functions in the $i$-th term of approximation of variable components of the model parameters $\tilde{a}_{\psi\psi}(t), \tilde{a}_{\psi\delta}(t)$;

$s_i$ is an $i$-th value of complex argument $s$;

$u(\omega), v(\omega)$ are real and imaginary component of the APFC of the open CS;

$u\ jv$ is a plane of the real and imaginary components of the APFC $w(s)$ of the opened CS;

$w(s)$ is a TF of the opened CS;

$w_z(s), w_m(s)$ are TF of CS;

$w_{z0}(s)$ is a first approximation of TF $w_z(s)$;

$w_{zk}(s)$ is a TF $w_z(s)$ at the $k$-th step of the iterations;

$\delta$ is an equivalent rotation angle of the steering wheel of the CS regulator's executive device;

$\eta_1$ is a specified value of the stability margin on the CP roots plane;

$\eta_{2k}$, $\eta_2$ are margins of stability on the CP roots plane at the $k$-th iteration step and after its end;

$\eta_a, \eta_{ph}$ are CS stability margin indicators by amplitude and by phase;

$\eta_{act}$, $\eta_{phct}$ are stability margin indicators when using the method of frozen coefficients;

$\psi, \dot{\psi}, \ddot{\psi}$ are yaw angle and its derivatives;

$\psi_g, \dot{\psi}_g$ are given values of the yaw angle and its derivative;

$\psi_0(s)$ is a first approximation of the image of the output signal of the CS;

$\omega_1$ is a value of the circular frequency at which the APFC of the open CS crosses a circle of unit radius.

## INTRODUCTION

The main requirements for the CS, as that's known are to ensure the specified parameters of the stability margin indicators and the accuracy of the trajectory. The fulfillment of these requirements is achieved by choosing the structure and parameters of the regulator and modeling the disturbed motion of the rocket in the vicinity of the nominal kinematic values.

At the first stage of CS development, a mathematical apparatus is used in the form of a system of linear differential equations with parameters that are assumed to be constant in the vicinity of a certain trajectory point [1], while as a result of fuel consumption, an increase in speed and flight altitude, the parameter values can change by tens of percent. That is, the so-called method of frozen coefficients is used, as a result, the dependence of the parameter on time is a piecewise-constant function. The advantage of this approach is the possibility of using it to solve the problems of analysis and synthesis of the mathematical apparatus of linear stationary systems, in particular, the Laplace transformation and obtaining the TF, based on which the accuracy and stability indicators are determined. The disadvantage is the presence of an error in the value of the model parameters, which is the largest at the extreme points of the selected time interval. This leads to the need to introduce reserve factors to obtain the specified values of the indicators guaranteed, which leads to an increase in the requirements for the power of the CS executive device and, as a result, to a decrease in the weight of the rocket's payload.

In this work, on the interval of the trajectory, where the time-varying system is matched by an equivalent stationary one, the variable component of the model parameter is approximated by exponential smoothing, which, thanks to the known properties of the Laplace transform, significantly simplifies the algorithm for obtaining the TF in comparison with approximation by other functions, for example, power series.

The TF includes a component that describes the influence of time instability of the model parameters on the CS indicators, but its coefficients on the selected trajectory interval do not depend on time, that is, it is a mathematical model of an equivalent stationary system.

Compared to the method of frozen coefficients, where the dependence of the parameter on time is a piecewise constant function and the largest error take place at the extreme points of the interval, the error of the parameter is determined only by the accuracy of exponential smoothing.

The TF of an equivalent stationary system makes it possible to determine the dependence of the CS indicators, particularly the margin of stability on the design parameters by using the mathematical apparatus of stationary systems.


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**The object of the study** is the control of the rotational movement of the rocket in the yaw plane.

**The subject of the study** is the stationary approximation of the LTV at a given time interval obtained by using the Laplace transform of non-time-constant components of the CS model.

**The purpose of the work** is to develop a methodical support for the construction of an algorithm for calculating the indicators of the margin of stability of the time-varying system of controlling the rotational motion of the rocket in the plane of yawing using the equivalent stationary approximation on a given time interval.

## 1 PROBLEM STATEMENT

The known in the theory of automatic control approach to determine the margin of stability uses the APFC of an open control system and based on the Nyquist criterion, according to which is performed an analysis of its location relative to the critical point with coordinates –1, j0 on the plane of real and imaginary parts of the APFC.

CS open at point A is a series connection of the regulator and CO (Fig. 1). The rotary motion of the rocket in the plane of yawing is taken as the CO, the input signal of which is perturbing acceleration *m* and the equivalent rotation angle δ of the regulator's executive device.

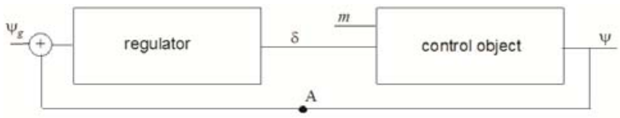


Figure 1 – Control sysems structural schema

The level of complexity of CS mathematical models depends on the research task. The order of the system of differential equations can be between 2 and 16.

For example, the disturbed motion of a typical three-stage space rocket in the channel of yawing, considering the first harmonic of fuel vibrations in four tanks and two-tone elastic oscillations of the rocket body, is described by a system of differential equations of the 16-th order [1].

The study of the stability of the rotational movement of the rocket at the first stage of design can be carried out without considering the fluctuations of the fuel in the tanks, the inertia of the executive device, the disturbed movement of the mass center and the final stiffness of the rocket body, since the frequency spectrum of the mentioned factors overlaps insignificantly. As a result the CO is considered as a rigid body, and the disturbed motion in one of the stabilization planes (for example, in the plane of yawing) is described by a second-order differential equation, the coefficients of which have the constant and time-variable components:

$$\ddot{\psi} = a_{\psi\psi}(t)\cdot\psi + a_{\psi\delta}(t)\cdot\delta + m = [\bar{a}_{\psi\psi} + \tilde{a}_{\psi\psi}(t)]\cdot\psi + [\bar{a}_{\psi\delta} + \tilde{a}_{\psi\delta}(t)]\cdot\delta + m\,. \quad (1)$$

In this work, on the example of the rotational motion of a "solid" rocket in one plane, the possibility of obtaining a stationary approximation of the LTV on a selected trajectory section by applying the Laplace transformation of equation (1) is considered, where the variable components of the model parameters $a_{\psi\psi}(t)$ and $a_{\psi\delta}(t)$ are given by the sum of exponential functions.

As a result, the TF of the open system and, accordingly, the APFC, based on which the indicators of the margin of stability are determined, do not depend on time, as in stationary systems.

This approach complements the methodical base of design work, as it makes it possible to establish trajectory intervals with constant LC coefficients, which has the consequence of reducing the level of complexity of the rocket motion control algorithm.

## 2 REVIEW OF THE LITERATURE

The issue of analysis and synthesis of LTV is an integral part of the theory of automatic control, the development of which is caused by the need to solve several technical problems, particularly, the design of CS for the movement of aircraft. For their research, with the aim of determining the LC that provides the specified indicators, various variants of the mathematical apparatus is used, for example, Lyapunov differential inequalities, matrix polynomials, Lyapunov – Krasovsky functionals, Lyapunov – Bregman functions, Lyapunov's parametric equations, models predictive control, differential equations with constant coefficients around a certain time.

Analysis of stability of LTV compared to stationary systems is much more complicated for several reasons. First, another formulation of the concept of stability, secondly, there is no obvious connection between the stability of the LTV and the eigenvalues of the matrix of the equations system. In addition, the result of the analysis largely depends on the state transition matrices, the possibility of determining which is obvious not always [2].

The construction of LF for LTV is related to the solution of a scalar differential equation, which contains both improper and double integrals [3]. For scalar LTV, a method of LF construction based on the use of the integral of the system parameter with a weight function on a finite interval is proposed. Conditions are imposed on the weight function so that LF is positively defined and uniformly bounded, and its time derivative according to the LTV equations is negatively defined, which is a criterion of stability.

New methods of LF construction for a certain class of LTV are proposed [4], Lyapunov's inverse theorem for asymptotic stability is proved. Its necessary and sufficient conditions are obtained based on the proved Lyapunov's differential inequalities [5].

With the use of Riccati equations and matrix inequalities, an algorithm for assessing the stability of LTV, whose disturbances are described by quadratic constraints, was developed [6].


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Obtaining the specified technical indicators of the LTV by using the Lyapunov stability theory is shown on the examples of spacecraft orientation systems [7, 8]; control of the disturbed movement of the aircraft in the pitch plane [9], the movement of the aircraft with vertical take-off and landing [10, 11], the glider in the presence of prohibited flight zones [12–14], guidance when meeting the conditions at the end of the flight interval [15].

The effectiveness of using Lyapunov's differential inequalities for the construction of the algorithm for the calculation of CL is shown, which provides a compromise between the requirements of speed and accuracy of stabilization, the properties of the transient process are established, and the assumption of a limited range of coefficient changes is removed.

The concept of building a dynamic controller in LTV feedback, when its parameters are known only approximately, has been developed [16]. The sufficient and necessary conditions for the possibility of solving the problem in the form of matrix inequalities are obtained, based on which the parameters of the controller are determined.

In the control system of the rocket rotational movement the model parameters deviation from the time-varying nominal values can amount to ten or more percent, therefore, to increase the efficiency of using the method of frozen coefficients, an algorithm for their refinement by using the data of measuring devices on the current values of part of the state vector coordinates is proposed [17]. Algorithms for specifying LTV parameters for various types of disturbances are also described in works [18–21].

The presence of non-linear links in the aircraft traffic CS complicates the task of obtaining the specified indicators, particularly, the stability margin. Linearization of non-linear links at certain points of the trajectory leads to LTV. The perespective method of predictive control [22, 23] was used for their research. An example of its application can be a predictive controller for solving the problem of meeting spaceships in the context of a limited three-body problem, which can be used to control the docking process with space stations between the Earth and the Moon [24].

The analysis of available sources shows that the problem of quantitative assessment of the stability margin of time-varying control systems, in particular systems for controlling the rocket rotational movement does not have a proper solution.

Based on the equivalent stationary approximation of the LTV on a certain trajectory section, this indicator can be defined as the reduced smallest distance from the selected point in the space of LC coefficients to the boundary of the stability region or on the plane of the CP roots as the distance from its imaginary axis to the nearest root, and also based on the criterion stability by Nyquist through analyzing the APFC of an open system as a amplitude margin and phase margin.

## 3 MATERIALS AND METHODS

As is known, the sequence of determining the CS stability margin includes the following actions: the choice of a mathematical model, for example, in the form of differential equations; their Laplace transformation and obtaining the TF; transition from TF to APFC of an open system under the condition of using the Nyquist stability criterion.

Representation of the variable component of parameters of the CS model (Fig. 1) by the sum of exponential functions has advantages from the point of view of the level of complexity of the transition from differential equations (1) to TF. This follows from the well-known properties of the Laplace transform of the time function $f(t)$ into a function of the complex variable *s*, which is called the image and can be written as:

$$L\{f(t)\} = \int_0^{\infty} f(t)\cdot e^{-s\cdot t}\cdot dt . \quad (2)$$

When the approximation of the variable component of the CS model parameter is carried out by the sum of, for example, six exponential functions, that is

$$\tilde{a}_{\psi\psi}(t) = \sum_{i=1}^{6} C_{\psi i}\cdot e^{r_{\psi i}\cdot t} , \quad (3)$$

then based on (2, 3) the Laplace transform of the separate component of equation (1) according to the image delay theorem will be as follows

$$L\{\psi\cdot\tilde{a}_{\psi\psi}(t)\} = L\{\psi\cdot\sum_{i=1}^{6} C_{\psi i}\cdot e^{r_{\psi i}\cdot t}\} = \sum_{i=1}^{6} C_{\psi i}\cdot\psi(s-r_{\psi i}) . \quad (4)$$

Therefore, relation (4) gives the Laplace transformation of the product of the yaw angle $\psi$ on the variable component of the model parameter, which has the consequence of simplifying the transition from the CS differential equation to the TF.

If in LC − the dependence of the equivalent rotation angle $\delta$ of the regulator's executive device steering wheel on the given and actual value of the yaw angle $\psi_g, \psi$ are taken into account with the coefficients $k_\psi, k'_\psi$ their time derivatives $\dot{\psi}_g, \dot{\psi}$, that is

$$\delta = (\psi_g + \psi)\cdot k_\psi + (\dot{\psi}_g + \dot{\psi})\cdot k'_\psi , \quad (5)$$

then in the CS model (1) there will also be products of the yaw angle $\psi$ and its derivative $\dot{\psi}$ on the variable component of the model parameter $a_{\psi\delta}(t)$ :


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$$\psi \cdot \tilde{a}_{\psi\delta}(t) = \psi \cdot \sum_{i=1}^{6} C_{\delta i} \cdot e^{r_{\delta i} \cdot t},$$

$$\dot{\psi} \cdot \tilde{a}_{\psi\delta}(t) = \dot{\psi} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot e^{r_{\delta i} \cdot t}. \quad (6)$$

The use of ratios (2, 4, 6) and the method of integration by parts establishes a connection between the individual terms of the CS equation (1) and their images:

$$L\{\psi \cdot \tilde{a}_{\psi\delta}(t)\} = \sum_{i=1}^{6} C_{\delta i} \cdot \psi(s - r_{\delta i}),$$

$$L\{\dot{\psi} \cdot \tilde{a}_{\psi\delta}(t)\} = \sum_{i=1}^{6} C_{\delta i} \cdot \psi(s - r_{\delta i}) \cdot (s - r_{\delta i}). \quad (7)$$

According to (1, 3–7), the CS equation can be written in the form:

$$\ddot{\psi} - \psi \cdot (\bar{a}_{\psi\psi} + \sum_{i=1}^{6} C_{\psi i} \cdot e^{r_{\psi i} \cdot t}) -$$
$$-(k_{\psi} \cdot \psi + k_{\psi}^{'} \cdot \dot{\psi}) \cdot (\bar{a}_{\psi\delta} + \sum_{i=1}^{6} C_{\delta i} \cdot e^{r_{\delta i} \cdot t}) =$$
$$= (k_{\psi} \cdot \psi_g + k_{\psi}^{'} \cdot \dot{\psi}_g) \cdot (\bar{a}_{\psi\delta} + \sum_{i=1}^{6} C_{\delta i} \cdot e^{r_{\delta i} \cdot t}) + m. \quad (8)$$

It's known the principle of superposition is valid for linear systems, according to which the result of the action of the input signal $\psi_g(t)$ or $m(t)$ can be determined independently. In order to build an algorithm for calculating indicators of the CS stability margin from two possible TFs

$$w_z(s) = \frac{L\{\psi(t)\}}{L\{\psi_g(t)\}} = \frac{\psi(s)}{\psi_g(s)}, \; w_m(s) = \frac{L\{\psi(t)\}}{L\{m(t)\}} = \frac{\psi(s)}{m(s)}, \quad (9)$$

in this work, the TF $w_z(s)$ is selected, which is determined by the Laplace transformation of equation (8) at zero initial values.

To obtain the TF, the differential equation (8) is transformed into an algebraic one with respect to the images of the actual $\psi(s)$ and specified $\psi_g(s)$ value of the yaw angle:

$$\psi(s) \cdot [s^2 - k_{\psi}^{'} \cdot \bar{a}_{\psi\delta} \cdot s - k_{\psi} \cdot \bar{a}_{\psi\delta} - \bar{a}_{\psi\psi} -$$
$$- \sum_{i=1}^{6} C_{\psi i} \cdot \frac{\psi(s - r_{\psi i})}{\psi(s)} - k_{\psi} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot \frac{\psi(s - r_{\delta i})}{\psi(s)} -$$
$$- k_{\psi}^{'} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot \frac{\psi(s - r_{\delta i}) \cdot (s - r_{\delta i})}{\psi(s)}] =$$
$$= \psi_g(s) \cdot [\bar{a}_{\psi\delta} \cdot (k_{\psi} + k_{\psi}^{'} \cdot s) +$$
$$+ k_{\psi} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot \frac{\psi_g(s - r_{\delta i})}{\psi_g(s)} +$$
$$+ k_{\psi}^{'} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot \frac{\psi_g(s - r_{\delta i}) \cdot (s - r_{\delta i})}{\psi_g(s)}]. \quad (10)$$

Equation (10) makes it possible to obtain the TF $w_z(s)$ in the form of a fractional-rational function of a complex-type argument *s*:

$$w_z(s) = \frac{\psi(s)}{\psi_g(s)} = \frac{P(s)}{Q(s)},$$

where the designations are accepted:

$$P(s) = \bar{a}_{\psi\delta} \cdot (k_{\psi} + k_{\psi}^{'} \cdot s) + k_{\psi} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot \frac{\psi_g(s - r_{\delta i})}{\psi_g(s)} +$$
$$+ k_{\psi}^{'} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot \frac{\psi_g(s - r_{\delta i}) \cdot (s - r_{\delta i})}{\psi_g(s)}, \quad (11)$$

$$Q(s) = s^2 - k_{\psi}^{'} \cdot \bar{a}_{\psi\delta} \cdot s - k_{\psi} \cdot \bar{a}_{\psi\delta} - \bar{a}_{\psi\psi} -$$
$$- \sum_{i=1}^{6} C_{\psi i} \cdot \frac{\psi(s - r_{\psi i})}{\psi(s)} - k_{\psi} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot \frac{\psi(s - r_{\delta i})}{\psi(s)} -$$
$$- k_{\psi}^{'} \cdot \sum_{i=1}^{6} C_{\delta i} \cdot \frac{\psi(s - r_{\delta i}) \cdot (s - r_{\delta i})}{\psi(s)}. \quad (12)$$

Iterations are necessary to obtain the TF $w_z(s)$, since the image of the output signal $\psi(s)$ is included in the last three terms of the equation (10) left part, which are a consequence of the time instability of the model parameters on the trajectory's selected section and considered as a disturbance in this work.

To obtain the first approximation of the image of the output signal $\psi_0(s)$ necessary for the iterations, the image of the signal at the input of the CS $\psi_g(s)$ is required, the choice of which does not affect the indicators of the stability margin. From the point of view of the complexity level of the algorithm, it can be taken as constant – single signal with accuracy up to the factor *d*, that is $\psi_g(t) = d \cdot 1(t)$. Then according to (2) $\psi_g(s) = d / s$.

When the disturbance is not taken into account, then in equation (10) terms with coefficients $C_{\psi i}, C_{\delta i}$ are assumed to be zero and the first approximation of the TF $w_z(s)$ will have the form


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$$w_{z0}(s)=\frac{\psi_0(s)}{\psi_g(s)}=\frac{\bar{a}_{\psi\delta}\cdot(k_\psi+k_\psi^{'}\cdot s)}{s^2-\bar{a}_{\psi\delta}\cdot k_\psi^{'}\cdot s-k_\psi\cdot\bar{a}_{\psi\delta}-\bar{a}_{\psi\psi}}. \quad (13)$$

The first approximation of the output signal's image

$$\psi_0(s)=\psi_g(s)\cdot w_{z0}(s)=d\cdot w_{z0}(s)/s.$$

Numerator (11) of TF $w_z(s)$

$$P(s)=\bar{a}_{\psi\delta}\cdot(k_\psi+k_\psi^{'}\cdot s)+$$
$$+s\cdot(k_\psi\cdot\sum_{i=1}^{6}\frac{C_{\delta i}\cdot}{s-r_{\delta i}}+k_\psi^{'}\cdot\sum_{i=1}^{6}C_{\delta i}) \quad (14)$$

according to equation (10) and relations for the terms of its right-hand side:

$$\frac{\psi_g(s-\alpha)}{\psi_g(s)}=\frac{s}{s-\alpha},\quad \frac{\psi_g(s-\alpha)\cdot(s-\alpha)}{\psi_g(s)}=s.$$

The LC coefficients (5) $k_\psi$ and $k_\psi^{'}$, which are included in (8, 10–14), are determined for the selected trajectory interval based on the given previous values of the margin of stability $\eta_1$ on the CP roots plane and the frequency $f_1$ of the rocket body oscillations in the transient process of disturbance compensation:

$$k_\psi=-(\eta_1^2+4\pi^2\cdot f_1^2+\bar{a}_{\psi\psi})/\bar{a}_{\psi\delta},\quad k_\psi^{'}=-2\eta_1/\bar{a}_{\psi\delta}. \quad (15)$$

Relations (15) are obtained from the fact that the roots of the denominator $Q_0$ of the first TF approximation (13) according to the values $\eta_1$ and $f_1$ are as follows:

$$s_{1,2}=-\eta_1\pm j\cdot 2\pi\cdot f_1.$$

Iterations to determine the denominator $Q(s)$ TF $w_z(s)$ – CP can be carried out according to the scheme:

$$Q_k(s)=Q_0(s)-\frac{1}{\psi_{k-1}(s)}\cdot\left[\sum_{i=1}^{6}C_{\psi i}\cdot\psi_{k-1}(s-r_{\psi i})-\right.$$
$$-k_\psi\cdot\sum_{i=1}^{6}C_{\delta i}\cdot\psi_{k-1}(s-r_{\delta i})-$$
$$\left.-k_\psi^{'}\cdot\sum_{i=1}^{6}C_{\delta i}\cdot\psi_{k-1}(s-r_{\delta i})\cdot(s-r_{\delta i})\right]; \quad (16)$$
$$w_{zk}(s)=\frac{P(s)}{Q_k(s)};\ \psi_k(s)=\psi_g(s)\cdot w_{zk}(s)=$$
$$=w_{zk}(s)\cdot d/s;\ k=\overline{1,n},$$

where the index $k$ is the number of the iteration step, $Q_0(s)$ is the denominator of the TF (13), in which the disturbance is not considered.

At each step of the iteration, an array $N$ of $l$ rows and two columns is created, in which the values of CP $Q_k(s)$ are entered, where the argument $s$ varies in a range sufficient to calculate the passage of the APFC in the vicinity of the critical point according to the Nyquist stability criterion. By processing this array with the use of MLS ($l$ equations with three unknown coefficients of the CP), the current coefficients $q_{2k}, q_{1k}, q_{0k}$ of the CP and, accordingly, the values $\eta_{2k}$, $f_{2k}$ are determined.

The number of iteration steps $n$ depends on the results of checking the achievement of the specified value of the difference of the modules selected to control the convergence of the values at the current and previous step, for example $|f_{2k}-f_{2k-1}|$ or $|\eta_{2k}-\eta_{2k-1}|$.

The result of the performed iterations is the indicator $\eta_2$ of the stability margin on the CP roots plane and TF (9) of the closed system

$$w_z(s)=\frac{P(s)}{q_2\cdot s^2+q_1\cdot s+q_0}, \quad (17)$$

which is necessary to determine the indicators of the stability margin according to the Nyquist criterion. These indicators are based on the TF of the system open at point A (Fig. 1),

$$w(s)=\frac{w_z(s)}{1+w_z(s)}=\frac{P(s)}{q_2\cdot s^2+q_1\cdot s+q_0+P(s)}=\frac{P(s)}{Q_a(s)}, \quad (18)$$

taking into account the location of the polynomial $Q_a(s)$ roots.

The formulation of the Nyquist criterion depends on the number of polynomial $Q_a(s)$ roots in the right half of the complex plane.

For example, when one of the roots is located in the right half of the plane, then the CS is stable, if in the frequency interval from zero to infinity the critical point $K$ in this criterion is semi-encircled by the open system's curve of APFC

$$w(j\omega)=\frac{P(j\omega)}{Q_a(j\omega)}=u(\omega)+j\cdot v(\omega), \quad (19)$$

where the polynomials $P, Q_a$ are represented in formulas (14, 17, 18).

Indicators of stability margin in terms of amplitude $\eta_a$ and phase $\eta_{ph}$ are determined based on the APFC of the open system (19).

The peculiarity of the application of the torque of the CS executive device to the rocket body is that the model


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parameter $a_{\psi\delta}$ (1) is less than zero, therefore, unlike the classical version of the Nyquist stability criterion, the coordinates of the critical point on the plane $u\ jv$ are as follows: $[+1 \quad j\cdot 0]$.

The margin of stability by amplitude $\eta_a$ is the distance from the point $K$ to the point $S$ of intersection of the APFC (19) and the circle of unit radius with the center $O$ of the plane $u\ jv$, and the margin of stability by phase $\eta_{ph}$ is the angle's value between the axis $Ou$ and the vector **OS** (Fig. 2).

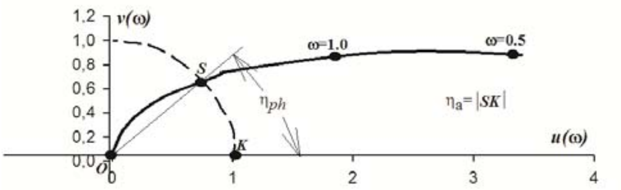


Figure 2 – Margin of stability by amplitude and by phase

The named indicators of the margin of stability are not independent, the relationship between them is determined by the following ratios:

$$\eta_a = \sqrt{2\cdot(1-\cos\eta_{ph})}\ ,\ \eta_{ph} = \arccos(1-\frac{\eta_a^2}{2})\ ,$$

which can be used for additional control of the obtained results.

The choice between $\eta_a$ and $\eta_{ph}$ in the process of studying the system's dynamic characteristics depends on the specific task.

To calculate the described indicators of the stability margin of the CS on the selected trajectory interval, the following data are required:

– constant components of the model parameters (1) $\bar{a}_{\psi\psi}, \bar{a}_{\psi\delta}$;

– coefficients of approximation of variable component of the model parameters by the sum of exponential functions (3, 6)

$$C_{\psi i}, r_{\psi i,}\ C_{\delta i}, r_{\delta i,}\ i=\overline{1,6};$$

– the preset values of the stability margin $\eta_1$ on the plane of the CP $Q(s)$ roots and the frequency $f_1$ of the rocket body oscillations in the transient process of the disturbance compensation.

The results of the calculations should be:

– LC coefficients (5) $k_\psi, k_\psi^{'}$;

– values of the stability margin $\eta_2$ on the CP $Q(s)$ roots plane and the frequency $f_2$ of the rocket body oscillations in the transient process of the disturbances compensation;

– indicators of the stability margin of the CS by amplitude $\eta_a$ and by phase $\eta_{ph}$.

Bringing these indicators closer to the desired values can be ensured by correcting $\eta_1$, $f_1$ or characteristics of the CS executive device.

## 4 EXPERIMENTS

The purpose of the experiments is to verify the described methodological support for the construction of the algorithm for determining the indicators of the CS stability margin on the example of the trajectory section of the rocket first stage, where the deviations of the model parameters from their average values can be 40%.

On this trajectory section, the variable components of the model parameters in the equation (1) are approximated by the sum of exponential functions (3, 6), the coefficients of which and exponents are given in the Table 1.

Table 1 – Approximation coefficients of the model parameters variable components

| $i$ | $r_{\psi i}$ | $C_{\psi i}$ | $r_{\delta i}$ | $C_{\delta i}$ |
|---|---|---|---|---|
| 1 | 0.14149 | 5365 | 0.04742 | –2.836 |
| 2 | 0.14266 | –17210 | 0.06852 | 10.296 |
| 3 | –0.30716 | 11.781 | 0.00621381 | 0.565 |
| 4 | 0.14383 | 24170 | 0.1209 | 0.0661 |
| 5 | –0.31185 | –11.579 | 0.5 | $-4.3\cdot10^{-9}$ |
| 6 | 0.14445 | –12320 | 0.07312 | –8.017 |

According to the sequence of the described actions, the following data are also required to build the algorithm for calculating the indicators of the stability margin of the rocket rotational motion control system in one plane:

– constant components $\bar{a}_{\psi\psi}, \bar{a}_{\psi\delta}$ of the model parameters in the equation (1);

– preset values of the stability margin $\eta_1$ on the plane of the CP roots and the frequency $f_1$ of the rocket body oscillations in the transient process of the disturbance compensation.

For the selected trajectory section, the coefficients CL $k_\psi$, $k_\psi^{'}$ are determined according to (15) with two variants of the previous values of the stability margin $\eta_1$ (Table 2).

Table 2 – Data for calculation of LC coefficients

| $\bar{a}_{\psi\psi}$ | $\bar{a}_{\psi\delta}$ | $\eta_1$ | $f_1$ |
|---|---|---|---|
| $s^{-2}$ | | $s^{-1}$ | Hz |
| 0.849 | –0.331 | 1.2 | 0.3 |
| | | 0.5 | |

The experiments were carried out in the Mathcad environment, in which the following data determination procedures were used:


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– polyroots(q) – the roots of the polynomial whose coefficients are entered in the array q;

– angle(a,b) – the angle between the abscissa axis and the vector with coordinates *a, b*;

– Minimize(f,x) – argument ***x***, at which the function ***f*(*x*)** is minimal;

– Re(f), Im(f) – real and imaginary component of the complex function *f*.

The main procedures that were necessary for the operation of the algorithm are as follows:

– fm(l,a,b,Q) – entry into the array *l* values of the argument *x* in the range *a...b* and the corresponding values of the function *Q(x)*. By processing this array using MLS, it is approximated by a polynomial of a given degree;

– fq(l,mq) – calculation of the coefficients $q_0$, $q_1$, $q_2$ of the polynomial *Q* (16) by processing array mq of *l* rows using the MLS, which is also filled by the fm procedure:

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} l & \sum_{i=1}^{l} s_i & \sum_{i=1}^{l} s_i^2 \\ \sum_{i=1}^{l} s_i & \sum_{i=1}^{l} s_i^2 & \sum_{i=1}^{l} s_i^3 \\ \sum_{i=1}^{l} s_i^2 & \sum_{i=1}^{l} s_i^3 & \sum_{i=1}^{l} s_i^4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \sum_{i=1}^{l} Q(s_i) \\ \sum_{i=1}^{l} s_i \cdot Q(s_i) \\ \sum_{i=1}^{l} s_i^2 \cdot Q(s_i) \end{bmatrix};$$

– ψf(s,$q_0$,$q_1$,$q_2$) – determination at the current step of the iteration of the image of the CS output signal depending on the complex argument *s* and the coefficients of the polynomial *Q* (12, 16) under the action of the disturbance $\psi_g(t) = 1(t)$,

$$\psi f(s, q_0, q_1, q_2) = \frac{P(s)}{(q_2 \cdot s^2 + q_1 \cdot s + q_0) \cdot s}; \qquad (20)$$

– Qd(s,$\psi_{cur}$) – calculation of the value of the component of the polynomial *Q* (16), which is due to the instability of the model parameters, depending on the coefficients $q_{0k}$, $q_{1k}$, $q_{2k}$ of the function $\psi_{cur}$ (20) at the current iteration step;

Q(s,ψcur) – calculation of the CP value (16) at the current iteration step:

$$Q(s, \psi_{cur}) = s^2 - \bar{a}_{\psi\delta} \cdot k_\psi' \cdot s - \bar{a}_{\psi\psi} - \\ -\bar{a}_{\psi\delta} \cdot k_\psi - Q_d(s, \psi_{cur});$$

– *u*(ω), *v*(ω) – the real and imaginary component of the APFC of the open system according to (18):

$$u(\omega) = \frac{\mathrm{Re}(w_z(j\omega)) + |w_z(j\omega)|^2}{1 + 2\,\mathrm{Re}(w_z(j\omega)) + |w_z(j\omega)|^2},$$

$$v(\omega) = \frac{\mathrm{Im}(w_z(j\omega))}{1 + 2\,\mathrm{Re}(w_z(j\omega)) + |w_z(j\omega)|^2};$$

– f1(ω) – function for calculation using the Minimize($f_1$,ω) procedure of the frequency $\omega_1$, at which the APFC crosses a circle of unit radius:

$$f_1(\omega) = \left| u(\omega)^2 + v(\omega)^2 - 1 \right|;$$

– $\eta_a(\omega_1)$, $\eta_{ph}(\omega_1)$ – formulas for calculating the stability margin according to the Nyquist criterion by amplitude and by phase:

$$\eta_a(\omega_1) = \sqrt{\left[ (u(\omega_1) - 1)^2 + v(\omega_1)^2 \right]},$$

$$\eta_{ph}(\omega_1) = \mathrm{angle}(u(\omega_1), v(\omega_1)).$$

Experiments with the use of the above tools made it possible to verify the possibility of using the sequence of actions described in section (3) to determine indicators of the CS stability margin at the selected trajectory section.

## 5 RESULTS

The advantage of representing the variable components of the model parameters as a sum of exponential functions is a simple transition from the CS differential equations (8) to their Laplace transformation, and the disadvantage is that iterations are necessary to obtain the TF. This can be seen from equation (10), in which the image ψ(s) of the CS output signal is included in the terms of the left part of the equation, which are due to the instability of the parameters.

The convergence of the iterative process of determining the instability influence of the model parameters on the stability margin η for the selected data example and two variants of the initial value $\eta_1$ is shown in Tables 3, 4.

In the third and fourth columns of the Tables 3, 4 are shown fragments of the array $N$ in which *l* values of CP $Q(s_i)$ are entered in the range of arguments sufficient to establish the position of the APFC of the open system relative to the critical point in the Nyquist criterion. Processing of this array using MLS gives the coefficients and roots of CP (16) after the current iteration step.

As follows from these Tables, for the selected data example, three iterations are enough so that the indicator $\eta_2$ of the CS stability margin, considering the instability of the model parameters, was calculated with an error of no more than 0.01 $s^{-1}$.

As is known, to calculate the parameters of the stability margin by amplitude $\eta_a$ and by phase $\eta_{ph}$ based on the Nyquist criterion, the APFC $w(j\omega)$ of the open system (19) is needed in the vicinity of the frequency range ω, in which its passage relative to the critical point with coordinates $[+1 \quad j \cdot 0]$ on the plane $u\ jv$ is determined.


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Table 3 – Convergence of iterations at $\eta_1 = 1.2\ s^{-1}$

| $k$ | $i$ | $s_i$ | $Q_k(s_i)$ | $\eta_2$ | $\eta_{2k}-\eta_{2\ k-1}$ |
|---|---|---|---|---|---|
| 1 | 1 | 0.4 | 3.01972 | 0.9064601 | –0.2935395 |
| | 2 | 0.73043 | 5.3342038 | | |
| | 3 | 1.06087 | 6.5131834 | | |
| | . | ….. | ….. | | |
| | $l$ | 8 | 82.30107 | | |
| | | $q_{21}$ | $q_{11}$ | $q_{01}$ | |
| | | 1.00444 | 1.82097 | 3.47350 | |
| 2 | 1 | 0.4 | 4.4387594 | 0.9059249 | $-5.35\cdot10^{-4}$ |
| | 2 | 0.73043 | 5.3424195 | | |
| | 3 | 1.06087 | 6.5218544 | | |
| | . | ….. | ….. | | |
| | $l$ | 8 | 82.3035688 | | |
| | | $q_{22}$ | $q_{12}$ | $q_{02}$ | |
| | | 1.00446 | 1.81992 | 3.48263 | |
| 3 | 1 | 0.4 | 4.4386312 | 0.9059502 | $-2.53\cdot10^{-5}$ |
| | 2 | 0.73043 | 5.342307 | | |
| | 3 | 1.06087 | 6.5217596 | | |
| | . | ….. | ….. | | |
| | $l$ | 8 | 82.3035647 | | |
| | | $q_{23}$ | $q_{13}$ | $q_{03}$ | |
| | | 1.00445 | 1.81997 | 3.48250 | |

Table 4 – Convergence of iterations at $\eta_1 = 0.5\ s^{-1}$

| $k$ | $I$ | $s_i$ | $Q_k(s_i)$ | $\eta_2$ | $\eta_{2k}-\eta_{2\ k-1}$ |
|---|---|---|---|---|---|
| 1 | 1 | 0.4 | 3.01972 | 0.3665767 | –0.1334233 |
| | 2 | 0.73043 | 3.5782648 | | |
| | 3 | 1.06087 | 4.4027546 | | |
| | . | ….. | ….. | | |
| | $l$ | 8 | 72.5998937 | | |
| | | $q_{21}$ | $q_{11}$ | $q_{01}$ | |
| | | 1.00346 | 0.73569 | 2.51093 | |
| 2 | 1 | 0.4 | 2.679288 | 0.2033659 | –0.1632108 |
| | 2 | 0.73043 | 3.1218419 | | |
| | 3 | 1.06087 | 3.8378597 | | |
| | . | ….. | ….. | | |
| | $l$ | 8 | 69.8009891 | | |
| | | $q_{22}$ | $q_{12}$ | $q_{02}$ | |
| | | 1.00446 | 1.81992 | 3.48263 | |
| 3 | 1 | 0.4 | 2.6741968 | 0.2046855 | 0.0013196 |
| | 2 | 0.73043 | 3.120315 | | |
| | 3 | 1.06087 | 3.8388745 | | |
| | . | ….. | ….. | | |
| | $l$ | 8 | 69.8023279 | | |
| | | $q_{23}$ | $q_{13}$ | $q_{03}$ | |
| | | 1.00376 | 0.41091 | 2.29259 | |

Table 5 – Indicators of the CS stability margin

| $\eta_1$ | $\eta_2$ | $\eta_a$ | $\eta_{acr}$ | $\eta_{ph}$ | $\eta_{phcr}$ |
|---|---|---|---|---|---|
| $s^{-1}$ | | – | | degrees | |
| 1.2 | 0.906 | 0.717 | 0.842 | 42.0 | 49.8 |
| 0.5 | 0.205 | 0.174 | 0.414 | 10.0 | 23.9 |

The formulation of the criterion depends on the roots of the CP of the open system, which can be determined, for example, by the Minimize procedure in the Mathcad environment. For the data example in the Tables 1, 2 and $\eta_1$=1.2 $s^{-1}$ they are equal to –0.963 and +1.075, i.e. one of the CP roots is in the right half of the roots plane. Therefore, according to the Nyquist criterion, the CS stability takes place under the condition that the curve of the APCH (19) is in the range frequency ω from zero to infinity (Fig. 2) makes a semicircle above the critical point $\boldsymbol{K}$ with coordinates $[-1\ \ j\cdot0]$.

The calculations performed according to the described algorithm show that the instability of the model parameters for the data example (Tables 1, 2) at $\eta_1$=1.2 $s^{-1}$ leads to a decrease of the stability margin in comparison with the results of the method of frozen coefficients for the selected trajectory section on the CP roots plane (16) by approximately 25% and, according to the Nyquist criterion, the stability margin in term of amplitude by 15%, in term of phase by 16%.

Since the relationships between the named indicators are not described by linear functions, the ratios between them in quantitative terms do not coincide.

When $\eta_1$=0.5 $s^{-1}$ the indicators of the stability margin on the CP roots plane decrease by approximately 59%, and according to the Nyquist criterion, the stability margin in terms of amplitude and phase decreases by 58% (Table 5).

The experiment results show the possibility of building an algorithm for calculating the stability margin indicators of a time-varying CS on a selected trajectory section obtaining an equivalent stationary CS using the Laplace transformation of the model parameters time-varying components given by the sum of exponential functions.

## 6 DISCUSSION

To use the described actions in the design work for the construction of the algorithm for calculating the stability margin indicators, it is necessary given by tables or graphs of the dependence of the model parameters on time. The flight path is divided into sections, on each of which are determined coefficients of approximation of the model parameters variable components by the sum of exponential functions.

Based on the CO characteristics and the CS executive device, the desired values of the CP roots are assigned.

The Laplace transformation of the variable components of the model parameters makes it possible to move from a differential equation with time-varying coefficients to a TF, which matches LTV with an equivalent stationary system on a selected trajectory section.

The proposed approach to determining the stability margin indicators of time-varying CS has the advantage that their error is the same for all points of the selected trajectory section, while when using the method of frozen coefficients, the error depends on the distance to the middle point of the trajectory section. This can give a possibility of increasing the size of the trajectory sections and, accordingly, reducing their number.

## CONCLUSIONS

**The scientific novelty** of the work consists in the development of a methodology for determining the indicators of the margin of stability of a time-varying rotary motion control system of a rocket by means of Laplace transformation of the variable component of the mathematical model parameters given by the sum of exponential functions.


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**The practical significance** of the obtained results is the expansion of the methodological basis for the design rocket motion control systems.

**Prospects for further research** is to assess the complexity level of the algorithm taking into account the inertia of the executive device and the disturbed movement of the mass center.

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# ЗАПАС СТІЙКОСТІ НЕСТАЦІОНАРНОЇ СИСТЕМИ УПРАВЛІННЯ ОБЕРТАЛЬНИМ РУХОМ РАКЕТИ

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## АНОТАЦІЯ

**Актуальність.** Система управління рухом ракети є нестаціонарною, оскільки в процесі польоту її параметри залежать від точки траєкторії і витрат палива. Показники запасу стійкості визначають в обмеженому околі окремих точок траєкторії з використанням алгоритмів, які розроблені тільки для лінійних стаціонарних систем, що призводить до необхідності введення коефіцієнтів запасу в апаратних засобах. В доступних джерелах розробці методів визначення кількісної оцінки запасу стійкості нестаціонарної системи управління належної уваги не приділяється.

**Мета роботи –** розробка методичного забезпечення побудови алгоритму розрахунку показників запасу стійкості нестаціонарної системи управління обертальним рухом ракети у площині рискання з використанням на вибраних дільницях траєкторії еквівалентного стаціонарного наближення.

**Метод.** Математична модель системи управління обертальним рухом ракети в одній площині прийнята у вигляді лінійного диференційного рівняння без врахування інерції виконавчого пристрою та інших збурювальних факторів. Ефект відхилення параметрів від їх середніх значень для певної дільниці траєкторії розглядається як збурення, що дає можливість переходу від нестаціонарної моделі до еквівалентної наближеної стаціонарної. Для оцінки показників запасу стійкості використаний критерій Найквіста, що спирається на аналіз частотної характеристики розімкненої системи, для визначення якої використовується математичний апарат перетворення Лапласа. З метою спрощення переходу від функцій часу у диференційному рівнянні збуреного руху до функцій комплексного змінного у перетворенні Лапласа змінні у часі параметри моделі подані у вигляді суми експоненціальних функцій.

**Результат.** Розроблене методичне забезпечення для побудови алгоритму визначення запасу стійкості системи управління обертальним рухом ракети на заданій дільниці траєкторії з непостійними у часі параметрами.

**Висновки.** На прикладі нестаціонарної системи управління обертальним рухом ракети показана можливість використання перетворення Лапласа для визначення показників запасу стійкості.

Отримані результати можуть бути використані на початковому етапі проектних робіт.

Наступний етап дослідження це оцінка рівня складності алгоритму при врахуванні інерції виконавчого пристрою та збуреного руху центру мас.

**КЛЮЧОВІ СЛОВА**: управління рухом ракети, лінійна нестаціонарна система, перетворення Лапласа.

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