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ABOUT OF THE ANNEALING METHOD USING FOR THE TRAVELING SALESMAN PROBLEM SOLUTION WITH THE FUZZY TIME PERCEPTION

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ABSTRACT

Context. The article considers a technique for the use of fuzzy numbers and the annealing method for solving the traveling salesman problem, which is formulated as the problem of finding a route to visit a given number of cities without repetitions with a minimum duration of movement. The task of formalizing the algorithm for solving the traveling salesman problem by the annealing method using fuzzy numbers for subjective time perception is posed. The use of fuzzy numbers to increase the accuracy to represent real-world circumstances is proposed.

Objective. The goal of the work is to develop an algorithm for solving the traveling salesman problem based on the implementation of the annealing method with fuzzy numbers representing the subjective time perception for traveling between the cities with the minimum perceived duration of movement along the route.

Method. This paper proposes a method for solving the traveling salesman problem by the annealing method with fuzzy numbers for subjective time perception. A scheme for formalizing the procedure for solving the traveling salesman problem with the minimal perceived duration of movement along the route is described. A variant of the original traveling salesman problem is proposed, which consists in using fuzzy numbers to represent the uncertainty and subjective time perception in traveling between cities as opposed to regular crisp numbers to show regular distance and/or time of traveling. The results of the proposed algorithm for calculating solutions to the traveling salesman problem with minimization of the perceived duration of movement are presented, the obtained solutions are compared with the solutions found by other heuristic methods.

Results. The method for solving the traveling salesman problem using the annealing method with fuzzy numbers for subjective time perception is developed. A variant of the original traveling salesman problem is proposed, which consists in using fuzzy numbers to represent the uncertainty and subjective time perception in traveling between cities as opposed to regular crisp numbers to show regular distance and/or time of traveling. The application of fuzzy numbers makes it possible to perform calculation over possibly uncertain or subjective data, making results more accurate in the case of realistic deviations from the expected mean values in distance coverage. The results of the proposed algorithm for calculating solutions to the traveling salesman problem with minimization of the perceived duration of movement are presented, the obtained solutions are compared with the solutions found by other heuristic methods.

Conclusions. The paper considers a method for formalizing the algorithm for solving the traveling salesman problem using fuzzy numbers for subjective time perception. The use of fuzzy numbers to increase the accuracy to represent real-world circumstances is proposed. The scheme for formalizing the procedure for solving the traveling salesman problem with the minimal perceived duration of movement along the route is described. A variant of the original traveling salesman problem is proposed, which consists in using fuzzy numbers to represent the uncertainty and subjective time perception in traveling between cities as opposed to regular crisp numbers to show regular distance and/or time of traveling.

KEYWORDS: traveling salesman problem, fuzzy numbers, simulated annealing, combinatorial optimization, subjective perception of time, imprecision, uncertainty.

ABBREVIATIONS

TSP is a traveling salesman problem.

NOMENCLATURE

p is a cyclic permutation of numbers;

j_i is a city number;

n is a number of cities;

d_{ij} is a travel time between all pairs of vertices;

D is a matrix of moving cost (distances or times);

i, j are the indexes;

I is a set of vertex indices;

X is a binary matrix of transitions between vertices;

x_{ij} are the elements of matrix X, which equal to 0 or 1;

v_i is a vertex of graph, $i=\overline{1,n}$;

t is a moment of time;

S is a set of all system state;

$f(s)$ is a state change function;

s_i is a system state on i-th step;

s_k is a new state (candidate);

$t_{\min}$ is minimal temperature;

$t_{\max}$ is an output temperature;

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t_i is a current temperature of annealing process;
$T(t)$ is a temperature change function;
$E(s)$ is an objective function value;
$\tilde{A}$ is a fuzzy set;
E is a set of numbers;
$\mu_{\tilde{A}}(x)$ is a membership function;
(a, b, c) is a triangle fuzzy number;
$F1(\tilde{A})$ is a rank of a fuzzy number $\tilde{A}$;
$g(x)$ is a weight function;
M is a random number.

INTRODUCTION

The way decisions are made in society in many cases depends on the emotional state of a person. Feelings are like a reference point that is determined by a goal that is influenced by various factors. Emotions can be the reason for behavior that is appropriate for a particular situation, even when it is not the most efficient, but allows you to avoid any consequences that may arise from exceeding a certain time limit.

Special attention should be paid to these factors in the processes of formation and improvement of many theoretical ideas in the field of modeling human behavior, one of which is the adaptation of physical and mathematical models to real life. This makes it possible to combine the power of computational methods with the peculiarities of human behavior. Such tasks are common in the context of the application of artificial intelligence methods and algorithms, the creation of decision-making support systems, the resolution of resource allocation issues taking into account the human factor, etc.

Time is an important resource in activities involving human participation. Estimation of time intervals is fundamental to understanding time frames, even though the exact boundaries of the interval may not be defined until the process reaches a certain stage. Thus, a period of time is usually defined by an indefinite interval that can be roughly predicted given the nature of the passage of time, if the given limits of the interval are taken as a given. To measure time intervals, they can be expressed in phrases such as “quick response”, “normal timing” or “long wait”. This means that when solving problems that require verbal terms to refer to time, it is important to take time variation into account. It is clear that emotions have a great influence on the understanding of time in processes that involve a person [1].

In order to find the most successful or effective solution to problems, it is necessary to take into account factors that affect human emotions and, therefore, the speed of time perception, resource allocation and calendar planning. The paper proposes to develop an approach to the formalization of accounting for the flow of time based on fuzzy numbers and to apply it to solving certain fuzzy optimization problems related to taking into account the fuzzy perception of time arising from the subjectivity and irregularity of the time count.

The object of study is the process of optimal route search for the fuzzy traveling salesman problem with a minimum duration of movement.

The subject of study is the algorithm for solving the fuzzy traveling salesman problem based on the combinatorial methods in combination with triangle fuzzy numbers.

The purpose of the work is to develop an algorithm for solving the fuzzy traveling salesman problem using one of the combinatorial methods of the approximate solution of the problem in combination with fuzzy numbers as a way to define subjective perception of time to travel between cities.

1 PROBLEM STATEMENT

The traveling salesman problem is one of the most famous computational optimization problems. The task is to find the shortest route that passes through each city exactly once for a given number of cities. The search for such a path was formulated as a mathematical problem in 1930 and is still one of the most intensively researched optimization problems [2].

The number of alternative paths for a TSP with n nodes, where the nodes are cities and the edges are the cost of moving between two cities, is $(n-1)!$. Therefore, even for small problems, such as the one presented with only 20 nodes, the number of alternative paths is about $1.2*10^{17}$, to which there is no adequate computing power to explore through exhaustive enumeration.

The traveling salesman problem is one of the famous combinatorial problem [3]. To reduce the problem to a general form, we number the cities by numbers (1, 2, 3, ..., n), and describe the traveling salesman’s route by a cyclic permutation of numbers $p=(j_1, j_2,..., j_n, j_1)$, where all $j_1,...,\ j_n$ are different numbers.

The set of cities can be considered as the vertices of some graph with given distances (or travel time) between all pairs of vertices d_{ij} that form the matrix $D=(d_{ij})$, $i,j=\overline{1,n}$. We assume that the matrix is symmetric. The formal problem then is to find the shortest route (in time or length) t that goes through each city and ends at the starting point. In this formulation, the problem is called the closed traveling salesman problem, which is a well-known mathematical integer programming problem.

Let us formulate a mathematical model of the TSP problem. Let $I=\{1,...,n\}$ be the set of vertex indices of the problem graph. The objective function is the total distance or time of the route, including all the vertices of the task graph. The parameters of the problem are the elements of the matrix $D=(d_{ij})$, $i,j \in I$.

Shift tasks are elements of the binary matrix of transitions between vertices $X=\{x_{ij}\}$, $i, j \in I$, which are equal to 1 if there is an edge (v_i, v_j) in the constructed route for the task, 0 otherwise [4]. The shortest route in terms of distance or time is optimal:

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$$\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} d_{ij} x_{ij} \to \min \quad (1)$$

with constraints

$$\sum_{j=1, j\neq i}^{n} x_{ij} = 1,\ i = \overline{1,n},$$

$$\sum_{i=1, i\neq j}^{n} x_{ij} = 1,\ j = \overline{1,n}, \quad (2)$$

$$v_i - v_j + n x_{ij} \le n-1,\ 1 \le i \ne j \le n.$$

The last inequality ensures the connectivity of the vertex traversal route; it cannot consist of two or more unconnected parts.

The dynamic traveling salesman problem (DTSP) is a TSP defined by a dynamic cost (distance) matrix as follows:

$$D = \{d_{ij}(t)\}_{n(t)\times n(t)},$$

where $d_{ij}(t)$ is the amount of moving cost from city (node) i to city j at time t. In this definition, the number of cities $n(t)$ and the cost matrix are time dependent. The traveling salesman's dynamic problem is to find a minimum-cost route that contains all $n(t)$ nodes.

In other words, having all $n(t)$ nodes $j_1,\ldots, j_{n(t)}$ and the corresponding cost matrix $D(t)=(d_{ij}(t))$, $i,j=\overline{1,n(t)}$, we need to find a route with the minimum cost containing all $n(t)$ points, where t is the moment of time, $d_{ij}(t)$ is the distance or time between the points i and j:

$$\sum_{i=1}^{n(t)} \sum_{j=1, j\neq i}^{n(t)} d_{ij}(t) x_{ij} \to \min \quad (3)$$

with above constraints (2).

The change in the cost matrix D over time is a continuous process. Practically, in order to build analytical models, it is necessary to discretize this process of changes. Thus, D becomes a series of optimization problems $D(t_k)=(d_{ij}(t_k))$, $i,j=\overline{1,n(t_k)}$, $k = 0, 1, 2, \ldots, m-1$, with time windows $[t_k, t_{k+1}]$, where $\{t_k\}_{i=0}^{m}$ is a sequence of time points.

2 REVIEW OF LITERATURE

Algorithms that allow solving the problem of finding the optimal route are divided into exact and heuristic. In the case of exact methods, the search for solutions is based on optimization methods such as linear programming, dynamic programming, or the branch and bound method [5]. However, it is convenient to use exact methods only for small-scale problems (for example, for the purpose of primary design of a small-sized transport network), since their implementation requires large computing power.

Heuristic methods do not guarantee finding an optimal solution, but are aimed at quickly finding a locally optimal solution. Traditionally, "trial and error" approaches, such as random search or greedy algorithm, are used to quickly explore the solution space and find a promising solution [6]. Heuristics are more flexible and can be applied to larger problems, but the solution they offer may not be optimal. Among such heuristic methods, attention should also be paid to methods that imitate biological (ant colony algorithm and genetic algorithm [7, 8]) or physical processes [9, 10].

One of the methods of solving the traveling salesman problem using the combinatorial optimization technique is the annealing method [9]. By analogy with the annealing process of various physical materials, in which by raising its temperature to a high level and then gradually lowering it, the algorithm accidentally disturbs the output path ("heating") for further gradual lowering of the "temperature" [10].

When modeling the annealing process, the analog of temperature is the level of randomness, with the help of which changes are made to the path, which in the future improves in its duration. When the "temperature" of the process is high, changes occur to avoid the danger of reaching a local minimum, followed by control at the optimal value as the "temperature" is successively reduced. "Temperature" decays in a series of steps on an exponential decay curve, with each step the temperature being lower than before.

3 MATERIALS AND METHODS

Let's describe an annealing method. The approach implemented in the simulated annealing method is borrowed from physical processes. It is based on the process of crystallization of a substance, which metallurgists found to increase the homogeneity of the metal.

As is known, metals have crystal lattices that determine the geometric position of the atoms of the substance. The set of positions of all atoms will be called the state of the system; each state corresponds to a certain energy level. The purpose of annealing is to bring the system to the state with the lowest energy. The lower the energy level, the "better" the crystal lattice, that is, the fewer defects it has and the stronger the metal.

During annealing, the metal is first heated to any temperature, which causes the atoms of the crystal lattice to leave their positions. Slow and controlled cooling then begins. Atoms tend to get into a state with lower energy, but with a certain probability they can go into a state with higher energy. This probability decreases with temperature. The transition to a worse state, oddly enough, helps as a result to find a state with less energy than the original one. The process ends when the temperature drops to the set value.

Such a complex scheme with probabilities of transition from point to point is necessary so that the algorithm

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does not get stuck on a local minimum, taking it for a global optimum. To get out of this situation, you need to increase the energy of the system from time to time. At the same time, the general tendency to search for the lowest energy remains. This is the essence of the simulated annealing method.

To describe the algorithm scheme for formalizing the simulated annealing method, we introduce the notation:

S – set of all system state;

$f(s)$ – state change function;

s_i – system state on i-th step;

s_k – new state (candidate);

$t_{\min}$, t_i, $t_{\max}$ – minimal, current and output temperature respectively;

$T(t)$ – temperature change function;

$E(s)$ – objective function value.

The algorithm starts working from the initial state s_1, with the initial temperature $t_1 = t_{\max}$ and with the specified minimum temperature $t_{\min}$.

For every steps with numbers i=1,2,... while $t_i > t_{\min}$ repeat:

1) $s_k = f(s_{i-1})$;

2) $\Delta E = E(s_k) - E(s_{i-1})$;

3) if $\Delta E < 0$, then $s_i = s_k$;

4) otherwise, acceptance of a new state occurs with some probability $\exp(-\Delta E / t_i)$;

5) choose a random number M on interval (0,1);

6) if $\exp(-\Delta E / t_i) > M$, perform the transition $s_i = s_k$, otherwise, go to the next step;

7) reduce the temperature t : $t_{i+1} = T(t_i)$;

8) return the last state s_i, $i = i+1$.

Fuzzy passage of time

In everyday life, expressions such as "almost six", "quite tall", "not short enough" are often used to define a certain size in an approximate format. As a result, this method of evaluation requires the formalization of insufficiently clearly defined evaluations for their practical application in mathematical models. For this purpose, you can use concepts that allow you to present the subjective or intuitive meaning of fuzzy concepts in a constructive way. One of these concepts of uncertainty formalization is fuzzy numbers [11].

Fuzzy numbers are used to obtain results in problems related to decision-making and analysis. Fuzzy numbers defined in the number space are an extension of real numbers and have their own properties that can be attributed to number theory. To understand fuzzy numbers and their subspecies – triangular and parabolic numbers, consider the concept of a fuzzy set.

Let E be a set with a finite or infinite number of elements. Let A be the set contained in E. Then the set of ordered pairs $(x, \mu_{\tilde{A}}(x))$ defines a fuzzy subset $\tilde{A}$ for E, where x – is a member of E, and $\mu_{\tilde{A}}(x)$ – degree of belonging of x to A. The set of elements from A for which $\mu_{\tilde{A}}(x)>0$ form the support of a fuzzy set.

A fuzzy number is a generalization of an ordinary real number. It refers to a connected set of possible values, where each possible value has its own weight between 0 and 1. Thus, a fuzzy number is a special case of a convex normalized fuzzy set in the space of real numbers. Among the possible types of fuzzy numbers, triangular and parabolic numbers are considered in the work.

A fuzzy number $\tilde{A} = (a, b, c)$ is called a triangular fuzzy number if its membership function looks like this:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x < a; \\ (x-a)/(b-a), a \le x \le b; \\ (c-x)/(c-b), b \le x \le c; \\ 0, x > c. \end{cases} \quad (4)$$

Above the triangular numbers (Fig. 1), you can determine the main arithmetic operations for further use in calculations.

Let $\tilde{A} = (a, b, c)$ and $\tilde{B} = (a1, b1, c1)$ be two triangular numbers. Then:

– The sum is defined as $\tilde{A} + \tilde{B} = (a+a1, b+b1, c+c1)$.

– The difference is defined as $\tilde{A} - \tilde{B} = A + (-\tilde{B}) = (a-c1, b-b1, c-a1)$, where $-\tilde{B} = (-c1, -b1, -a1)$ is defined as the opposite of B.

In other words, opposite triangular numbers and their sum and difference are also triangular numbers. It is also worth noting that the results of inversion and multiplication of triangular numbers do not preserve this property and do not always represent triangular numbers.

The parabolic number (Fig. 1) is given similarly and has the same properties, but has a different membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x < a, \\ -((x-b)/(a-b))^2 + 1, a \le x \le b, \\ -((x-b)/(c-b))^2 + 1, b \le x \le c, \\ 0, x > c. \end{cases} \quad (5)$$

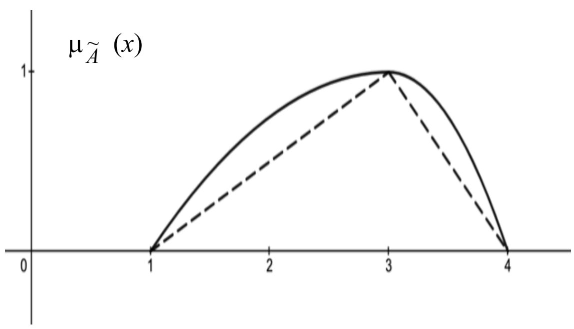

Figure 1 – Example of triangle and parabolic fuzzy number

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Although uncertainty information can be formalized using fuzzy numbers, the decision-making procedure must be precise. For example, the final output of fuzzy systems and the selection of appropriate solutions should be justified on the basis of the value characterized by the confidence (importance) indicator. To obtain a clear value, methods of calculating ranks (defuzzification) of fuzzy numbers are used, which are essentially "crisp" representative numbers, and can be used as generalized values for further calculations. One of the methods for calculating the rank of a fuzzy number is the Jaeger method, which calculates the Jaeger rank of the first type in the form [12]:

$$F1(\tilde{A}) = \frac{\int_0^1 g(x)\mu_{\tilde{A}}(x)dx}{\int_0^1 \mu_{\tilde{A}}(x)dx}, \tag{6}$$

where $g(x)$ is a weight function that measures the importance of the value of x. If $g(x) = x$, the index can be considered as the geometric center $\tilde{A}$, as shown in Figure 1. The support of the fuzzy number in this case is the segment [0, 1]. If the reference sets of the fuzzy numbers being compared do not coincide with [0, 1], then they can be scaled by dividing each of the numbers by max[sup $S_{\tilde{A}_i}$], where $S_{\tilde{A}_i}$ denotes the i-th reference set fuzzy number. Using this scaling procedure will give a factor of 1/ max[sup $S_{\tilde{A}_i}$] (1, if no scaling is used). The limits of integration in this case will be min[inf $S_{\tilde{A}_i}$] and max[sup $S_{\tilde{A}_i}$], respectively.

If $g(x) = x$ and the fuzzy number is triangular, the index $F1$ reduces to a simpler form:

$$F1(\tilde{A}) = \frac{1}{3}(a+b+c), \tag{7}$$

and in the case of a parabolic number:

$$F1(\tilde{A}) = \frac{1}{8}(3a+2b+3c), \tag{8}$$

where $a = inf\ S_{\tilde{A}_i}$, $\mu_{\tilde{A}}(b) = 1$, $c = \sup\ S_{\tilde{A}_i}$.

This method of defuzzification is also called the method of the center of gravity (COG) [13] (Fig. 2). Among other well-known methods, it is also worth noting the bisector of area (BOA) method [14], according to which there is such a value of x that a vertical line drawn through it divides the fuzzy number into two equal parts by their area.

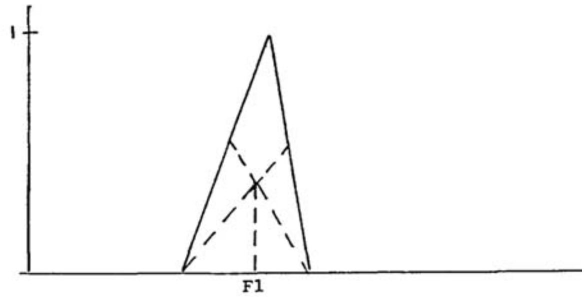

Figure 2 – Defuzzification by method of the gravity center

4 EXPERIMENTS

In the course of the study, the described algorithm was implemented based on the annealing method using fuzzy numbers to represent the subjective perception of the passage of time on road sections between cities. A multi-threaded Python implementation is proposed for numerical calculations. In the process of work, the method of calculating the rank of fuzzy numbers is chosen and the rank values of different methods (use of the peak abscissa, BOA, COG) are compared with the average value of random route passes. It was concluded that the best result was obtained by the center of gravity (COG) method. A comparison of the route estimation methods is given in the table, where the random route characterizes the time of travel along the route taking into account the average speed, the calculated route is the duration estimate obtained by the chosen method (see Table 1).

Table 1 – Rezults of the route estimation

Method	Random route	Calculated route
The peak abscissa	5367.78	5046.0
BOA (triangular FN)	5332.54	5291.72
BOA (parabolic FN)	5369.91	5341.70
COG (triangular FN)	5332.40	**5332.86**
COG (parabolic. FN)	5369.67	**5368.72**

During the program processing, three possible approaches to finding solutions are compared (using crisp, triangular, and parabolic numbers, respectively). As initial conditions, the TSPLib library was used, which has known TSP conditions in its catalog in the form of arrays of coordinates or matrices of the conditional distances between cities (e.g. u16, fr4, etc.). Fuzzy initial conditions were randomly generated with possible deviation from the expected value in either direction. To test the proposed approach, the time of the best constructed results for each type of fuzzy numbers was compared with the average value of the time taken for 10^5 random passes along the constructed route.

5 RESULTS

The results of the numerical experiments are shown in the Table 2, in which the best solutions are defined with actual time for trip for the different conditional distances between cities.

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Table 2 – Rezults of the method's comparison

Task	Fuzzy number type	Estimated time	Actual time
u16	crisp	6859.0	7247.38
	triangular	6859.0	7129.38
	parabolic	6859.0	**7128.92**
fr4	crisp	5046.0	5369.57
	triangular	5071.0	5369.30
	parabolic	5070.0	**5363.88**
pr76	crisp	108273.0	115105.75
	triangular	108894.0	**114102.55**
	parabolic	109295.0	114563.41
rd100	crisp	8185.0	8653.50
	triangular	7975.0	**8390.85**
	parabolic	8049.0	8447.22
rd400	crisp	18070.0	19087.33
	triangular	18089.0	19008.33
	parabolic	17808.0	**18713.72**

Thus, it was concluded that the use of fuzzy numbers in the annealing algorithm allows to obtain constructive results when solving the traveling salesman problem with fuzzy input parameters.

6 DISCUSSION

Given that the "cost" of travel between cities in time measurement can vary depending on the situation, a more accurate representation of such cost can be given in the form of triangular or parabolic numbers. If the subjective perception of time is chosen as the value, the relative duration of the trip between cities may vary depending on the factors affecting the path – traffic jams, bad weather, etc. Note that even in a simpler perception of the dynamic duration of the road between cities, when the actual time required to cover the path at the recommended average speed is measured, the same factors change the given duration, and therefore it makes sense to represent the studied travel time in the form of triangular or parabolic numbers.

Using one of the combinatorial methods of the approximate solution of the traveling salesman problem in combination with fuzzy numbers (and the corresponding method of calculating their rank), it is possible to achieve an effective result from the construction of the optimal path taking into account the dynamic features of roads between destinations. At the same time, better calculations can be obtained when using fuzzy parabolic numbers, since their essence is closer to reality. For the subjective overestimation or underestimation of the perception of the passage of time, the rule is valid: the greater the possible deviation in perception, the less likely it is to be obtained. For the numerical implementation of the actions of the annealing algorithm on fuzzy numbers that determine the time perception of the duration of movement between cities, operations according to the above schemes are used, and the different routes formed at the same time are compared with each other by finding and comparing the ranks of fuzzy numbers using one of the specified methods.

CONCLUSIONS

This paper investigates the use of fuzzy numbers and the annealing method to find a solution to the traveling salesman problem, which involves finding the shortest route for a given set of cities. Fuzzy numbers are used to model the inaccuracy and uncertainty of input data, and an annealing method is proposed to find solutions. The solutions obtained on the basis of the developed program in the Python language were analyzed. A comparison of the results of the TSP problem using crisp and fuzzy numbers using the annealing method was carried out. The results of numerical experiments are given, which show that the use of fuzzy numbers, in particular triangular and parabolic, with the annealing method leads to a significant improvement in the results of the TSP problem compared to the use of crisp numbers. This approach can be applied to real-world optimization problems involving imprecise or uncertain data and can be useful for optimizing processes with subjective time perception. A conclusion was made about the need for further research using the theory of fuzzy numbers, in particular in the direction of the correct choice of the type of numbers in accordance with the conditions of the task. Another direction of research involves further development of the proposed methodology for solving fuzzy dynamic traveling salesman problems and the use of other effictive (for example, genetic), approximate and greedy methods.

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ПРО ВИКОРИСТАННЯ МЕТОДУ ВІДПАЛУ ДЛЯ РОЗВ'ЯЗАННЯ ЗАДАЧІ КОМІВОЯЖЕРА З НЕЧІТКИМ СПРИЙНЯТТЯМ ЧАСУ

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АНОТАЦІЯ

Актуальність. Інтеграція нечітких чисел в алгоритми має вирішальне значення для вдосконалення обчислювальних методологій. Нечіткі числа з властивою їм неточністю пропонують більш реалістичне уявлення про явища реального світу. Адаптація та інноваційні алгоритми для включення нечітких чисел є важливими для вирішення складних проблем, коли дані можуть бути неточними або неоднозначними. Це вдосконалення допомагає більш обгрунтовано приймати рішення зважаючи на тонкощі реального світу, що у свою чергу сприяє прогресу в різних сферах і дозволяє проводити дослідження у контексті суб'єктивного сприйняття часу.

Ціль. Мета роботи – розробити алгоритм розв'язання задачі комівояжера з використанням нечітких чисел для формалізації невизначеності та неточності вхідних даних, пов'язаної з впливом суб'єктивності в оцінках тривалості необхідних проміжків часу.

Метод. У статті розглянуто метод відпалу з нечітким представленням часу для розв'язання нечіткої задачі комівояжера, що формулюється як задача знаходження маршруту відвідування заданої кількості міст без повторень з мінімальною тривалістю руху з нечіткими числами, що представляють час, необхідний для подолання відстаней між містами. Поставлено та вирішено задачу формалізації алгоритму розв'язання проблеми комівояжера на основі методу відпалу з використанням нечітких чисел. Запропоновано можливі методи апроксимації нечітких чисел в контексті поставленої задачі. Розроблено конструктивний алгоритм розв'язання задачі. Проведено обчислювальні експерименти.

Результати. Розроблено метод розв'язання задачі комівояжера з використанням методу відпалу та нечітких чисел. Запропоновано використання нечітких чисел для формалізації невизначеності та неточності вхідних даних, пов'язаної з впливом суб'єктивності в оцінках тривалості необхідних проміжків часу. Представлено результати розрахунків за допомогою запропонованого алгоритму в задачах комівояжера з мінімізацією суб'єктивної тривалості руху, показано можливі методи апроксимації нечітких чисел та їх порівняння в контексті поставленої задачі, проведено порівняння отриманих розв'язків із розв'язками, знайденими за допомогою інших евристичних методів.

Висновки. У статті розглянуто метод формалізації алгоритму розв'язання задачі комівояжера з використанням алгоритму методу відпалу та нечітких чисел. Запропоновано використання нечітких чисел для формалізації невизначеності та неточності вхідних даних, пов'язаної із впливом суб'єктивності в оцінках тривалості необхідних проміжків часу. Описано схему формалізації процедури використання методу відпалу з нечіткими числами, що представляють суб'єктивне представлення часу, необхідного для подолання відстаней між містами.

КЛЮЧОВІ СЛОВА: задача комівояжера, нечіткі числа, метод відпалу, комбінаторна оптимізація, суб'єктивне сприйняття плину часу, неточність, невизначеність.

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