UDC 62.505:629.524

STEWART PLATFORM MULTIDIMENSIONAL TRACKING CONTROL SYSTEM SYNTHESIS

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ABSTRACT

Context. Creating guaranteed competitive motion control systems for complex multidimensional moving objects, including unstable ones, that operate under random controlled and uncontrolled disturbing factors, with minimal design costs, is one of the main requirements for achieving success in this class devices market. Additionally, to meet modern demands for the accuracy of motion control processes along a specified or programmed trajectory, it is essential to synthesize an optimal control system based on experimental data obtained under conditions closely approximating the real operating mode of the test object.

Objective. The research presented in this article aims to synthesize an optimal tracking control system for the Stewart platform's working surface motion, taking into account its multidimensional dynamic model.

Method. The article employs a method of a multidimensional tracking control system structural transformation into an equivalent stabilization system for the motion of a multidimensional control object. It also utilizes an algorithm for synthesizing optimal stabilization systems for dynamic objects, whether stable or not, under stationary random external disturbances. The justified algorithm for synthesizing optimal stochastic stabilization systems is constructed using operations such as addition and multiplication of polynomial and fractional-rational matrices, Wiener factorization, Wiener separation of fractional-rational matrices, and the calculation of dispersion integrals.

Results. As a result of the conducted research, the problem of defining the concept of analytical design for a Stewart platform's optimal motion control system has been formalized. The results include the derived transformation equations from the tracking control system to the equivalent stabilization system of the Stewart platform's working surface motion. Furthermore, the structure and parameters of the main controller transfer function matrix for of this control system have been determined.

Conclusions. The justified use of the analytical design concept for the Stewart platform's working surface optimal motion control system formalizes and significantly simplifies the solution to the problem of synthesizing complex dynamic systems, applying the developed technology presented in [1]. The obtained structure and parameters of the Stewart platform's working surface motion control system main controller, which is divided into three components W_1, W_2, and W_3, improve the tracking quality of the program signal vector, account for the cross-connections within the Stewart platform, and increase the accuracy of executing the specified trajectory by increasing the degrees of freedom in choosing the controller structure.

KEYWORDS: synthesis, transfer function matrix, tracking control system, quality functional, Stewart platform.

ABBREVIATIONS

MFD is a method of matrix fraction description;

WS is a working surface.

NOMENCLATURE

C is a non-negative definite polynomial weight matrix of size $m{\times}m$, which bounds the variance of the control signal u;

E_{2n} is the $2n{\times}n$ unit matrix;

G_0+G_+ is a stable fractional-rational matrix, which is the stable part of the result of the separation of the matrix G;

G_n is a gain coefficient of the disturbance spectral density matrix in the controlled object $S^{/}_{\psi_{ob}\psi_{ob}}$;

K_g is a gain coefficient of the feedback matrix characterizing the dynamics of the object P_0^{-1};

M_1, M_0 is an (extended) polynomial matrix of dimensions 2n×m and n×m, respectively, that determines the sensitivity of the object to changes in control signals;

m is the number of signals at the output of the control system;

n is the local system inputs number;

$O_{m\times n}$ is a zero matrix of size $m{\times}n$;

P_1, P_0 is an (extended) polynomial matrix of dimensions $2n{\times}2n$ and $n{\times}n$, respectively, that characterizes the dynamics of the control object;

R is a positively definite polynomial weight matrix of size $n{\times}n$, which determines the influence of the stabilization error variance on the criterion e;

r_0 is a vector of program signals;

$S^{/}_{r_0 r_0}$ is a transposed spectral density matrix of the vector r_0;

$S^{/}_{uu}$ is a transposed spectral density matrix of control signal deviations;

$S^{/}_{x_{\varepsilon1} x_{\varepsilon1}}$ is a transposed spectral density matrix of the vector $x_{\varepsilon 1}$ at the output of the extended control object;

$S^{/}_{\psi_{ob}\psi_{ob}}$ is a transposedmatrix of spectral densities of the disturbing influence;

$S^{/}_{\xi_0\xi_0}$ is a transposed spectral density matrix of the extended disturbance vector ξ_0;

DOI 10.15588/1607-3274-2024-3-20

$S^{/}_{\varphi_0\psi}$ is a transposed mutual spectral density matrix between vectors φ_0 and ψ;

T_0+T_+ is a stable fractional-rational matrix, which is the stable part of the result of the separation of the matrix T;

u is an m-dimensional vector of control signals;

W_0, W_1, W_2, W_3 are transfer function matrices of the main controller and its components;

x_1 is a vector of signals at the output of the control system;

x_ε is extended vector of reactions;

z_1, - z_{11} are auxiliary transfer functions;

z_{22}, z_{21} are fractional rational matrices;

Φ is a block matrix of transfer functions of size $n\times(n+m)$;

α is a measurement's noise variance coefficient, values: α=0.0018 rad^2;

ε_x is a tracking error;

φ_1, φ_r are vectors of measurement noise;

ψ_{ob} is a vector of centred stationary random disturbances in the control object.

INTRODUCTION

Research results on the methods of designing control systems for mechanisms with a parallel structure based on the Stewart platform [2], taking into account the principles of automatic control theory, have determined that regardless of the application area, all the Stewart platform working surface (WS) motion control systems are multidimensional closed-loop control systems operating under the influence of random disturbances.

In the article [3], the Stewart platform dynamics model is identified and its transfer function, as well as the transfer function of the shaping filter, is determined. It has been determined that the considered mechanism is a multidimensional stable mechanical filter for both control signals and disturbances in the working area of the mechanism. Analysis of the Stewart platform dynamics' model identification results shows that the primary influence on the motion of the moving platform center of mass is the change in control inputs. However, neglecting the impact of disturbances reduces the positioning accuracy of the platform. Therefore, for the synthesis of the control system, methods should be applied that allow for determining the structure and parameters of the multidimensional controller, taking such influences into account.

Given the modern requirements for the accuracy of motion control processes of a moving object along a specified or programmed trajectory, it is necessary to synthesize the optimal structure and parameters of the object's control system, taking into account both real controllable and uncontrollable stochastic disturbing factors [4]. Also, in the process of synthesizing the optimal controller structure, it is necessary to evaluate and consider multidimensional dynamic models of the object itself, its basic parts, as well as the controllable and uncontrollable disturbing factors that affect the object in its real motion.

This work object of study is a Stewart platform's working surface motion multidimensional tracking control system. The Stewart platform is a spatial mechanism with a parallel kinematic structure, consisting of six identical kinematic chains (actuators) [5]. Such mechanisms include processing centers (machines), coordinate measuring centers, vibration platforms (testing rigs), motion simulators, and stabilization platforms. The Stewart platform has six degrees of freedom for the motion of its moving platform. By programmatically adjusting the lengths of the Stewart platform actuators, it is possible to control the position of the moving base, move it in vertical and horizontal directions, and rotate it in three planes.

The subject of study is the algorithm for converting the tracking system into an equivalent stabilization system, as well as the algorithm for synthesizing the Stewart platform's working surface motion control system.

The purpose of the work is to obtain the structure and parameters of an optimal controller for the Stewart platform's working surface motion control system, using a justified multidimensional objects optimal stochastic sta-bilization systems synthesizing algorithm.

1 PROBLEM STATEMENT

As a result of the conducted research and the structural schemes analysis of Stewart platform WS motion control system when used for various types of technological tasks such as positioning, stabilization, motion simulators of moving objects, etc. [5], and taking into account the principles of automatic control theory, it has been established that regardless of the application area, all motion control systems of the Stewart platform WS can be classified as multidimensional dual-loop tracking systems (Fig. 1) [2].

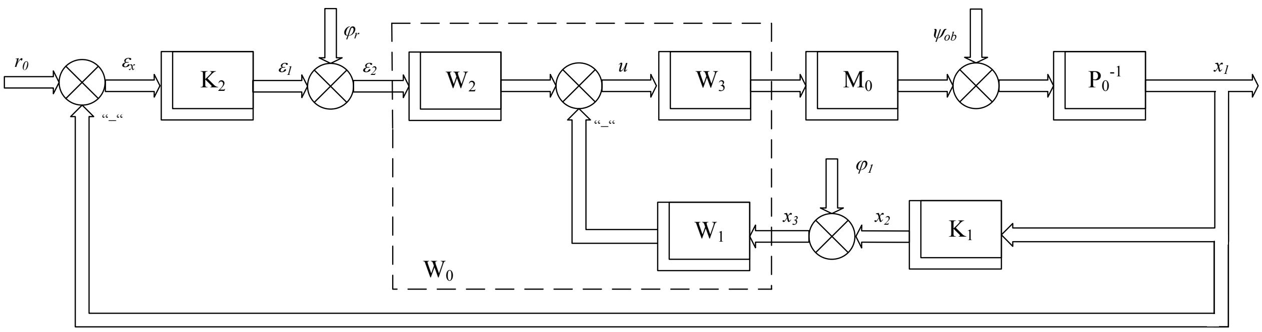

Figure 1 – Structural diagram of a multidimensional dual-loop tracking control system

DOI 10.15588/1607-3274-2024-3-20

We will also consider that the vector of output coordinates x_1 is fully measured using a system of imperfect sensors, whose dynamics are determined by the transfer matrix K_1. At the output of the sensors, there is an n-dimensional vector of centered stationary random noises φ_1, with a fractional-rational matrix spectral density $S_{\varphi_1\varphi_1}$. The system input is an n-dimensional vector of the motion program signal r_0. The program signal setter is described by the transfer function matrix K_2 of size $n\times n$. The stationary random noise of the program signal setter is characterized by the n-dimensional vector φ_r.

The study of research results, which are given in the sources [1, 6–8], made it possible to set the task of defining the Stewart platform WS optimal motion control system analytical design concept.The mentioned concept involves transforming the structural diagram in Fig. 1 into an equivalent structural diagram of a multidimensional stabilization system [6], taking into account the rules of structural diagrams and linear systems transformation [7]. This consolidation formalizes and significantly simplifies the solution of synthesizing complex dynamic systems, such as the Stewart platform's WS motion control system, using the developed technology presented in [1]. This technology utilizes an algorithm for synthesizing optimal systems for stochastic stabilization of motion in multidimensional controlled objects, which is robust even in the presence of stationary random external disturbances. It ensures enhanced reliability in computation results, combining the simplicity of computational algorithms with the capabilities and physical transparency of algorithms described in the monograph [6].

2 REVIEW OF THE LITERATURE

The technologies for synthesizing optimal linear time-invariant multidimensional control systems in the frequency domain [6–10] review has shown that the fundamental creating such systems method can be considered as the synthesizing optimal multidimensional stabilization systems presented in [8] method. It is based on the the Frobenius formula for polynomial matrix inversion use and involves complex computations in forming special-purpose polynomial matrices. All of this limits the effectiveness of applying the algorithms from [8] to solve the synthesis task, especially as the order and dimensions of the controlled object increase. At the same time, this monograph has proven that the structure and parameters of these service matrices do not affect the choice of the optimal regulator and the effectiveness of its use in the system; they only determine the course and complexity of the computational synthesis processes.

In the monograph [9], a new procedure for determining the aforementioned service matrices is justified based on the factorization of a properly constructed block polynomial matrix. It has allowed the author to significantly simplify the basic synthesis algorithm. At the same time, the relationships obtained in [9] allow for the synthesis of an optimal multidimensional stabilization system designed to operate under random disturbances in the form of a white noise vector and with ideal measurement of the output coordinates of the object, such as the Stewart platform.

In the monograph [6], a new method for synthesizing optimal multidimensional stabilization systems for dynamic objects, including unstable ones, is justified. This method is designed to operate under stationary random external disturbances with "non-ideal" measurements of the object's output coordinates. The algorithms based on this method involve selecting special-purpose polynomial matrices from physical considerations, which significantly simplifies their formation process. At the same time, repeated application of this method for creating stabilization systems has shown that as the dimensionality of the controlled object increases, problems of catastrophic loss of computational accuracy arise when performing computer calculations with limited bit-length precision.

3 MATERIALS AND METHODS

The synthesizing an optimal tracking control system for the Stewart platform's WS motion, as a multidimensional controlled object, task is formulated as follows. Suppose we have an n-dimensional linear controlled object (Fig. 1), whose motion is described by a system of ordinary differential equations, represented under zero initial conditions in the Laplace-transformed form:

$$P_0 x_1 = M_0 u + \psi_{ob}\,. \tag{1}$$

Supplement the object equation (1) with the error equation:

$$\varepsilon_x = r_0 - x_1\,,$$

so we can write the following system of equations:

$$\begin{cases} P_0 x_1 = M_0 u + \psi_{ob} \\ \varepsilon_x = r_0 - x_1 \end{cases},$$

or for better understanding, let's rewrite this system of equations as follows:

$$\begin{cases} P_0 x_1 + O_n \varepsilon_x = M_0 u + \psi_{ob} \\ E_n x_1 + E_n \varepsilon_x = O_n + r_0 \end{cases}. \tag{2}$$

Write the system of equations (2) in vector-matrix form:

$$\begin{bmatrix} P_0 & O_n \\ E_n & E_n \end{bmatrix}\begin{bmatrix} x_1 \\ \varepsilon_x \end{bmatrix} = \begin{bmatrix} M_0 \\ O_{n\times m} \end{bmatrix} u + \begin{bmatrix} \psi_{ob} \\ r_0 \end{bmatrix},$$

introduce new notations:

$$P_1 = \begin{bmatrix} P_0 & O_n \\ E_n & E_n \end{bmatrix},\ M_1 = \begin{bmatrix} M_0 \\ O_{n\times m} \end{bmatrix},\ \psi_r = \begin{bmatrix} \psi_{ob} \\ r_0 \end{bmatrix}, \tag{3}$$

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$$x_\varepsilon = \begin{bmatrix} x_1 \\ \varepsilon_x \end{bmatrix},$$

Given the notation (3), equation (1) can be written as follows:

$$P_1 x_\varepsilon = M_1 u + \psi_r , \tag{4}$$

As seen in Fig. 1, the input to the sensors K_1 and K_2 are the output coordinate vector of the controlled object x_1 and the tracking system error ε, respectively, while the output of the sensors K_1 and K_2 yield the vectors x_2 and ε_1. Then, the following equation can be written:

$$\begin{bmatrix} x_2 \\ \varepsilon_1 \end{bmatrix} = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} x_1 \\ \varepsilon_x \end{bmatrix}, \tag{5}$$

introduce the notation:

$$K_0 = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix},\ x_{\varepsilon_1} = \begin{bmatrix} x_2 \\ \varepsilon_1 \end{bmatrix}. \tag{6}$$

The sensors K_1 and K_2 have noises φ_1 and φ_r, which are multidimensional stationary-centered random processes with known spectral density matrices and cross-spectral densities. As seen in Fig. 1, the vectors x_3 and ε_2 act at the input of the regulator W_0 of the tracking system, so the following equation can be written:

$$\begin{bmatrix} x_3 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \varepsilon_1 \end{bmatrix} + \begin{bmatrix} \varphi_1 \\ \varphi_r \end{bmatrix}, \tag{7}$$

introduce the notation:

$$x_{\varepsilon_2} = \begin{bmatrix} x_3 \\ \varepsilon_2 \end{bmatrix},\ \varphi_0 = \begin{bmatrix} \varphi_1 \\ \varphi_r \end{bmatrix}. \tag{8}$$

Taking into account equation (5) and notation (6–8), we obtain:

$$x_{\varepsilon_2} = K_0 x_\varepsilon + \varphi_0 .$$

According to the block diagram in Fig. 1, the equation of the control signal u can be defined as follows:

$$u = W_3 \left(-W_1 x_3 + W_2 \varepsilon_2 \right),$$

and in matrix form

$$u = W_3 \begin{bmatrix} -W_1 & W_2 \end{bmatrix} \begin{bmatrix} x_3 \\ \varepsilon_2 \end{bmatrix},$$

where W_0 is the transfer function of the controller of the double-loop tracking system:

$$W_0 = W_3 \begin{bmatrix} -W_1 & W_2 \end{bmatrix}. \tag{9}$$

Then

$$u = W_0 \left(K_0 x_\varepsilon + \varphi_0 \right). \tag{10}$$

Therefore, the two-loop tracking system (Fig. 1) is structurally equivalent to the stabilization system depicted in Fig. 2, described by the equations of the object (4) and the controller (10).

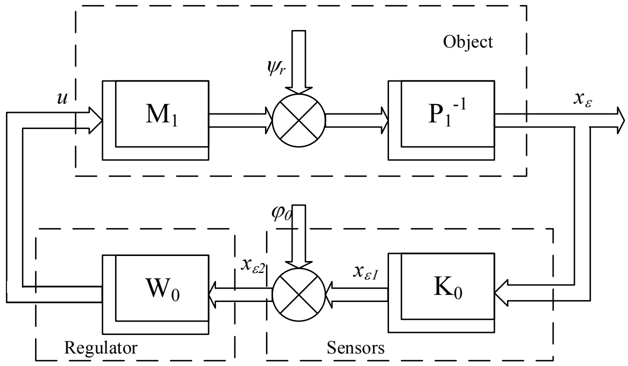

Figure 2 – Structural diagram of a multi-dimensional stabilization system

At the first stage of transformations, the structural diagram (Fig. 2) is reduced to the output x_{ε_1} of the sensors K_0 (Fig. 3), and the resulting system of differential equations is equivalent to the relationships (1):

$$P x_{\varepsilon_1} = M u + \psi , \tag{11}$$

in which the following notations are adopted

$$P = K_{10} P_1 K_0^{-1},\ M = K_{10} M_1,\ x_{\varepsilon_1} = K_0 x_\varepsilon, \quad \psi = K_{10} \psi_r . \tag{12}$$

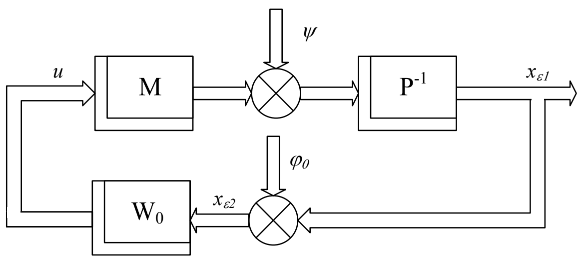

Figure 3 – Result of the structural transformations first stage

To determine the polynomial matrix K_{10} with the minimum possible order of elements, it is proposed to use a combination of algorithms for left-sided pole removal [12] and MFD decomposition [10] of the fractional-rational matrix K_0^{-1} and the product of matrices $P_1 K_0^{-1}$.

DOI 10.15588/1607-3274-2024-3-20

In this process, the matrix K_{10} should be found as a result of the MFD decomposition [10] of the following product:

$$K_{10}^{-1}P = P_1 K_{20}^{-1},$$

where K_{20} is the result of the left-side removal of the poles of the sensor transfer function matrix [12]

$$K_{20}^{-1}K_2 = K_0,$$

and between the determinants of polynomial matrices K_{10} and K_{20} there is an identity

$$|K_{10}| = |K_{20}|,$$

which is a consequence of the MFD decomposition of fractional-rational matrices.

In the second stage, the structural diagram (Fig. 3) is transformed into a standard form (Fig. 4), where the input of the stabilization system is affected by an extended disturbance vector ξ

$$\xi = (E_n, P)\xi_0, \qquad (13)$$

where the vector ξ_0 is the result of the vertical concatenation of the vectors ψ and φ_0

$$\xi_0 = \begin{bmatrix} \psi \\ \varphi_0 \end{bmatrix}, \qquad (14)$$

a vector x_{ε_2} acts as the output of the system.

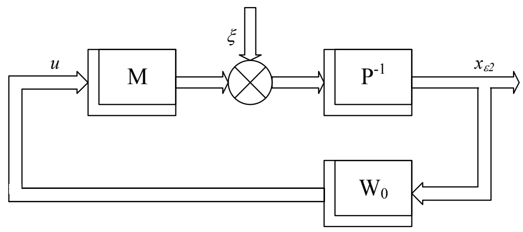

Figure 4 – Structural diagram of a typical stabilization system

By analogy with [6], the relationship between the vectors ξ_0 and x_ε is defined as follows:

$$x_{\varepsilon_2} = K_0^{-1}\left[F_{x_{\varepsilon_2}}^{\xi}(E_{2n}, P) - (O_{2n}, E_{2n})\right]\xi_0, \qquad (15)$$

where $F_{x_{\varepsilon_2}}^{\xi}$ is the matrix transfer function of the closed "object + regulator" system from the extended disturbance vector ξ to the output signal vector x_{ε_2}. The control signal vector u in the closed system also depends on the extended disturbance vector ξ

$$u = F_u^{\xi}(E_{2n}, P)\xi_0, \qquad (16)$$

where F_u^{ξ} is the matrix transfer function of the closed "object + regulator" system from the extended disturbance vector ξ to the control vector u.

In works [6, 8], based on equation (11), it has been proven that there is a relationship between matrices F_u^{ξ} and $F_{x_{\varepsilon_2}}^{\xi}$, which is characterized by the following relation:

$$PF_{x_{\varepsilon_2}}^{\xi} - MF_u^{\xi} = E_{2n}. \qquad (17)$$

Additionally, it has been demonstrated that the structure and parameters of these matrices depend on the matrix of transfer functions of the regulator W_0:

$$F_u^{\xi} = W_0(P - MW_0)^{-1}, \qquad (18)$$

$$F_{x_{\varepsilon_2}}^{\xi} = (P - MW_0)^{-1}. \qquad (19)$$

Thus, the structural transformations (Fig. 2–4) and equations (15), and (16) reduce the task of synthesizing an optimal stabilization system to the following: it is necessary to determine the structure and parameters of the regulator W_0 transfer function matrix by known polynomial and fractional-rational matrices M_1, P_1, K_0, $S_{\psi_{ob}\psi_{ob}}$, and $S_{\varphi_1\varphi_1}$. The inclusion of the regulator W_0 transfer function in the feedback circle to the control object ensures the stability of the stabilization system (Fig. 2) and delivers a minimum to the following quality criterion:

$$e = \left\langle x_{\varepsilon_1}^{/} R x_{\varepsilon_1} \right\rangle + \left\langle u^{/} C u \right\rangle. \qquad (20)$$

By substituting definitions (5) and (6) into the quality criterion of the stabilization system (20), we determine the functional of the quality criterion for the two-loop tracking system as follows:

$$e = \left\langle x_{\varepsilon}^{/} \left(K_0^{-1}\right)^{/} \begin{bmatrix} O_n \\ E_n \end{bmatrix} R [O_n, E_n] K_0^{-1} x_\varepsilon \right\rangle + \left\langle u^{/} C u \right\rangle, \qquad (21)$$

introduce the notation

$$R_1 = \left(K_0^{-1}\right)^{/} \begin{bmatrix} O_n \\ E_n \end{bmatrix} R [O_n, E_n] \left(K_0^{-1}\right).$$

Unlike in the stabilization system where R is a coefficient, in the tracking system, R_1 equals a matrix 2×2:

DOI 10.15588/1607-3274-2024-3-20

$$R_1 = \begin{bmatrix} O_n & O_n \\ O_n & K_{2*}^{-1} R K_2 \end{bmatrix}. \quad (22)$$

The task of synthesizing the regulator in the tracking system is to ensure, the first, the stability of the closed-loop tracking system and, the second, minimum of the system functional (21) by selecting the optimal structure of the regulator W_0.

To solve this problem, rewrite the functional (20) in the frequency domain:

$$e = \frac{1}{j} \int_{-j\infty}^{j\infty} tr\left(S'_{x_{\varepsilon 1} x_{\varepsilon 1}} R_1 + S'_{uu} C \right) ds. \quad (23)$$

Define the matrix of varied transfer functions Φ as follows

$$F_u^{\xi} = z_{22}\Phi + z_{21}, \quad (24)$$

where

$$z_{22} = \left(B + AP^{-1}M\right)^{-1}, \; z_{21} = -z_{22}AP^{-1}, \quad (25)$$

A and B are polynomial matrices found as a result of representing the auxiliary block matrix H:

$$H = \begin{bmatrix} O_{2n} & P & -M \\ P_* & -R & O_{2n\times m} \\ -M_* & O_{m\times 2n} & -C \end{bmatrix}, \quad (26)$$

in the form of the product of two factors (block polynomial matrices) V and Σ:

$$H = V_* \Sigma V, \quad (27)$$

where

$$V = \begin{bmatrix} E_{2n} & -S & N \\ O_n & P & -M \\ O_{m\times 2n} & A & B \end{bmatrix}, \quad \Sigma = \begin{bmatrix} O_{2n} & E_{2n} & O_{2n\times m} \\ E_n & O_{2n} & O_{2n\times m} \\ O_{m\times 2n} & O_{m\times 2n} & -E_m \end{bmatrix}, \quad (28)$$

provided that the determinant |V| is a Hurwitz polynomial.

The algorithm for factorizing matrix (27) was firstly proposed and detailed in [9]. Substituting expressions (26) and (28) into equation (27) establishes the existence of the following relationships:

$$\begin{gathered} P_* S + S_* P + A_* A = R, \\ M_* S + N_* P - B_* A = O_{m\times 2n}, \\ P_* N + S_* M - A_* B = O_{2n\times m}, \\ M_* N + N_* M + B_* B = C. \end{gathered} \quad (29)$$

In this case, the performance criterion (23) with equations (13–17) and (24) transforms into the functional:

$$\begin{aligned} e = \frac{1}{j} \int_{-j\infty}^{j\infty} tr\Bigg\{ \Bigg[\left[\begin{pmatrix} E_n \\ P_* \end{pmatrix} z_{21*} M_* P_*^{-1} RG + \begin{pmatrix} E_n \\ P_* \end{pmatrix} P_*^{-1} \times \right. \\ \times RG \Big] S'_{\xi_0\xi_0} + \Phi_* z_{22*} M_* P_*^{-1} RG S'_{\xi_0\xi_0} \begin{pmatrix} E_n \\ P_* \end{pmatrix} - \\ - \begin{pmatrix} O_n \\ E_n \end{pmatrix} RG S'_{\xi_0\xi_0} + \left[z_{21*} C z_{21} + z_{21*} C z_{22} \Phi + \Phi_* \times \right. \\ \times z_{22*} C z_{21} + \Phi_* z_{22*} C z_{22} \Phi \Big] (E_n, P) S'_{\xi_0\xi_0} \begin{pmatrix} E_n \\ P_* \end{pmatrix} \Bigg\} ds, \end{aligned} \quad (30)$$

where "*" is the sign of the Hermitian matrix conjugation [13], G is a fractional-rational matrix equal to

$$G = P^{-1} M z_{21} (E_n, P) + P^{-1} M z_{22} \Phi (E_n, P) + \\ + P^{-1}(E_n, P) - (O_n, E_n).$$

The matrix $S'_{\xi_0\xi_0}$ is defined as the result of applying the Wiener-Khinchin theorem to the vector (14) in the form:

$$S'_{\xi_0\xi_0} = \begin{bmatrix} S'_{\psi\psi} & S'_{\varphi_0\psi} \\ S'_{\psi\varphi_0} & S'_{\varphi_0\varphi_0} \end{bmatrix}, \quad (31)$$

where matrix $S'_{\psi\psi}$ is the transposed matrix of spectral densities of the equivalent disturbances vector, which, considering expression (12), is equal to:

$$S'_{\psi\psi} = K_{10} \cdot S'_{\psi_r \psi_r} \cdot K_{10*}, \quad (32)$$

$$S'_{\varphi_0\psi} = K_{10} \cdot S'_{\varphi_1 \psi_r}. \quad (33)$$

The search for the algorithm to determine the structure and parameters of the transfer function matrix W, as in [6, 8, 9], can be accomplished by minimizing the functional (30) on the class of robust and physically realizable variable matrices Φ using the Wiener-Kolmogorov procedure. According to this procedure, the first variation of the functional (30) has been found:

$$\delta e = \frac{1}{j} \int_{-j\infty}^{j\infty} tr\left[\delta\Phi_* \frac{\partial}{\partial \Phi_*} tr(*) + \frac{\partial}{\partial \Phi} tr(*)_* \delta\Phi \right] ds, \quad (34)$$

DOI 10.15588/1607-3274-2024-3-20

$$\frac{\partial}{\partial \Phi_*} tr(*) = z_{22*}\left(M_* P_*^{-1} R P^{-1} M + C\right) z_{22} \Phi \times$$
$$\times (E_n, P) S'_{\xi_0\xi_0} \begin{pmatrix} E_n \\ P_* \end{pmatrix} + z_{22*}\left(M_* P_*^{-1} R P^{-1} M z_{21} + \right.$$
$$\left. + M_* P_*^{-1} R P^{-1} + C z_{21}\right)(E_n, P) S'_{\xi_0\xi_0} \times \quad (35)$$
$$\times \begin{pmatrix} E_n \\ P_* \end{pmatrix} - z_{22*} M_* P_*^{-1} R (O_n, E_n) S'_{\xi_0\xi_0} \begin{pmatrix} E_n \\ P_* \end{pmatrix}.$$

Define the matrix D as the result of the left-sided factorization [11] of generalized disturbances spectral density's transpose matrix.

$$DD_* = (E_n, P) S'_{\xi_0\xi_0} \begin{pmatrix} E_n \\ P_* \end{pmatrix}. \quad (36)$$

Assume that the fractional-rational matrix T equals:

$$T = z_{22*}\left(M_* P_*^{-1} R P^{-1} M z_{21} + M_* P_*^{-1} R P^{-1} + C z_{21}\right) D, \quad (37)$$

and the matrix G:

$$G = -z_{22*} M_* P_*^{-1} R \left(S'_{\psi\varphi_0} + S'_{\varphi_0\varphi_0} P_*\right) D_*^{-1}. \quad (38)$$

Since the relationships (29) hold, the expression (37) is reduced to the form:

$$T = z_{22*}\left(M_* P_*^{-1} S_* - N_*\right) D,$$

and the partial derivative (35) is simplified and represented as

$$\frac{\partial}{\partial \Phi_*} tr(*) = \Phi D D_* + T D_* - G D_*.$$

Thus, the first variation of the quality functional (34) becomes equal to:

$$\delta e = \frac{1}{j} \int_{-j\infty}^{j\infty} tr\left[\delta\Phi_* \left(\Phi D D_* + T D_* - G D_*\right) + \right.$$
$$\left. + \left(D D_* \Phi_* + D T_* - D G_*\right) \delta\Phi\right] ds. \quad (39)$$

As seen from the monograph [6], the matrix of variable functions Φ, which meets the conditions of stability and physical realizability and minimizes the functional (30), considering expression (39), should be determined based on the following relationship:

$$\Phi = -\left(T_0 + T_+ + G_0 + G_+\right) D^{-1}. \quad (40)$$

Substituting the result (40) into expression (24) and solving equation (18) for the regulator's transfer function matrix, taking into account relationship (25), allows us to determine that:

$$W_0 = (B + \Phi M)^{-1}(-A + \Phi P). \quad (41)$$

4 EXPERIMENTS

The initial data for synthesizing the optimal structure of the Stewart platform's WS motion two-loop tracking control systems consists of its dynamic models, as a control object, as well as the spectral density of the acting disturbance, which were determined based on the results of field tests under conditions close to the real operating mode of the experimental sample of the Stewart platform, using special algorithms [3]. Thus, the dynamics of the Stewart platform (Fig. 1) are described by the matrices:

$$P_0 = \begin{bmatrix} z_2 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_2 \end{bmatrix}, \quad (42)$$

where $z_1 = \left(s^2 + 0.63s + 0.15\right)\left(s^2 + 2s + 1.09\right)$, $z_2 = \left(s^2 + 0.11s + 0.04\right) z_1$,

$$M_0 = \begin{bmatrix} 0.013(s+5.4)(s+0.83)(s+0.15)\left(s^2+0.3s+0.067\right) & -0.016(s-2.1)(s+0.14)(s-0.027)\left(s^2+1.89s+1\right) & -0.008(s-2.1)(s+0.76)(s+0.19)\left(s^2+0.24s+0.085\right) \\ 0.01(s+2.1)\left(s^2+0.22s+0.03\right)\left(s^2+1.35s+0.83\right) & -0.004(s+9.3)\left(s^2+1.95s+1\right)\left(s^2+0.067s+0.09\right) & 0.004(s+1.95)\left(s^2+0.22s+0.055\right)\left(s^2+1.06s+0.67\right) \\ 0.05(s+0.96)\left(s^2+0.5s+0.14\right)\left(s^2-0.018s+0.2\right) & 0.006(s+0.19)\left(s^2+1.4s+0.54\right)\left(s^2+1.23s+4.09\right) & 0.025(s+0.94)\left(s^2-0.2s+0.067\right)\left(s^2+0.59s+0.17\right) \end{bmatrix}, \quad (43)$$

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the spectral density of the acting disturbance $S'_{\psi_0\psi_0}$:

$$S'_{\psi_{ob}\psi_{ob}} = 10^{-4} z_4 \begin{bmatrix} 9 & 5.3 & 3.2 \\ 5.3 & 9 & 6.7 \\ 3.2 & 6.7 & 34.4 \end{bmatrix}, \quad (44)$$

where $z_3 = (s+0.45)(s+0.55)$,

$$z_4 = \left| \frac{(s^2+0.1s+0.04)(s^2+0.6s+0.15)(s^2+2s+1)}{z_3} \right|^2 .$$

Since the measurement of the output coordinate vector x_1 and the program signal vector r_0 is performed by inertialess measuring devices, which according to their technical specifications are proportional elements with a transfer coefficient equal to 1, the following equations are satisfied:

$$K_{10} = K_{20} = E_{3\times3}, K_0 = K_1 = K_2 = E_{3\times3},$$

according to the definitions (12) and (32), (33) following equations take place

$$P = P_1,\ M = M_1,\ S'_{\psi\psi} = S'_{\psi_r\psi_r},\ S'_{\varphi_0\psi} = S'_{\varphi_0\psi_r},\quad x_2 = x_1,\ \psi = \psi_r . \quad (45)$$

To find the auxiliary matrix H, it is necessary to determine the polynomial weight matrices R_1, which defines the impact of stabilization error variance on the criterion value (20), and C, which limits the variance of the control signal u.

Based on the methodology for determining weight matrices presented in the works [14], we obtain the polynomial weight matrix R_1, according (22):

$$R_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.16 & -0.002 & -0.06 \\ 0 & 0 & 0 & -0.002 & 0.24 & -0.165 \\ 0 & 0 & 0 & -0.06 & -0.165 & 1.17 \end{bmatrix}, \quad (46)$$

the polynomial weight matrix C is equal to:

$$C = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}. \quad (47)$$

Substituting matrices (42), (43), (46), (47) into expression (26) allows us to determine the auxiliary polynomial matrix H. Factorization of this matrix based on algorithm [9] allowed us to determine the following blocks of the matrix V (28), necessary for further synthesis:

$$A = O_{12\times12},\ B = E_{3\times3}, \quad (48)$$

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0797 & -0.00107 & -0.031519 \\ 0 & 0 & 0 & -0.00107 & 0.11927 & -0.0825 \\ 0 & 0 & 0 & -0.031519 & -0.0825 & 0.58515 \end{bmatrix} \quad (49)$$

$$N = \frac{10^{-3}}{z_5} \left[\begin{matrix} 0.555(s-9.269)(s+0.867)(s+0.00885)(s^2+0.39s+0.083) \\ 2.9(s-0.8)(s+0.337)(s-0.19)(s^2+1.24s+0.39) \\ -27.579(s+0.9399)(s^2+0.5398s+0.1419)(s^2-0.16s+0.22) \\ 0.555(s-9.269)(s+0.867)(s+0.00885)(s^2+0.39s+0.083) \\ 2.9(s-0.8)(s+0.337)(s-0.19)(s^2+1.24s+0.39) \\ -27.579(s+0.9399)(s^2+0.5398s+0.1419)(s^2-0.16s+0.22) \end{matrix} \right.$$
$$\left. \begin{matrix} 1.4946(s-1.419)(s-0.29)(s+0.159)(s^2+1.8s+0.96) \\ 0.93919(s+3.8)(s+2.185)(s+0.2466)(s^2+0.4747s+0.1616) \\ -4.57(s+0.195)(s^2+1.29s+0.5287)(s^2+1.567s+3.496) \\ 1.4946(s-1.419)(s-0.29)(s+0.159)(s^2+1.8s+0.96) \\ 0.93919(s+3.8)(s+2.185)(s+0.2466)(s^2+0.4747s+0.1616) \\ -4.57(s+0.195)(s^2+1.29s+0.5287)(s^2+1.567s+3.496) \end{matrix} \right. \quad (50)$$

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$$\left.\begin{matrix} 1.4(s-0.856)(s+0.8166)(s+0.06667)(s^2+0.35s+0.1) \\ 1.5936(s-0.7858)(s^2+0.31s+0.02536)(s^2+1.239s+0.395) \\ -14.688(s+0.918)(s^2-0.256s+0.05877)(s^2+0.5895s+0.16) \\ 1.4(s-0.856)(s+0.8166)(s+0.06667)(s^2+0.35s+0.1) \\ 1.5936(s-0.7858)(s^2+0.31s+0.02536)(s^2+1.239s+0.395) \\ -14.688(s+0.918)(s^2-0.256s+0.05877)(s^2+0.5895s+0.16) \end{matrix}\right],$$

where

$$z_5 = \left|s^2+0.113s+0.0439\right|^2\left|s^2+0.625s+0.15\right|^2 \times$$
$$\times\left|s^2+2.005s+1.087\right|^2.$$

After substituting matrices (48) into expression (25), the fractional-rational matrices z_{22} and z_{21} are found to be:

$$z_{21} = O_{12\times12},\ z_{22} = E_{3\times3}. \quad (51)$$

To determine the matrix $S'_{\xi_0\xi_0}$ (31), we consider definition (45), as well as the fact that the extended vector of output signals ψ and the extended vector of measurement device noises φ_0 are not correlated with each other. Thus,

$$S'_{\varphi_0\psi} = S'_{\varphi_1\psi_{ob}} = O_{3\times3}.$$

Based on this, we can determine:

$$S'_{\xi_0\xi_0} = \begin{bmatrix} Q_n & O_{3\times3} & O_{3\times3} & O_{3\times3} \\ O_{3\times3} & DS_{rr} & O_{3\times3} & O_{3\times3} \\ O_{3\times3} & O_{3\times3} & R_n & O_{3\times3} \\ O_{3\times3} & O_{3\times3} & O_{3\times3} & gE \end{bmatrix},$$

where

$$S'_{\psi\psi} = \begin{bmatrix} Q_n & O_{3\times3} \\ O_{3\times3} & DS_{rr} \end{bmatrix},$$

DS_{rr} – the of amplification coefficients matrix of the input signal r_0 spectral density matrix, taken from [3], Q_n – the weighted covariance matrix of the input signals ψ extended vector, disturbance ψ_{ob}, and program signal r_0.

According to [7], the weighted covariance matrix Q_n is equal to:

$$Q_n = K_g^{-1}(G_n - \alpha\Omega)K_g^{-1},$$

$$Q_n = 10^{-5}\begin{bmatrix} 0.0665 & 0.0441 & 0.0266 \\ 0.0441 & 0.0646 & 0.0555 \\ 0.0266 & 0.0555 & 0.2594 \end{bmatrix}.$$

$$S'_{\varphi_0\varphi_0} = \begin{bmatrix} R_n & O_{3\times3} \\ O_{3\times3} & gE \end{bmatrix},$$

where gE – the matrix, which is determined according to recommendations [14], as a quantity approaching zero, R_n – the weighted covariance matrix of the measurement's noises extended vector $R_n = \left\langle \varphi_0\varphi_0' \right\rangle$.

According to (8), the measurement's noises extended vector φ_0 consists of the program signal measurement's noises φ_r, whose values approach zero, and of the output signal measurement's noises φ_1, whose values are determined by the root-mean-square deviation of the feedback sensor values. As the feedback sensor, an inertial navigation system [16] is used, with the following root-mean-square deviation values along the axes:

$$\varphi_1 = \frac{\pi}{180}\begin{bmatrix} 0.769 & 0.847 & 1.344 \end{bmatrix}'.$$

Thus, R_n is equal:

$$R_n = \begin{bmatrix} 0.0018 & 0 & 0 \\ 0 & 0.0022 & 0 \\ 0 & 0 & 0.0056 \end{bmatrix}.$$

5 RESULTS

Based on the above experimental data, the product DD* is obtained using expression (36). Define the matrix D as the result of the left-sided factorization [11] product DD*:

DOI 10.15588/1607-3274-2024-3-20

$$D=\begin{bmatrix} 0.042426\cdot z_8 & 0.0032097(s+0.39)z_6 & 0.00063363(s+0.45)z_6 \\ 0.0035306(s+0.39)z_6 & 0.046669z_8 & 0.0019254(s+0.418)z_6 \\ 0.0011156(s+0.45)z_6 & 0.0030819(s+0.418)z_6 & 0.074699z_8 \\ -0.042426 & 0 & 0 \\ 0 & -0.046669 & 0 \\ 0 & 0 & -0.074699 \end{bmatrix.$$
$$\begin{matrix} 0.00078229(s+0.3158)z_7 & 0.00048716(s+0.41)z_6 & -0.0005479(s+0.1556)z_7 \\ 0.0002854(s+0.39)z_6 & 0.0016987(s+0.3)\cdot z_7 & 0.00068386(s+0.412)z_6 \\ 9.018\cdot 10^{-5}(s+0.45)z_6 & 0.00058573(s+0.42)z_6 & 0.007968(s+0.334)z_7 \\ 0.52486 & 0 & 0 \\ 0.042654 & 0.23899 & 0 \\ 0.18619 & -0.013884 & 0.1671 \end{matrix}\Bigg], \quad (52)$$

where $z_6=(s^2+0.5689s+0.19)(s^2+2s+1.09)$, $z_7=(s^2+0.55s+0.16)(s^2+2s+1.09)$, $z_8=(s^2+0.69s+0.166)(s^2+2s+1.09)\times$ $\times(s^2+0.27s+0.1)$.

Taking into account the obtained results (48–52), as well as the fact that the extended vector of output signals ψ and the extended vector of measurement device noises φ_0 are not correlated, we will determine the fractional-rational matrices T and G, which are subject to separation, based on equations (37) and (38). The matrix G has only negative poles, so the result of the separation G_0+G_+ equals O_{3x3}. The result of the T matrix separation is as follows:

$$T_0+T_+=\frac{10^{-4}}{z_9}\begin{bmatrix} 6.187(s+0.178)(s^2+0.5986s+0.15) & 12.18(s+0.089)(s^2+0.605s+0.148) \\ 7.98(s+0.225)(s^2+0.62s+0.156) & 26.64(s+0.114)(s^2+0.617s+0.152) \\ 1.3696(s+0.096)(s^2+0.597s+0.146) & 1.578(s-0.0034)(s^2+0.5897s+0.14) \end{bmatrix.$$
$$\begin{matrix} 43.5(s-0.0249)(s^2+0.61s+0.148) & 0 \\ 160.7(s+0.036)(s^2+0.6186s+0.15) & 0 \\ 0.128(s-7.7)(s^2+0.5778s+0.127) & 0 \end{matrix} ,$$
$$\begin{matrix} 2.286(s+0.085)(s^2+0.61s+0.148) & 18.32(s-0.0277)(s^2+0.625s+0.15) \\ 0 & 69.91(s+0.0377)(s^2+0.625s+0.15) \\ 0 & -0.217(s+1.786)(s^2+0.625s+0.15) \end{matrix}\Bigg]$$

where $z_9=(s^2+0.113s+0.0439)(s^2+0.625s+0.15)$.

Thus, using equation (40) and the results of the separation T_0+T_+, G_0+G_+, the varying matrix Φ can be determined.

So, as a result of applying algorithm (41) using the obtained matrix Φ and definitions (3), (45), and (48), we find the transfer function matrix of the controller for the two-loop tracking system in the form:

$$W_0=\frac{1}{z_{10}}\begin{bmatrix} 854.69(s+1.447)(s^2+8.117s+26.65)(s^2+6.165s+34.87) \\ 269.13(s+1.49)(s^2+5.712s+14.1)(s^2+8.98s+130.7) \\ -204.87(s+1.46)(s^2+7.5s+24.4)(s^2+5.03s+137.3) \end{bmatrix.$$
$$\begin{matrix} 665.55(s+1.674)(s^2+10.1s+46.47)(s^2+4.286s+26.75) \\ -323.24(s+15.62)(s-3.042)(s+1.579)(s^2+6.54s+29.76) \\ -347.88(s+1.65)(s^2+3.137s+29.15)(s^2+12.6s+76.73) \end{matrix} \quad (53)$$

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$$\begin{matrix} 906.75(s+0.946)(s^2+10.23s+32.73)(s^2+5.456s+23.9) \\ 865.26(s+0.9466)(s^2+10.08s+34.07)(s^2+5.716s+17.9) \\ -798.6(s+0.9462)(s^2+10.12s+33.55)(s^2+6.867s+27.39) \end{matrix}$$

$$\begin{matrix} -2.0366(s^2+10.4s+33.5)(s^2+5s+21.28)(s^2+7.135s+145.1)/s \\ 0.090625(s+58.39)(s-51.5)(s^2+10.4s+34.15)(s^2+4.06s+14.95)/s \\ -9.79(s+11.1)(s-1.9)(s^2+10.46s+33.67)(s^2+5.07s+21.4)/s \end{matrix}$$

$$\begin{matrix} -7.6496(s^2+7.9s+26.64)(s^2+5.7s+30.76)(s^2+10.14s+62.3)/s \\ 6.226(s+13.4)(s-3.6)(s^2+5.9s+20.1)(s^2+7.995s+33.49)/s \\ 1.649(s^2+7.318s+25.36)(s^2+7.426s+45)(s^2+11.2s+185.6)/s \end{matrix} ,$$

$$\left.\begin{matrix} -6.11(s^2+10.88s+33.05)(s^2+4.869s+19.66)(s^2+7.656s+46.29)/s \\ 7.8247(s^2+10.89s+30.7)(s^2+4.3s+13)(s^2+8.16s+45.12)/s \\ 1.1987(s+10.85)(s+32.6)(s^2+4.79s+18.36)(s^2+9.877s+234.8)/s \end{matrix}\right]$$

where $z_{10}=(s+8.538)(s+0.97)(s^2+10.48s+33.15)\times$ $\times(s^2+4.918s+24.3)$.

The structure and parameters of the transfer function matrices of the optimal multidimensional controller (Fig. 1) for the tracking system (9) were determined. According to [11], there exists a pair of matrices W_3 and $[-W1\ W2]$ that form a left coprime factorization of the matrix W_0. Using the methodology from [17], which allows for the computation of the normalized left coprime factorization of a matrix, the components of the transfer function matrices of the optimal multivariable controller W_0 (9) can be written as follows:

$$W_1=\frac{1}{z_{11}}\left[\begin{matrix} -854.69(s+1.2)(s^2+6.898s+21.5)(s^2+6.06s+20.6) \\ -269.13(s+0.75)(s^2+9.155s+29.4)(s^2+5.339s+32.76) \\ 204.87(s-1.336)(s^2+6.878s+16.39)(s^2+4.865s+14) \end{matrix}\right.$$

$$\begin{matrix} -665.55(s+2.697)(s^2+4.36s+11)(s^2+7.267s+26.86) \\ 323.24(s+0.64)(s^2+7.1s+25.85)(s^2+11.09s+54.87) \\ 347.88(s+2.76)(s^2+4.815s+13.1)(s^2+6.59s+23.8) \end{matrix} , \quad W_2=\frac{1}{s}\begin{bmatrix} -2.0366 & -7.6496 & -6.11 \\ 0.09 & 6.226 & -7.8247 \\ -9.79 & 1.649 & 1.1987 \end{bmatrix},$$

$$\left.\begin{matrix} -906.75(s+0.9598)(s^2+5.76s+16.37)(s^2+8.309s+33.8) \\ -865.26(s+0.9937)(s^2+6.079s+17.86)(s^2+8.4s+35.2) \\ 798.6(s+0.9468)(s^2+5.68s+16)(s^2+8.36s+34.6) \end{matrix}\right]$$

$$W_3=\frac{1}{z_{10}}\left[\begin{matrix} (s^2+9.43s+32.57)(s^2+5.51s+25.2)(s^2+8.625s+55.26) \\ -0.159(s-262.9)(s^2+5.1s+19.39)(s^2+8.919s+33.28) \\ -0.213(s+220.1)(s^2+9.635s+34.4)(s^2+5.7s+24.79) \end{matrix}\right.$$

$$\begin{matrix} -0.159(s+78.46)(s^2+6.618s+19.46)(s^2+6.37s+41.75) \\ (s+9.99)(s-0.2656)(s^2+5.879s+17.4)(s^2+7.88s+36.98) \\ 0.052825(s+25.17)(s^2+6.395s+18.8)(s^2-19.59s+347.5) \end{matrix} ,$$

$$\left.\begin{matrix} -0.213(s-81.35)(s^2+10.79s+33.96)(s^2+4.8s+20.36) \\ 0.052825(s+363.6)(s^2+10.96s+34)(s^2+3.767s+13.1) \\ (s+10.1)(s-0.96)(s^2+10.68s+33.5)(s^2+4.938s+20.8) \end{matrix}\right]$$

where
$z_{11}=(s^2+5.65s+15.87)(s^2+7.767s+29.11)\times$ $\times(s^2+8.589s+36.66)$.

The main controller is divided into three components: W_1, W_2 and W_3. This distribution of the controller enhances the tracking quality of the program signal vector and accounts for cross-connections within the Stewart platform. Additionally, it allows for increased precision in following the specified trajectory by increasing the degrees of freedom in selecting the controller structure.

DOI 10.15588/1607-3274-2024-3-20

6 DISCUSSION

The justified transformations (3)–(16) form the basis for developing an information technology to convert the structure of a multidimensional two-loop tracking system into a multidimensional stabilization system. This transformation will enable the assessment of tracking quality and control costs in a two-loop multidimensional tracking system under random and regular influences using standardized approaches for stabilization system analysis.

The developed rules for calculating the transfer function matrices of the controller (41) provide a theoretical basis for defining an information technology for synthesizing an optimal multidimensional two-loop tracking system, which ensures the highest possible tracking accuracy along a random trajectory with an acceptable level of control costs. The main limitation of using the relation (41) is related to the requirement for the stationary and centricity of the multidimensional useful signals and disturbances acting at the system inputs.

The implementation of the obtained transfer function matrices of the controller (Fig. 1) using microprocessor technology requires the representation of the matrix equation as follows:

$$u = W_3 \begin{bmatrix} -W_1 & W_2 \end{bmatrix} \begin{bmatrix} x_3 \\ \varepsilon_2 \end{bmatrix},$$

in the form of a finite-difference equation.

Additionally, the availability of algorithms for calculating the matrix of optimal transfer functions (24) from the extended disturbance vector ξ to the control signal vector u allows for the synthesis of an optimal quadratic criterion (20) neuron-phase regulator.

CONCLUSIONS

The work involved synthesizing the optimal structure and parameters of a multidimensional tracking control system the Stewart platform's WS motion control system, considering a multidimensional dynamic model that includes the object itself, its basic components, controlled and uncontrolled disturbances acting on it in conditions close to real operating modes.

The scientific novelty of the obtained results lies in the application of the tracking system reduction algorithm to an equivalent stabilization system, which allowed for the use of a justified method to synthesize an optimal multidimensional stabilization system for a dynamic object operating under the influence of multidimensional stationary random useful signals, disturbances, and measurement noise.

The practical significance of the obtained results lies in determining the Stewart platform's WS motion control system main controller structure and parameters. Its integration into the feedback loop ensures the stability of the closed-loop control system. The main controller is distributed into three components: W_1, W_2, and W_3, which improves the quality level of tracking the program signals vector and allows for the consideration of cross-couplings within the Stewart platform. It also provides the capability to increase the accuracy of trajectory execution by increasing the number of freedom degrees in the controller structure, as the controller consists of three transfer function elements.

Perspectives for further research. Considering that the stabilization system synthesis algorithm forms the basis for developing any closed-loop control system, it is worthwhile to consider the next step as the development of an information technology for analytical design of optimal multidimensional tracking systems under random influences.

ACKNOWLEDGEMENTS

The authors express their sincere gratitude to Professor Blokhin L. M. for numerous suggestions and recommendations for improving approaches to synthesizing optimal multidimensional stabilization systems for dynamic objects, which are widely used in aviation and technology production.

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Received 07.06.2024.
Accepted 13.08.2024.

УДК 62.505:629.524

СИНТЕЗ БАГАТОВИМІРНОЇ СЛІДКУВАЛЬНОЇ СИСТЕМИ КЕРУВАННЯ ПЛАТФОРМИ СТЮАРТА

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АНОТАЦІЯ

Актуальність. Створення гарантовано конкурентоспроможних систем керування рухами складних багатовимірних рухомих об'єктів, у тому числі нестійких, які функціонують в умовах дії випадкових контрольованих та неконтрольованих збурюючих факторів, з мінімальними витратами на проектування є однією з головних вимог досягнення успіху на ринку даного класу пристроїв. Також важливо, для досягнення сучасних вимог до точності процесів керування рухом рухомого об'єкта на заданій або програмованій траєкторії руху необхідно синтезувати оптимальну системи керування на підставі експерементальних даних отриманих в умовах наближених до реального режима функціонування дослідного зразка об'єкту.

Мета роботи. Метою дослідження, результати якого представлені у цій статті, є виконання синтезу оптимальної слідкувальної системи керування рухом робочої поіерхні платформи Стюарта з врахуванням її багатовимірної моделі динаміки.

Метод. У статті використано метод структурного перетворення багатовимірної слідкувальної системи керування до еквівалентної системи стабілізації руху багатовимірних об'єктів керування. Також викорестано алгоритм синтезу оптимальної системи стабілізації динамічних об'єктів, як стійких, так ні, в умовах дії стаціонарних випадкових зовнішніх збурень. Обгрунтований алгоритм синтезу оптимальних стохастичних систем стабілізації, побудований за допомогою операцій додавання, множення поліноміальних та дробово – раціональних матриць, вінеровської факторизації, вінеровської сепарації дробово – раціональних матриць, знаходження дисперсійних інтегралів.

Результати. В результаті проведених досліджень формалізовано задачу визначеня концепції аналітичного конструювання оптимальної системи керування рухом РП платформи Стюарта. Результати включають визначені рівняння перетворення з слідкуючої системи керування до еквівалентної системи стабілізації руху робочої поіерхні платформи Стюарта. Також визначено структуру і параметри матриці передавальних функцій гововного регулятора оптимальної слідкувальної системи керування рухом робочої поіерхні платформи Стюарта.

Висновки. Обгрунтоване використання концепції аналітичного конструювання оптимальної системи керування рухом РП платформи Стюарта формалізує і істотно спрощує розв'язання задачі синтезу складних динамічних систем та застосування для цього розробленої технології, представленої у [8]. Отримані структура та параметри головного регулятора системи керування рухом РП платформи Стюарта, який розподілений на три складові W_1, W_2 та W_3, сприяє поліпшенню рівень якості слідкування за вектором програмних сигналів і дозволяє врахувати перехресні зв'язки всередині платформи Стюарта, підвищує точності виконання заданої траєкторії за рахунок збільшення кількості ступенів свободи при виборі структури регулятора.

КЛЮЧОВІ СЛОВА: синтез, матриця передавальних функцій, слідкуюча система керування, функціонал якості, платформа Стюарта.

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DOI 10.15588/1607-3274-2024-3-20