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THE STATES' FINAL PROBABILITIES ANALYTICAL DESCRIPTION IN AN INCOMPLETELY ACCESSIBLE QUEUING SYSTEM WITH REFUSALS AND WITH INPUT FLOW OF REQUIREMENTS' GROUPS

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ABSTRACT

Context. The basis for the creation and management of real queuing systems (QS) is the ability to predict their effectiveness. For the general case of such systems with refusals, with limited approachability of service devices and with a random composition of group requirements in the input flow, the prediction of their performance remains an unsolved problem.

Objective. The research has the aim to find an analytical representation for final probabilities in the above-mentioned case of Markov QS, which allows us to predict the efficiency of its operation depending on the values of the parameters in its structure and control.

Method. For the above-mentioned types of QS, the state probabilities can be described by a system of Kolmogorov's differential equations, which for the stationary case is transformed into a homogeneous system of linearly dependent algebraic equations. For real QS in communication systems, the number of equations can be estimated by the degree set and amount to several thousand, which gives rise to the problem of their recording and numerical solution for a specific set of operating conditions parameters values. The predictive value of such a solution does not exceed the probability of guessing the numerical values of the QS operating conditions parameters set and for parameters with a continuous value, for example, for random time intervals between requests, is zero.

The method used is based on the analytical transition to the description of QS states groups with the same number of occupied devices. At the same time, the desire to obtain the final probabilities of states in a form close to the Erlang formulas remains. The influence of the above-mentioned QS properties can be localized in individual recurrent functions that multiplicatively distort Erlang formulas.

Results. For the above-mentioned types of QS, analytical calculation formulas for estimating the QS states final probabilities have been found for the first time, which makes it possible to predict the values of all known indicators of system efficiency. In this case, the deformation functions of the states groups' probability distribution in QS have a recurrent form, which is convenient both for finding their analytical expressions and for performing numerical calculations.

When the parameters of the QS operating conditions degenerate, the resulting description automatically turns into a description of one of known QS with failures, up to the Erlang QS.

Conclusions. The analytical calculation expressions found for the final probabilities of the above-mentioned QS turned out to be applicable to all types of Markov QS with failures, which was confirmed by the results of a numerical experiment. As a result, it became possible to practically apply the obtained analytical description of the considered QS for operational assessments of developed and existing QS effectiveness in the possible range of their operating conditions.

KEYWORDS: Markov queueing systems, requirements' groups.

ABBREVIATIONS

AAMF is an anti-aircraft missile fire zone;

MAA is an enemy means of air attack;

$M_L/M/n$ is a designation of a QS with a Poisson input flow of requirements groups with random composition, with maximum number (L) of requirements in a group (M_L), with an exponential distribution of random service time for each requirement (M), with number (n) of identical service channels (devices);

$M/M/n$ is a Kendall-Basharin classification for QS with failures;

No. i, j is a cell address in Table 1: i-row number, j-column number;

QS is a designation for queuing system;

SAM is a designation for an anti-aircraft missile system;

NOMENCLATURE

b_r is a probability designation of occurrence in the input flow of QS a request consisting of exactly r requirements;

C_n^m is a number of combinations from n to m;

d_k^q is a probability of transition to the k-th level of the model graph of queuing system by jumping through q tiers of the graph;

$f_1()$ is a notation for the input density function of the demand flow;

$f_2()$ is a notation for the service duration distribution density function;

F_k is a probability deformation function, which deforms the Erlang probability P_k;

I is a designation for the intensity of the requirements input flow in QS;

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$i_1,..,i_n$ is a designation for the numbers of specific service devices;

$k_1 k_2$ are the numbers of channels that become occupied when moving from previous states to the state under consideration;

L is a designation for the maximum total requirements number in one request;

$M_{b.d}$ is a designation for the mathematical expectation of devices number engaged in servicing;

M_{in} is a designation for the mathematical expectation of the number of requirements in one request at the QS input;

n is a designation for the number of devices/channels in QS;

N_{missed} is a designation for the aircraft number mathematical expectation that have penetrated the air defense system with impunity;

$N_{total.En}$ is an enemy aircraft total number in a blow;

P_k is a notation for the QS state probability in which are occupied exactly k devices;

p_{ijk} is a designation for such a QS state probability in which are engaged in servicing devices with numbers i, j, k;

$P_{service}$ is a designation for the probability of requirements service in QS (QS performance indicator);

$Q_{i_1 ... i_m}$ is a probability of the requirement falling within the service area of channels with $i_1,..,i_m$ numbers;

r is a designation for requirements number in one request;

s is a number of air defense missile systems not engaged in firing at enemy aircraft;

$S_{i_1 ... i_k}$ is one of possible QS states, in which channels with numbers $i_1,..,i_m$ are busy servicing;

T is a mathematical expectation of requirement's duration service time in service device;

v_i is a designation for the requirement starting service by one of the devices probability, given that i available devices are occupied already;

v_i^1 is a designation for the intensity of requirement starting service by one of the devices, given that i available devices are occupied already;

z_J is a total area of j-fold overlap of service channel accessibility zones;

z_Σ is a total service area of requirements flow in QS;

λ is a designation for the input requests flow parameter in QS;

λ_r is a designation for the requests partial flow parameter which consists of exactly r requirements in each request at the input of the QS;

μ is a designation for channel performance in QS;

ρ is a load factor of a service device in QS of $M/M/n$ type;

α_k^q is an intensity of the transition QS into a group of states with k occupied channels by a jump over q group states QS;

γ_i is a channel number occupied in the previous and current states;

β_i is a busy channel service number in a queuing system;

π_j is a probability of a requirement falling into the total accessibility zone of exactly j service channels;

ξ_m is a designation for the channels' groups maximum number m out of n channels' total number;

ρ_q is a designation for the QS load factor by the partial λ_q incoming flow of requests.

INTRODUCTION

To ensure life, every person has to periodically satisfy their needs for food, clothing, communication services, transportation services, banking services, medical and other services. Needs for such services often arise at unexpected (random) moments in time. For a group of people within individual regions, such needs accumulate, generating continuous flows of typical demands for specific services.

For the noted conditions, the properties of such random-time flows of events were investigated by A. Ya. Khinchin [1], and he proved the statement about the asymptotic emergence of a set of mathematical features in these flows, which received the name "simplest" flow of events.

Due to the repetitive nature of service requirements, systems for processing and satisfying such requirements are created in each specific area of activity to meet them. The creation of systems requires preliminary calculations of many parameters values for each system, which determined the need for the emergence of calculation models, called queuing system (QS) models. For conditions with the simplest input flow of demands, Markov models of QS with failures are known, where if all devices are busy at the moment the next demand arrives, this demand is rejected and leaves the QS without being serviced.

There are many different conditions for the receipt and servicing of requirements for which the necessary QS models are developed. Thus, at the moment of admission, a patient at a clinic may not find a free doctor of the required specialization, a driver at a gas station may not find the required type of fuel, an enemy aircraft may fly through the air defense group's fire zone with impunity. In all such cases, at the moment of the requirement receipt, there may be free service devices in the system, but these devices are not available for service.

In some cases, the system input may receive requirements not one by one, but in groups with a composition that is not known in advance.

Thus, during an epidemic or during military operations, patients may be admitted to medical

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institutions in groups. Enemy aircraft, as a rule, carry out missions as part of tactical groups that may enter the fire zone of an air defense group.

As a result, shock loads arise in mass service systems and the efficiency of the systems decreases.

When developing such systems, it becomes necessary to have a calculation scheme (a model) of its parameters in which both properties – the flow of groups of requirements and the partial availability of service channels – must be taken into account simultaneously.

For any real QS with the noted features, the development of necessary model is possible in the form of a system of Kolmogorov differential equations and the corresponding algebraic equations for the stationary operating mode of the QS. The solution of equations system is possible only by numerical methods. In this case, the structure of equations must correspond to the value of operating conditions parameters.

It is possible to guess the future values of some random parameters of working conditions with a probability strictly equal to zero, which reduces the predictive value of calculations and makes it relevant to develop an analytical description of the required model.

The first analytical description of a single-link switching non-fully accessible QS model was obtained in the theory of teletraffic [2] (the third Erlang formula). For the case of group requests entering the system, an analytical model was developed in [3]. For a multi-channel non-fully accessible *M*/*M*/*n* QS, an analytical model was developed in [4] and can be useful for developing an analytical model that simultaneously takes into account two noted features of the QS operating conditions.

The research object is a stationary service process in a queuing system $M_L/M/n$ with refusals, with the entry into the system requirements groups with a previously unknown composition and with incomplete accessibility of service devices for the incoming flow of requirements.

The research subject is the probability distribution law of states groups in incompletely accessible QS of the $M_L/M/n$ type in a stationary mode.

The research purpose is to obtain an analytical description of the final probabilities of states groups in incompletely accessible QS of $M_L/M/n$ type, with a simultaneous assessment of its correctness.

1 PROBLEM STATEMENT

A flow of requests (requirement groups) with intensity I and density $f_1(t) = Ie^{-It}$ enters non-fully accessible QS of $M_L/M/n$ type.

Each service channel in non-fully accessible QS can be a part of one or more channel groups. Each group of r ($r = 1, ..., L$) requirements represents a service request and must be serviced using r service devices.

One of the accessible device groups is selected to service the next request in the input flow. In the selected channel group, any free channel is assigned to service each requirement in the request with equal probability.

In case of insufficient number of free channels some of requirements from this request are denied service and leave the system. The service duration of each requirement is random and has an exponential distribution $f_2(t) = \mu e^{-\mu t}$.

The considered distribution densities allow us to assert the possibility of describing the desired model in the class of Markov processes with discrete states and continuous time.

For the sake of brevity, we will further use the well-known statements [1, pp. 14, 40, 41] on the properties of a stationary non-ordinary flow of requirements.

A stationary flow of time moments without after effects, in which groups of events appear, is called a non ordinary or universal stationary flow and has the properties of the simplest flow. A non ordinary flow includes requests of r requirements ($r = 1, 2, ..., L$) in a request.

Such a flow can be defined by specifying the probability distribution law (b_r) of the occurrence of exactly r requirements in any group (in any request) of the input flow. The intensity of requirements groups flow turns out to be greater than the flow parameter ($I, \lambda < I$), which contains piecemeal flows with parameters λ_r :

$$\lambda_r = \lambda b_r,\ r = \overline{1, L};\ M_{in} = \sum_{r=1}^{L} r \cdot b_r;$$
$$\lambda = \sum_{r=1}^{L} \lambda_r;\ I = \sum_{r=1}^{L} r \cdot \lambda_r. \tag{1}$$

In a non ordinary flow of requirements at the QS input, time intervals between requests are random and satisfy the conditions of A. Ya. Khinchin limit theorem [1], have an exponential distribution with the parameter λ :

$$f(t) = \lambda e^{-\lambda t}, \qquad t > 0. \tag{2}$$

The following well-known [4] logic of operation of the same QS, but with the simplest input flow of requirements, can be used as the basis for constructing the $M_L/M/n$ model with incomplete accessibility of service devices, an example of which, for the variant of a city polyclinic operation, is given in [4].

To demonstrate the physical processes, involved in constructing a model for servicing the simplest flow of requirements by a non-fully accessible system, [4] examines a simplified example of the operation of three SAM systems group, located on the ground (Fig. 1) when firing at the simplest flow of enemy aircraft.

Each SAM system (No. 1, No. 2, No. 3) has a firing zone, projected onto the earth’s surface, in the form of a

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circle (Fig. 1 Q_1, Q_2, Q_3). In the general fire zone of SAM systems there are areas with mutual overlapping fire zones of neighboring SAM systems, which in the service system form a group of devices.

Each SAM system (service device) can be a member of several service groups. To indicate the probability that the next requirement (enemy aircraft) will be accessible to a specific group of *m* devices, a designation indicating the numbers of these devices is used $Q_{i_1 \ldots i_m}$.

In the example (Fig. 1) such probabilities for SAM systems fire zones are indicated by symbols Q_1, Q_2, Q_3, Q_{12}, Q_{23}, Q_{13}, Q_{123}.

When an aircraft enters such an area, the aircraft can be fired upon (servicing the requirement) by any of the adjacent SAM systems.

The group's maximum number ξ_j from *j* service devices out of the total number of *n* service devices coincides with the number of combinations C_n^j from *n* by *j* (3):

$$\xi_j = C_n^j = \frac{n!}{j! \cdot (n-j)!}, \; 0 \le j \le n. \tag{3}$$

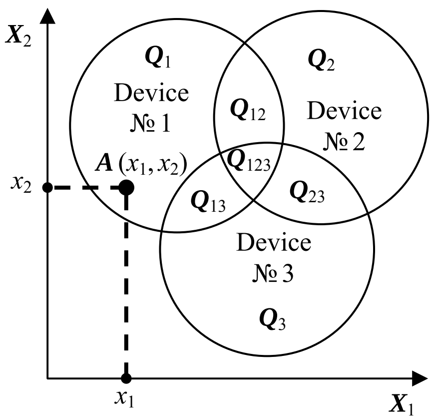

Figure 1 – Visualization of the principles for choosing an affordable device to serve the next requirement [4]

Within a constant number *j*, the observed probability remains approximately the same for different groups of devices:

$$Q_{i_1 \ldots i_j} = l_j. \tag{4}$$

The probability l_J estimation method for the example of a city polyclinic operation is given in [4]. For a simplified example of SAM systems grouping, the relative value π_j of the total area z_J of the region with *j*-fold overlap of fire zones can be used:

$$\pi_j = \frac{z_j}{z_\Sigma}; \quad 0 \le j \le n. \tag{5}$$

On the other hand, the probability that the next requirement (enemy aircraft) will be accessible for servicing by any of groups of *j* devices (SAM systems), is equal to the probabilities sum:

$$\pi_j = \sum_{i=1}^{\xi_j} l_j = \xi_j \cdot l_j = C_n^j \cdot l_j, \quad 0 \le j \le n. \tag{6}$$

From expressions (3) and (4) we find the probability estimate l_J:

$$l_j = \frac{z_j}{C_n^j \cdot z_\Sigma} = \frac{\pi_j}{C_n^j}, \quad 0 \le j \le n. \tag{7}$$

In [4] an example of a *M*/*M*/*n* QS graph (Fig. 2) with incomplete availability of service channels is considered in detail and a basic equality [4, formulas (23), (29)] is found for finding the final probabilities of group states of a non-fully accessible QS with refusals (8):

$$\begin{aligned} & k \cdot P_k = P_{k-1} \cdot C_{n-k+1}^1 \cdot \rho \cdot v_{k-1}, \qquad k = 1, 2, ..., n; \\ & \rho = \frac{I}{\mu}; \text{ then } \; k\mu P_k = P_{k-1} \cdot C_{n-k+1}^1 \cdot \alpha_k^1, \; k = \overline{1,n}; \\ & v_k^1 = I \cdot \sum_{j=1}^{n} \left(l_j \cdot \sum_{i=0}^{j-1} \frac{1}{C_{i+1}^1} C_{k-1}^{j-1-i} C_{n-k}^i \right), \; 0 \le k \le n. \end{aligned} \tag{8}$$

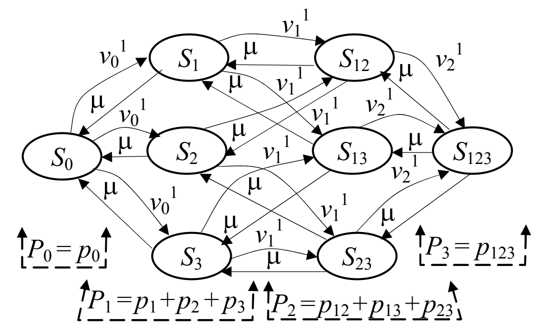

Figure 2 – Graph of *M*/*M*/3 QS model with refusals and with incomplete accessibility of service devices [4]

In [4], the correctness of the obtained expressions for the final probabilities was also proven by their automatic transition to the well-known Erlang formulas [5] for the *M*/*M*/*n* system with refusals.

In [3, formula (17)], a basic equality was found for searching for the final probabilities P_k of group states in QS with a non ordinary requirements groups' input flow:

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$$\rho_i = \frac{1}{\mu}\sum_{j=1+i}^{L}\lambda_j, \ 0 \le i < L;$$

$$\boldsymbol{k}P_k = \rho_0 P_{k-1} + \rho_1 P_{k-2} + ... + \rho_{k-1}P_0; \quad (9)$$

$$\mu \boldsymbol{k} P_k = \sum_{j=1}^{k} P_{k-j} \cdot \sum_{q=j}^{L}\lambda_j .$$

In [3] the correctness of the obtained expressions for the final probabilities is also shown by their automatic transition to the well-known Erlang formulas [5] for the *M*/*M*/*n* system with refusals.

The problem is to find analytical expressions for final probabilities P_k, ($k = 1, 2, …, n$) of states groups' in the service process, in which in the $M_L/M/n$ system, with incomplete accessibility of service devices there are exactly *k* requirements. The marked probabilities of states group's in QS, in which exactly *k* channels are occupied with servicing, allow us to find the servicing probability $P_{service}$ of requirements.

Thus, the problem of predicting the efficiency of real queuing systems with partially accessible devices with refusals arises in the presence of a requirements groups' flow with random composition at the input of the system.

2 REVIEW OF THE LITERATURE

The first model of a queuing system was developed by A.K. Erlang [5] in 1909 to describe the operation of a telephone station. The process of each requirement servicing from the next subscriber consisted of connecting his communication channel to the communication channel with another subscriber. After the end of the communication session, the channels were freed and it became possible to use them for service requests from other subscribers. If a requirement was received from a subscriber at a time when all communication channels were busy, such a requirement from the subscriber was denied service. The moments of requirements receipt time and end of communication sessions were not known in advance and were considered random in the model.

Processes of servicing requirements with an unknown start and end time also occur in other systems and areas of human activity. These include logistic systems [6], production systems [7], telecommunication networks [2], systems for management in medicine [8], systems for traffic management [9], and systems for the defense of objects from air blows [10] as well as socio-economic systems [11]. In each of these areas, the Erlang model could be used either directly or with modifications that were necessitated by the peculiarities of the processes in a particular area. Thus, the closest to the description of processes in the management of medical institutions and systems for protecting objects from air strikes were models [4] and [3], which separately took into account the properties of incomplete accessibility of service devices and the group composition of requirements in service requests, respectively. Each of these properties necessitated a transformation of the Erlang model, which made it possible to assess the directions in changes in the efficiency of the corresponding systems. However, both of the noted properties can simultaneously occur in service processes, which necessitated further modification of the Erlang model [5].

3 MATERIALS AND METHODS

For the sake of certainty, we will consider the construction of the desired service model using the example of repelling attack of enemy aircraft of different composition and purpose (flow of requests at the input of the service system) by group of *n* SAM systems deployed on the ground, which, in this case under consideration, is *n*-channel not fully accessible queuing system. In the general zone of anti-aircraft missile fire of the SAM systems group there are areas with *j*-th layer of fire zone (Fig.1). At each such section, the enemy aircraft can be fired upon by any of the *j* SAM systems (serviced by any of the *j* devices). Each section can correspond to its own set of specific SAM systems (service devices).

The enemy MAA strike passes through the SAM systems grouping fire zone. The enemy MAA combat formations contain MAA groups of different purposes according to $r = 1, ..., L$ MAA in the group (a non ordinary flow of requirements at the input of the queuing system).

If a group, consisting of *r* MAAs (*r*-group) enters a section of the AAMF zone with *j*-th layering, in which *s* SAM systems are free $s \le j$, then at $r > s$ exactly *s* the MAAs will begin to be fired upon (will be serviced), and $r - s$ MAAs will pass through the AAMF zone without impact (are denied in service). There is reason to assume that the probability of the *r*-group MAA getting into any of the AAMF regions with the *j*-th layering is the same and equal to l_j. Then the probability π_j of getting into one of the zones with the *j*-th layering is found (3)–(7), taking into account all the accessible zones:

$$\pi_j = \xi_j \cdot l_j, \quad 0 \le j \le n. \quad (10)$$

In turn, from (10) and (3) it follows:

$$l_j = \frac{\pi_j}{C_n^j}, \quad 0 \le j \le n. \quad (11)$$

For the noted conditions, the combat model of the SAM systems group can be represented by a model of incompletely accessible queuing system of a non ordinary flow of requests.

Statement for a mathematical problem. A non ordinary Poisson flow of requirements, grouped into requests, with the parameter λ enters QS $M_L/M/n$ with non-fully accessible channels and with the vector of probabilities $\{\pi_j\}$ for suitability of requirements to service devices. The flow of requirements is a superposition of independent partial flows, each of which is characterized

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by a constant number r of requirements in a request ($r = 1, ..., L$) and a parameter λ_r of the exponential distribution of time intervals between neighboring requests:

$$\lambda_r = \lambda \cdot b_r, \quad r = 1, ..., L . \tag{12}$$

Each request (group of requirements) can be assigned to any of the service zones, which has its own individual set of channels with specific numbers $j_1, ..., j_k$ capable of servicing requirements in that zone. Each channel spends on average T minutes on a service cycle and has a productivity of $\mu = T^{-1}$.

Any of the accessible free channels is selected for servicing without any preferences, that is, with a probability inversely proportional to the number of free accessible service channels. Some of the requirements, for which there are free service channels in the zone under consideration, are started to be serviced and remain in the system until the end of the service cycle. Requirements, for which there are no free channels in the zone under consideration, leave the system unserviced (are denied service). One channel can service only one requirement at a time.

At any given moment in time, the system can be in one of the possible states $S_{i_1 ... i_k}$ with specific numbers $i_1, .., i_k$ and the number k of channels occupied by servicing. The set of all possible states with k occupied channels forms a group state S_k of the system, is called a tier of the model graph and contains C_n^k possible internal states.

The problem is to determine analytical expressions for the final probabilities P_k, $k = 0, ..., n$ of the service system's group states S_k .

Solution. A fragment of the model's graph, under consideration, with the number of service channels $n = 4$ is shown in Figure 3, where only representative arcs of typical states $S_{i_1 ... i_k}$ for each tier of the model graph are shown. The intensities of transitions along these arcs are marked in the breaks of the arcs. The return from the state $S_{i_1 ... i_k}$ of the k-th tier to the ($k-1$)-th tier occurs in the same states from which it was possible to get to the states $S_{i_1 ... i_k}$ of the k-th tier.

Further, for the convenience of analysis, the hypothesis is adopted that $L \geq n$, which does not reduce the generality of the analysis, since it is always possible to include in the flow of requirements the missing probabilities b_r, setting them equal to zero.

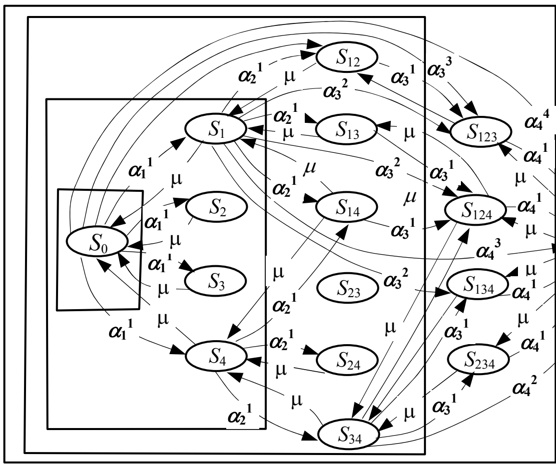

Figure 3 – Fragment of the service model graph of a non ordinary flow of requirements by a non-fully accessible QS with refusals

For the intensities of transitions to states of tiers with a large value of k, the notation α_k^q is used, in which the lower index denotes the absolute number of the tier to which the transition arc leads, and the upper index indicates the number of tiers traversed by the arc, including the tier of the final state of the model. For example, the designation α_3^2 shows the intensity with which transitions to a specific state of the 3rd tier follow from a specific state of the 1st tier (Fig. 3, an example of a transition from state S_1 to state S_{134}).

In order to understand the order of transition intensities formation, we will use the designation $Q_{i_1 ... i_k}$ of a requirements group probability of falling into the service area of specific group of channels and compose expressions for transition intensities. Let us temporarily introduce notations, indicating the numbers of occupied channels in the initial state of the model graph and in the final state.

$$\begin{gathered}
\alpha_{0\to1}^1 = \lambda_1 (Q_1 + \frac{1}{2} \cdot (Q_{12} + Q_{13} + Q_{14}) + \\
+ \frac{1}{3} \cdot (Q_{123} + Q_{124} + Q_{134}) + \frac{1}{4} \cdot Q_{1234}) + Q_1 \cdot \sum_{r=2}^{L} \lambda_r ; \\
\alpha_{1\to12}^1 = \lambda_1 (Q_2 + Q_{12} + \frac{1}{2} \cdot (Q_{23} + Q_{24}) + \\
+ \frac{1}{2} \cdot (Q_{123} + Q_{124}) + \frac{1}{3} \cdot Q_{234} + \frac{1}{3} \cdot Q_{1234}) + \\
+ (Q_2 + Q_{12}) \cdot \sum_{r=2}^{L} \lambda_r ;
\end{gathered} \tag{13}$$

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$$\alpha^1_{12\to123}=\lambda_1(Q_3+Q_{23}+Q_{13}+\frac{1}{2}Q_{34}+$$
$$+Q_{123}+\frac{1}{2}\cdot(Q_{134}+Q_{234})+\frac{1}{2}\cdot Q_{1234})+$$
$$+(Q_3+Q_{13}+Q_{23}+Q_{123})\cdot\sum_{r=2}^{L}\lambda_r;$$
$$\alpha^1_{123\to1234}=\lambda_1(Q_4+Q_{14}+Q_{24}+Q_{34}+$$
$$+Q_{124}+Q_{134}+Q_{234}+Q_{1234})+(Q_4+Q_{14}+$$
$$Q_{24}+Q_{34}+Q_{124}+Q_{134}+Q_{234}+Q_{1234})\cdot\sum_{r=2}^{L}\lambda_r.$$

Taking into account (4), that is, the hypothesis about the equal probability of requests (groups of requirements) getting into any service zone with the same number j of accessible service channels, we find that the intensities of transitions between the states of two different tiers are the same for all arcs of one direction, connecting the states of these tiers and are equal:

$$\alpha^1_{0\to1}=\alpha^1_{0\to2}=\ldots=\alpha^1_1=$$
$$=\lambda_1\cdot(l_1+\frac{3}{2}l_2+l_3+\frac{1}{4}l_4)+l_1\cdot\sum_{r=2}^{L}\lambda_r;$$
$$\alpha^1_{1\to12}=\alpha^1_{1\to13}=\ldots=\alpha^1_2=$$
$$=\lambda_1\cdot(l_1+2l_2+\frac{4}{3}l_3+\frac{1}{3}l_4)+(l_1+l_2)\cdot\sum_{r=2}^{L}\lambda_r;$$
$$\alpha^1_{12\to123}=\alpha^1_{12\to124}=\ldots=\alpha^1_3=\lambda_1\cdot(l_1+ \quad (14)$$
$$+\frac{2}{5}l_2+2l_3+\frac{1}{2}l_4)+(l_1+2l_2+l_3)\cdot\sum_{r=2}^{L}\lambda_r;$$
$$\alpha^1_{123\to1234}=\alpha^1_{124\to1234}=\ldots=\alpha^1_4=\lambda_1\cdot(l_1+$$
$$+3l_2+3l_3+l_4)+(l_1+3l_2+3l_3+l_4)\cdot\sum_{r=2}^{L}\lambda_r.$$

The obtained expressions (14) in the general case take the following form:

$$\alpha^1_k=\lambda_1\cdot\sum_{j=1}^{n}l_j\cdot\sum_{i=0}^{j-1}\frac{1}{C^1_{i+1}}\cdot C^{j-1-i}_{k-1}\cdot C^i_{n-k}+$$
$$+(\sum_{j=0}^{k-1}l_{j+1}\cdot C^j_{k-1})\cdot\sum_{r=2}^{L}\lambda_r\,. \quad (15)$$

Reasoning similarly for the graph arcs that transfer the service process through one state, we find:

$$\alpha^2_{0\to12}=\lambda_2(Q_{12}+\frac{1}{3}\cdot Q_{123}+\frac{1}{3}Q_{124}+$$
$$+\frac{1}{6}Q_{1234})+Q_{12}\cdot\sum_{r=3}^{L}\lambda_r\,;$$
$$\alpha^2_{1\to123}=\lambda_2(Q_{23}+Q_{123}+\frac{1}{3}Q_{234}+ \quad (16)$$
$$+\frac{1}{3}Q_{1234})+(Q_{12}+Q_{123})\cdot\sum_{r=3}^{L}\lambda_r\,;$$
$$\alpha^2_{12\to1234}=\lambda_2(Q_{34}+Q_{134}+Q_{234}+$$
$$+Q_{1234})+(Q_{34}+Q_{134}+Q_{234}+Q_{1234})\cdot\sum_{r=3}^{L}\lambda_r.$$

Taking into account equality (4), expressions (16) will take the following form:

$$\alpha^2_{0\to12}=\alpha^2_{0\to13}=\ldots=\alpha^2_2=$$
$$=\lambda_2\cdot(l_2+\frac{2}{3}l_3+\frac{1}{6}l_4)+l_2\cdot\sum_{r=3}^{L}\lambda_r;$$
$$\alpha^2_{1\to123}=\alpha^2_{1\to124}=\ldots=\alpha^2_3=$$
$$=\lambda_2\cdot(l_2+\frac{4}{3}l_3+\frac{1}{3}l_4)+(l_2+l_3)\cdot\sum_{r=3}^{L}\lambda_r; \quad (17)$$
$$\alpha^2_{12\to1234}=\alpha^2_{13\to1234}=\ldots=\alpha^2_4=$$
$$=\lambda_2\cdot(l_2+2l_3+l_4)+(l_2+2l_3+l_4)\cdot\sum_{r=3}^{L}\lambda_r\,.$$

However, in this case, the general expression for the transition intensity α^2_k is not obvious. To derive such an expression, we introduce a model of one state at the k-th level of the graph (Fig. 4).

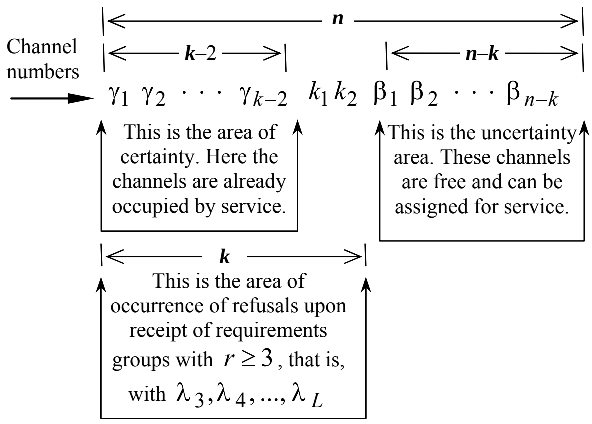

Figure 4 – Model of a single state with k busy channels on the k-th tier of none-full-accessible QS's graph

In this state $\gamma_1\gamma_2\ldots\gamma_{k-2}k_1k_2$, the service channels are busy and the $\beta_1\beta_2\ldots\beta_{n-k}$ channels are free. The

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edge with the desired transition intensity α_k^2 leads from the state with $\gamma_1\gamma_2...\gamma_{k-2}$ busy channels (the symbol γ_i can correspond to any number of the real service channel, provided that it occurs once in the set $\gamma_1\gamma_2...\gamma_{k-2}$) to the marked state $\gamma_1\gamma_2...\gamma_{k-2}k_1k_2$.

Thus, in the analyzed state there are three types of channels:

γ_i – channels occupied in the previous and current states;

k_1k_2 – channels occupied during the transition from previous states to the state under consideration;

β_i – unoccupied channels.

The transition to the state $\gamma_1\gamma_2...\gamma_{k-2}k_1k_2$ can be caused by a partial flow λ_2 of requests consisting of 2 requirements, as well as by flows λ_r of requests with a large number of requirements ($r \geq 3$) in the request.

In the last case, these requests should not fall into areas with channel accessibility $\beta_1\beta_2...\beta_{n-k}$ (Fig. 4), since this will lead to the inclusion of β-channels and bypassing state $\gamma_1\gamma_2...\gamma_{k-2}k_1k_2$ in consideration.

From the composition of each mentioned request λ_r of flows ($r \geq 3$) only 2 requirements will get into the service channels k_1k_2, the remaining $(r-2)$ requirements of each such request will be lost. Let us denote the intensity of transitions caused by this part of the flows with "refusals" by the symbol α_{k**}^2.

At the same time, the flow requests λ_2 can also fall into the areas of β-channels. In this case, the probability of including the channels k_1k_2 with a simultaneous transition to the analyzed state $\gamma_1\gamma_2...\gamma_{k-2}k_1k_2$, although it decreases, but exists and should be considered.

We will first conduct the analysis for the flow of requests with the parameter λ_2, finding the intensity of transitions α_{k*}^2, and present the result of this analysis in a compact notation:

$$\begin{aligned}\alpha_{k*}^2 = \lambda_2 \cdot [l_2 \cdot 1 + l_3 \cdot \sum_{i=0}^{1} \frac{1}{C_{i+2}^2} \cdot C_{k-2}^{1-i} \cdot C_{n-k}^i + \\ + l_4 \cdot \sum_{i=0}^{2} \frac{1}{C_{i+2}^2} \cdot C_{k-2}^{2-i} \cdot C_{n-k}^i + \cdots + \\ + l_k \cdot \sum_{i=0}^{k-2} \frac{1}{C_{i+2}^2} \cdot C_{k-2}^{k-2-i} \cdot C_{n-k}^i + \cdots + \\ + l_n \cdot \sum_{i=0}^{n-2} \frac{1}{C_{i+2}^2} \cdot C_{k-2}^{n-2-i} \cdot C_{n-k}^i].\end{aligned} \quad (18)$$

It can be noted that the last term is nonzero only at $i = n-k$. The final expression for the transition intensity α_{k*}^2 will take the following form:

$$\alpha_{k*}^2 = \lambda_2 \cdot \sum_{j=1}^{n} l_j \cdot \sum_{i=0}^{j-2} \frac{1}{C_{i+2}^2} \cdot C_{k-2}^{j-2-i} \cdot C_{n-k}^i . \quad (19)$$

Let us consider the procedure for determining the second, "refusal" component α_{k**}^2 of the desired intensity.

When requests for streams with parameters $\lambda_r, r \geq 3$ are received, the transition to the state $\gamma_1\gamma_2\cdots\gamma_{k-2}k_1k_2$ is possible only when such a request enters an area accessible for servicing by channels k_1k_2 and inaccessible for β-channels.

The number of such accessible areas for each value l_j ($j = 1, ..., n$) of the overlap coefficients of the service devices accessibility zones is equal to:

$$\alpha_{k**}^2 = \left[\sum_{j=0}^{k-2} l_{j+2} \cdot C_{k-2}^j \right] \cdot \sum_{r=3}^{L} \lambda_r . \quad (20)$$

Combining (19) and (20) for $k = 2$ we find the desired transition intensity:

$$\begin{aligned}\alpha_k^2 = \lambda_2 \cdot \sum_{j=1}^{n} l_j \cdot \sum_{i=0}^{j-2} \frac{1}{C_{i+2}^2} \cdot C_{k-2}^{j-2-i} \cdot C_{n-k}^i + \\ + \left[\sum_{j=0}^{k-2} l_{j+2} \cdot C_{k-2}^j \right] \cdot \sum_{r=3}^{L} \lambda_r .\end{aligned} \quad (21)$$

Reasoning similarly and comparing expressions (15) and (21), we find the desired expression for the transition intensity in the general case:

$$\begin{aligned}\alpha_k^q = \lambda_q \cdot \sum_{j=1}^{n} l_j \cdot \sum_{i=0}^{j-q} \frac{1}{C_{i+q}^q} \cdot C_{k-q}^{j-q-i} \cdot C_{n-k}^i + \\ + \left[\sum_{j=0}^{k-q} l_{j+q} \cdot C_{k-q}^j \right] \cdot \sum_{r=q+1}^{L} \lambda_r , \ q = \overline{1,n}; \ k = \overline{1,n}.\end{aligned} \quad (22)$$

It should be noted that the arc with intensity α_k^q leaves the state $\gamma_1\gamma_2...\gamma_{k-q}$ belonging to the tier of the model graph with number $k-q$, and enters the state $\gamma_1\gamma_2...\gamma_{k-q}k_1k_2...k_k$ of the graph k-th tier.

To determine the final probabilities in the researched non-fully accessible QS with a non ordinary input flow of requirements groups, we will use the well-known [12]

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property of Markov graphs. According to this property, in steady state, the total flow of movements along the ribs entering any closed contour on the graph is equal to the total flow of transitions along the edges leaving this contour. In this case, the flow of transitions along a separate edge of the graph is equal to the product of the intensity of transitions along this edge, for example μ for the state S_{34} in Fig. 3, by the probability of the state from which this edge comes out. Thus, for a state S_{34}, the flow of transitions along the outgoing edge can be obtained in the form: $\mu \cdot p_{34}$.

Next, we will use the noted property and, for the system of inserted contours in Fig. 3, we will compose equations for the balance of transition flows. In this case, for the probabilities of group (tiered) states, we use the probability symbol P, and for the probabilities of states within each tier, we use the probability symbol p and take into account the ratio of group probabilities and probabilities of states within each tier (Fig. 3):

$$\begin{aligned} &P_0 = p_0; \quad P_1 = p_1 + p_2 + p_3 + p_4; \\ &P_2 = p_{12} + p_{13} + p_{14} + p_{23} + p_{24} + p_{34}; \\ &P_3 = p_{123} + p_{124} + p_{134} + p_{234}; \\ &P_4 = p_{1234}. \end{aligned} \tag{23}$$

For the inner contour enclosing the vertex S_0, we obtain:

$$\begin{aligned} &\mu p_1 + \mu p_2 + \mu p_3 + \mu p_4 = \\ &= (4\alpha_1^1 + 6\alpha_2^2 + 4\alpha_3^3 + 1\alpha_4^4) \cdot p_0. \end{aligned} \tag{24}$$

Taking into account (23) and the number of channels $n = 4$, equality (24) can be represented in the following form

$$\mu P_1 = P_0 \cdot (C_n^1\alpha_1^1 + C_n^2\alpha_2^2 + C_n^3\alpha_3^3 + C_n^4\alpha_4^4). \tag{25}$$

For the remaining contours in Fig. 3, we omit the procedure for the transition from the probabilities of internal states to the probability of the group state, noting that such a transition did not require any hypotheses about the values of the internal states probabilities, we obtain:

$$\begin{aligned} &2\mu P_2 = P_1(3\alpha_2^1 + 3\alpha_3^2 + 1\alpha_4^3) + \\ &+ P_0(6\alpha_2^2 + 4\alpha_3^3 + 1\alpha_4^4) = P_1(C_{n-1}^1\alpha_2^1 + C_{n-1}^2\alpha_3^2 + \\ &+ C_{n-1}^3\alpha_4^3) + P_0(C_n^2\alpha_2^2 + C_n^3\alpha_3^3 + C_n^4\alpha_4^4); \end{aligned} \tag{26}$$

$$\begin{aligned} &3\mu P_3 = P_2(2\alpha_3^1 + 1\alpha_4^2) + P_1(3\alpha_3^2 + 1\alpha_4^3) + \\ &+ P_0(4\alpha_3^3 + 1\alpha_4^4) = P_2(C_{n-2}^1\alpha_3^1 + C_{n-2}^2\alpha_4^2) + \\ &+ P_1(C_{n-1}^2\alpha_3^2 + C_{n-1}^3\alpha_4^3) + P_0(C_n^3\alpha_3^3 + C_n^4\alpha_4^4); \end{aligned} \tag{27}$$

$$\begin{aligned} &4\mu P_4 = P_3 \cdot 1\alpha_4^1 + P_2 \cdot 1\alpha_4^2 + P_1 \cdot 1\alpha_4^3 + P_0 \cdot 1\alpha_4^4 = \\ &= P_3 \cdot C_{n-3}^1\alpha_4^1 + P_2 \cdot C_{n-2}^1\alpha_4^2 + P_1 \cdot C_{n-1}^1\alpha_4^3 + P_0 \cdot C_n^1\alpha_4^4 \end{aligned} \tag{28}$$

In the general case, the equation for the balance of transition flows will take the following form:

$$k \cdot \mu \cdot P_k = \sum_{j=1}^{k} P_{k-j} \cdot \sum_{q=j}^{n-k+j} C_{n-k+j}^q \cdot \alpha_{k-j+q}^q. \tag{29}$$

Next, we will take into account the need for subsequent verification of correctness of the resulting analytical description of an incompletely accessible QS with input flow of non ordinary groups requirements, by degenerating them into already known and verified models, we will look for expressions for the final probabilities of group states in a form convenient for such verification:

$$\begin{aligned} &P_k = P_0 \cdot \frac{(\rho_1)^k}{k!} \cdot F_k; \; k = \overline{1, n}; \\ &P_0 = \left(\sum_{k=0}^{n} \frac{(\rho_1)^k}{k!} \cdot F_k \right)^{-1}. \end{aligned} \tag{30}$$

where for the functions F_k we'll need formulas:

$$\rho_q = \frac{\lambda_q}{\mu}, \quad q = \overline{1, n}. \tag{31}$$

To find an analytical calculation formula for the probability P_k deformation function F_k, we present the expression for the transition intensity (22) taking into account the transition probability d_k^q, and then we obtain:

$$\begin{aligned} &\alpha_k^q = \lambda_q \cdot [\sum_{j=1}^{n} l_j \cdot \sum_{i=0}^{j-q} \frac{1}{C_{i+q}^q} \cdot C_{k-q}^{j-q-i} \cdot C_{n-k}^i + \\ &+ \left(\sum_{j=0}^{k-q} l_{j+q} \cdot C_{k-q}^j \right) \cdot \frac{1}{\lambda_q} \cdot \sum_{r=q+1}^{\mathrm{L}} \lambda_r] = \lambda_q \cdot d_k^q, \\ &q = \overline{1, n}; \quad k = \overline{1, n}. \end{aligned} \tag{32}$$

Next, we substitute the expression for the final probabilities (30) into equality (29), and we obtain:

$$\begin{aligned} &k \cdot \mu \cdot P_0 \cdot \frac{(\rho_1)^k}{k!} \cdot F_k = \sum_{j=1}^{k} P_0 \cdot \frac{(\rho_1)^{k-j}}{(k-j)!} \cdot F_{k-j} \cdot \\ &\cdot \sum_{q=j}^{n-k+j} C_{n-k+j}^q \cdot \lambda_q \cdot d_{k-j+q}^q. \end{aligned} \tag{33}$$

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After elementary transformations we obtain:

$$F_k = \sum_{j=1}^{k} F_{k-j} \cdot \frac{(k-1)!}{(\rho_1)^j (k-j)!} \cdot \sum_{q=j}^{n-k+j} C_{n-k+j}^{q} \cdot \rho_q \cdot d_{k-j+q}^{q}, \quad k = \overline{1, n}. \qquad (34)$$

In order to find the value of a function F_0, we substitute the value $k = 1$ into expression (29), and then we get:

$$\mu \cdot P_1 = P_0 \cdot \sum_{q=1}^{n} C_n^q \cdot \lambda_q \cdot d_q^q. \qquad (35)$$

Then we substitute the probability value P_k from equality (30) into equality (35) at $k = 1$, and we obtain:

$$\mu \cdot P_0 \cdot \rho_1 \cdot F_1 = P_0 \cdot \sum_{q=1}^{n} C_n^q \cdot \lambda_q \cdot d_q^q. \qquad (36)$$

From equality (36) and (31) we find:

$$F_1 = \frac{1}{\rho_1} \cdot \sum_{q=1}^{n} C_n^q \cdot \rho_q \cdot d_q^q. \qquad (37)$$

On the other hand, from equality (36) with the value $k = 1$ we obtain:

$$F_1 = F_0 \cdot \frac{1}{\rho_1} \cdot \sum_{q=1}^{n} C_n^q \cdot \rho_q \cdot d_q^q. \qquad (38)$$

Equating the right-hand sides of equalities (37) and (38)

$$\frac{1}{\rho_1} \cdot \sum_{q=1}^{n} C_n^q \cdot \rho_q \cdot d_q^q = F_0 \cdot \frac{1}{\rho_1} \cdot \sum_{q=1}^{n} C_n^q \cdot \rho_q \cdot d_q^q, \qquad (39)$$

we find the value of the function F_0:

$$F_0 = 1. \qquad (40)$$

To check the correctness of expressions (22), (30) and (34) of the obtained analytical description of the non-fully accessible $M_L/M/n$ QS with a non ordinary input flow of requirements groups, we'll research the asymptotic transition of its description into the specification of a similar fully accessible $M_L/M/n$ QS with a non ordinary input flow of requirements groups, and then into the description of a similar non-fully accessible QS but with a simplest requirements flow.

When the analytical description of a non-fully accessible QS with a non ordinary input flow of requirements groups degenerates into a fully accessible QS, all values of accessibility probabilities for individual groups of service channels become equal to zero, except for the probability l_n:

$$l_k = \frac{\pi_k}{C_n^k} = 0, \quad k < n; \quad l_n = \frac{\pi_n}{C_n^n} = \pi_n = 1. \qquad (41)$$

Then expression (22) degenerates into equality (42):

$$\alpha_k^q = \lambda_q \cdot l_n \cdot \sum_{i=0}^{n-q} \frac{1}{C_{i+q}^q} \cdot C_{k-q}^{n-q-i} \cdot C_{n-k}^{i} + [l_k] \cdot \sum_{r=q+1}^{L} \lambda_r, \quad q = \overline{1, n}; \quad k = \overline{1, n}. \qquad (42)$$

The product of the combinations under the sum sign in equality (42) is different from zero and equal to one only when the value $i = n - k$. Then expression (42) takes the following form:

$$\alpha_i^q = \lambda_q \cdot l_n \cdot \frac{1}{C_{i+q}^q} + l_i \cdot \sum_{r=q+1}^{L} \lambda_r. \qquad (43)$$

When substituting the value $i = k - j + q$ into equality (43), we obtain:

$$\alpha_{k-j+q}^q = \lambda_q \cdot l_n \cdot \frac{1}{C_{n-k+j}^q} + l_k \cdot \sum_{r=q+1}^{L} \lambda_r. \qquad (44)$$

Next, we substitute the value α_{k-j+q}^q from equality (44) into equality (29) and, taking into account conditions (41), we find:

$$k \cdot \mu \cdot P_k = \sum_{j=1}^{k} P_{k-j} \cdot \sum_{q=j}^{n-k+j} C_{n-k+j}^{q} \cdot \alpha_{k-j+q}^q = \sum_{j=1}^{k} P_{k-j} \cdot \sum_{q=j}^{n-k+j} C_{n-k+j}^{q} \cdot [\lambda_q \cdot \frac{1}{C_{n-k+j}^{q}} + l_k \cdot \sum_{r=q+1}^{L} \lambda_r]. \qquad (45)$$

In the internal sum in equality (44), we select the last term with the value $q = n - k + j$ and take into account the conditions (41). Then expression (45) will take the following form:

$$k \cdot \mu \cdot P_k = \sum_{j=1}^{k} P_{k-j} \cdot \left(\sum_{q=j}^{n-k+j-1} [\lambda_q \cdot \frac{C_{n-k+j}^q}{C_{n-k+j}^q} + 0] + C_{n-k+j}^{n-k+j} \cdot 0 \cdot \sum_{r=q+1}^{L} \lambda_r \right). \qquad (46)$$

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Finally, taking into account the accepted hypothesis $L = n$, we find:

$$k \cdot \mu \cdot P_k = \sum_{j=1}^{k} P_{k-j} \cdot \sum_{q=j}^{L} \lambda_q \,. \qquad (47)$$

Expression (47) completely coincides with the basic equality for finding the final probabilities P_k of group states in the $M_L/M/n$ QS with a non ordinary input flow of requirements groups (8).

Thus, when a found description of $M_L/M/n$ QS with incomplete approachability of service devices degenerates into the same system, but fully accessible, the found description is automatically transformed into a known description of QS $M_L/M/n$ with complete approachability of service devices. The noted phenomenon testifies in favor of correctness of the obtained analytical description in relation to simpler $M_L/M/n$ QS, which is a simplified version in relation to considered non-fully accessible QS with a non ordinary input flow.

The *M*/*M*/*n* QS, with incomplete approachability of service devices and with an ordinary (simplest) input flow of requirements [4, formula (29)], taking into account the designations of variables, is described by the equation (8) of transitions flows balance (48):

$$k\mu P_k = P_{k-1} \cdot C_{n-k+1}^{1} \cdot \alpha_k^1, \quad k = \overline{1,n} \,. \qquad (48)$$

In equalities (1) the probability $b_1 = 1$ and all other probabilities become equal to zero $b_r = 0, r = 2, ..., L$.

Then the parameters of the partial flows of requirements will take the form $\lambda_1 = I;\ \lambda_r = 0,$ for $r \geq 2$, which will lead to changes in the found analytical description (22), where for all values $q \geq 2$ $\alpha_k^q = 0$.

On the right-hand side of (29), the inner sum turns out to be nonzero only when $j = 1$, and expression (29) is transformed to the form:

$$k \cdot \mu \cdot P_k = \sum_{j=1}^{k} P_{k-j} \cdot \sum_{q=j}^{n-k+j} C_{n-k+j}^{q} \cdot \alpha_{k-j+q}^{q} = \left. \vphantom{\sum} \right|_{j=1} = P_{k-1} \cdot C_{n-k+1}^{1} \cdot \alpha_k^1 \,. \qquad (49)$$

which coincides with the description (48) of a non-fully accessible *M*/*M*/*n* QS, with refusals and with an ordinary (simplest) input flow of requirements. At the same time, expression (22) for α_k^q is transformed to the form:

$$\alpha_k^1 = I \cdot \sum_{j=1}^{n} l_j \cdot \sum_{i=0}^{j-1} \frac{1}{C_{i+1}^1} \cdot C_{k-1}^{j-1-i} \cdot C_{n-k}^{i}, \quad k = \overline{1,n}, \qquad (50)$$

which completely coincides with the expression [4, formula (29)] (see here formula (8)) for a non-fully accessible *M*/*M*/*n* QS, with refusals and with a simplest input flow.

As a result, when the found description of a non-fully accessible QS with a non ordinary input flow of requirements groups and with refusals degenerates into a non-fully accessible system with refusals, but with a simplest input flow, the found description is automatically transformed into a known description of a non-fully accessible *M*/*M*/*n* QS with refusals and with an ordinary (simplest) input flow of requirements. The noted phenomenon testifies to the correctness of the obtained analytical description in relation to the simpler *M*/*M*/*n* QS, which is a simplified version in relation to considered incompletely accessible QS with a non ordinary input flow. Thus, the application of the mathematical apparatus for analysis the groups of states of Markov graphs [12] made it possible to obtain analytical formulas for calculating values of states' final probabilities in process of servicing a non ordinary flow of requirements groups in a non-fully accessible system with refusals, which is a general case of previous types of QS.

4 EXPERIMENTS

In order to test the operability of analytical description of the incompletely accessible $M_L/M/n$ model, we use an example from the topical sphere – of important objects air defense (Fig. 5) by a grouping of four single-channel SAM systems (Table 1, No. 1, 2–3) – "service devices", which should prevent a planned air blow of $N_{total.En} = 15$ enemy aircraft (Table 1, No. 2–8, 2–3) with a duration of 7,5 minutes and at intensity of 2 aircraft per minute (Table 1, No. 2–8, 2–3). Let's assume that shelling an aircraft in a fire zone ends with its destruction.

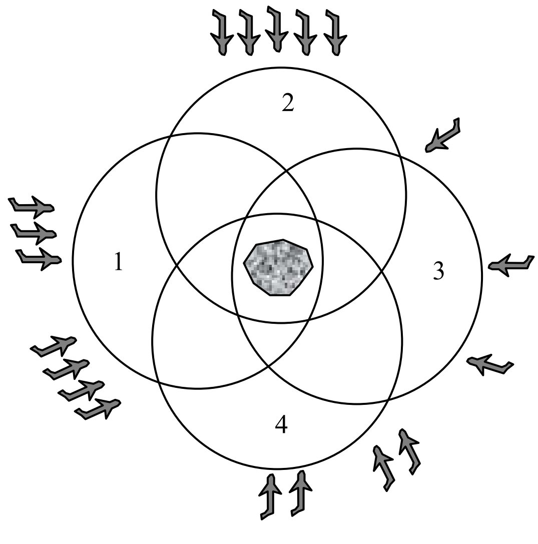

Figure 5 – An example of setting the task of assessing the objects' air defense effectiveness by the SAM systems' using the $M_5/M/4$ model of an incompletely accessible QS with input flow of requirement groups

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To destroy an object covered, at least five aircraft are required. Therefore, the task of SAM systems grouping is considered fulfilled in the case when less than five aircraft can break through to the object ($N_{missed} < 5$).

In QS with incomplete approachability of service devices, refusal to service the next requirement is possible with any number of free devices. Therefore, to estimate the service probability, it is necessary to use information about the mathematical expectation of the occupied devices number $M_{b.d.}$ and the performance μ of QS' one channel:

$$M_{b.d.} = \sum_{k=0}^{n} k \cdot P_k ; \; P_{service} = \frac{\mu \cdot M_{b.d.}}{I} . \qquad (51)$$

We also note that the mathematical expectation of the enemy aircraft number that broke through to the target (N_{missed}) can be found (52):

$$N_{missed} = N_{total.En} \cdot \left(1 - P_{service}\right). \qquad (52)$$

To perform the calculations, we use the initial data (Table 1 No. 1–24, 2–9), and the set of formulas (1), (41), (14), (17), (31). To calculate the missing expressions for the transition intensities, we use equality (22), and obtain:

$$\alpha_3^3 = \lambda_3 (l_3 + \frac{1}{4} l_4) + l_3 \cdot \sum_{r=4}^{L} \lambda_r ;$$
$$\alpha_4^3 = \lambda_3 (l_3 + l_4) + l_4 \cdot \sum_{r=4}^{L} \lambda_r ; \qquad (53)$$
$$\alpha_4^4 = \lambda_4 (l_4) + l_4 \cdot \sum_{r=5}^{L} \lambda_r .$$

Next, we use equalities (40) and (34). We find the missing expressions for the deformation functions F_2–F_4 of the final probabilities P_k of QS states group using the general recurrent expression (34), we obtain:

$$F_2 = F_1 \cdot \frac{1}{\lambda_1} \cdot (3\alpha_2^1 + 3\alpha_3^2 + 1\alpha_4^3) + + \frac{(\mu)}{(\lambda_1)^2} \cdot (6\alpha_2^2 + 4\alpha_3^3 + 1\alpha_4^4) ; \qquad (54)$$

$$F_3 = F_2 \cdot \frac{1}{\lambda_1} \cdot (2 \cdot \alpha_3^1 + 1 \cdot \alpha_4^2) + F_1 \cdot \frac{2\mu}{(\lambda_1)^2} \cdot \cdot (3 \cdot \alpha_3^2 + 1 \cdot \alpha_4^3) + F_0 \cdot \frac{2\mu^2}{(\lambda_1)^3} \cdot (4 \cdot \alpha_3^3 + 1 \cdot \alpha_4^4) ; \qquad (55)$$

$$F_4 == F_3 \cdot \frac{1}{(\lambda_1)^1} \cdot \alpha_4^1 + F_2 \cdot \frac{3 \cdot (\mu)^1}{(\lambda_1)^2} \cdot \alpha_4^2 + + F_1 \cdot \frac{6 \cdot (\mu)^2}{(\lambda_1)^3} \cdot \alpha_4^3 + F_0 \cdot \frac{6 \cdot (\mu)^3}{(\lambda_1)^4} \cdot \alpha_4^4 . \qquad (56)$$

We will calculate the probabilities of QS states group according to (30). To control the correctness of the obtained model $M_L / M / n$ with incomplete approachability of service devices, we'll use the well-known formulas of Erlang model [5]:

$$P_k = P_0 \cdot \frac{\rho^k}{k!} ; \; k = \overline{1, n} ; \; P_0 = \left(\sum_{k=0}^{n} \frac{\rho^k}{k!} \right)^{-1} . \qquad (57)$$

Table 1 and Figures (Fig. 6–Fig. 9) present the calculation results..

For ease of analysis, Table 1 and Figures 6–9 introduce an abbreviated designation for the compared QS models, which allows us to establish a significant difference in the probability distribution of group states of the model developed in this research (Devel.) and known models (Req.gr., Inacces. and Erlang).

The known models do not provide for simultaneous consideration of the essential features of real QS – incomplete accessibility of service devices and the group composition of the input flow of requirements with an unknown composition of groups in advance, which leads to a significant distortion of the predicted results for QS operation.

Thus, (Fig. 9 and Table 1 No. 37) the excess in the assessment of the expected value of the efficiency indicator $P_{service}$, relative to the value obtained using the developed model, turns out to be at least one and a half times greater than the more realistic assessment using the developed model.

As a result, all known models allow us to assume that the SAM group will let through less than 5 enemy aircraft and thus reliably perform its task (Table 1 No. 38).

However, simultaneous consideration of the incomplete accessibility of service devices and the group composition of requirements in the input flow more objectively shows the significant inability of the SAM group to perform its task, since more than eight enemy aircraft can pass "without service" to the protected object (Table 1 No. 38, 12).

At the same time, the result of the experiment demonstrates the automatic degeneration of the deformation functions (F_0–F_4) into the functions of incomplete accessibility (Table 1 No. 27–31, 13) and in the function of non ordinary (Table 1 No. 27–31, 14), as well as in a single value (Table 1 No. 27–31, 15) with an automatic transition to the description of the final probabilities QS (Table 1 No. 32–36) of group states in the model with incomplete accessibility of service channels, in the model with group arrival of requirements and in the Erlang model.

The calculation expressions of the deformation functions F_0–F_4 are recurrent, which allows automating the calculation of their values for specific operating conditions and the specific configuration of QS.

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Table 1 – Comparative assessment of service probability and task performance efficiency by the SAMS group (Fig. 5) using the developed and previous types of QS models with refusals

Names and values of models' parameters											Types of models			
#	Name	Value	#	Name	Value	#	Name	Value	#	Name	Devel.*	Inacces.*	(Req.gr.)*	Erlang*
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	n	4	13	μ	1	15	α_1^1	0.087	27	F_0	1	1	1	1
2	I	2	14	ρ_1	0.133	16	α_2^1	0.120	28	F_1	5	1	5	1
3	λ	0.667	15	L	5	17	α_3^1	0.164	29	F_2	40.1	0.9	55	1
4	a_1	0.200	16	λ_1	0.133	18	α_4^1	0.340	30	F_3	372.204	0.486	912.5	1
5	a_2	0.200	17	λ_2	0.133	19	α_2^2	0.033	31	F_4	3408.55	0.247	19637.5	1
6	a_3	0.200	18	λ_3	0.133	20	α_3^2	0.056	32	P_0	0.451	0.178	0.360	0.143
7	a_4	0.200	19	λ_4	0.133	21	α_4^2	0.144	33	P_1	0.301	0.356	0.240	0.286
8	a_5	0.200	20	λ_5	0.133	22	α_3^3	0.021	34	P_2	0.161	0.321	0.176	0.286
9	π_1	0.4	21	l_1	0.1	23	α_4^3	0.061	35	P_3	0.066	0.115	0.130	0.190
10	π_2	0.3	22	l_2	0.05	24	α_4^4	0.037	36	P_4	0.020	0.029	0.093	0.095
11	π_3	0.16	23	l_3	0.04	25	$M_{b.d}$	0.9	37	$P_{service}$	0.452	0.731	0.678	0.905
12	π_4	0.14	24	l_4	0.14	26	M_{in}	3	38	N_{missed}	8.23	4.04	4.84	1.43

*Devel. – The developed QS $M_L/M/n$ with refusals with not fully accessible service channels and with input flow of requirements' groups.
*Req.gr. – QS $M_L/M/n$ with input flow of requirements' groups and with refusals. The functions F_j automatically degenerate into known non-ordinary functions [3, formula (18)].
*Inacces. – QS $M/M/n$ with not fully accessible service channels and with refusals. The functions F_j automatically degenerate into known functions of incomplete accessibility [4, formula (28)].
*Erlang – QS $M/M/n$ with refusals. The functions F_j automatically become equal to one [5]

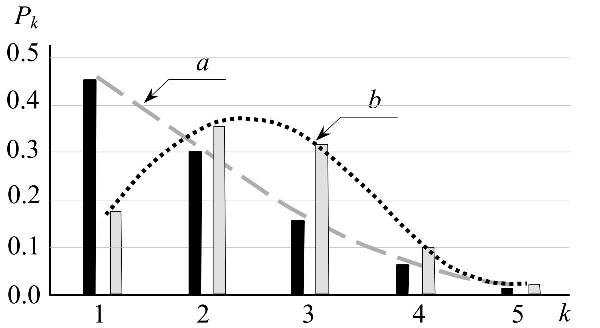

Figure 6 – Final probabilities P_k of states' groups in queuing systems with refusals and under identical conditions:
a) the developed $M_L/M/n$ model;
b) the well-known $M/M/n$ model. *Inacces. – see (*) in Table 1,
($b_1 = 1; b_r = 0; \lambda_1 = I; \lambda_r = 0$, for $r \geq 2$)

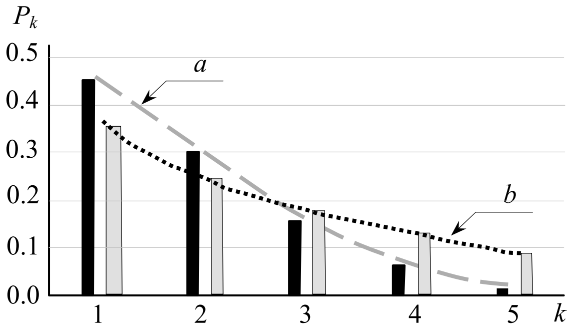

Figure 7 – Final probabilities P_k of states' groups in queuing systems with refusals and under identical conditions:
a) the developed $M_L/M/n$ model;
b) the well-known $M_L/M/n$ model. *Req.gr. – see (*) in Table 1, ($l_n = \pi_n = 1; l_k = 0, k < n.$, (41))

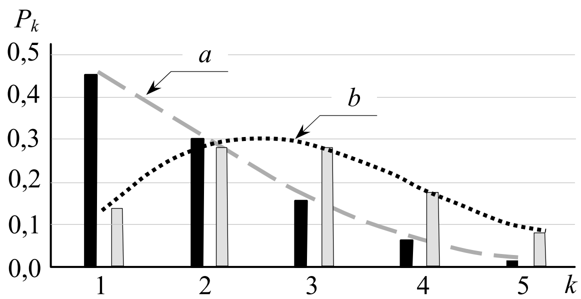

Figure 8 – Final probabilities P_k of states' groups in queuing systems with refusals and under identical conditions:
a) the developed $M_L/M/n$ model;
b) the well-known $M/M/n$ model. *Erlang – see (*) in table 1,
($l_n = \pi_n = 1; l_k = 0, k < n$;
$b_1 = 1; b_r = 0; \lambda_1 = I; \lambda_r = 0$, for $r \geq 2$)

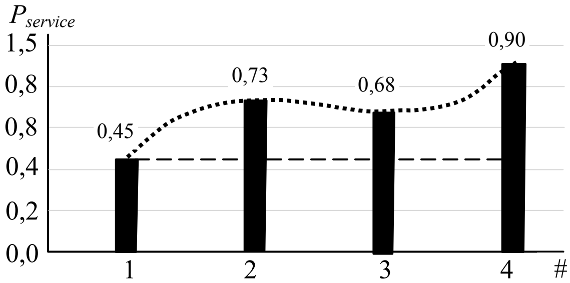

Figure 9 – Predicted value of performance indicator for a group of SAM systems (Fig. 5), using QS models:
1) the developed $M_L/M/n$ model;
2) $M/M/n$ model *Inacces. – see (*) in table 1;
3) $M_L/M/n$ model. *Req.gr. – see (*) in table 1;
4) $M/M/n$ model. *Erlang – see (*) in table 1.

5 RESULTS

In the course of this research, analytical formulas for calculating the values of final probabilities in Markov QS $M_L/M/n$ with incomplete approachability of service devices were obtained for the first time, which makes it possible to evaluate and predict the values of all known

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indicators of system efficiency. In this case, the deformation functions (F_k) of the states groups' probability distribution in QS have a recurrent form (34), which is convenient both for finding their analytical expressions and for performing numerical calculations. When the parameters of the QS operating conditions degenerate, the resulting description automatically goes into a description of one of a known QS: with refusals and with incomplete accessibility of service devices ($M/M/n$); into a description of a known QS with refusals, with full accessibility of service devices and with non-ordinary input flow of requirements groups ($M_L/M/n$), into a description of a known Erlang QS with refusals. During the numerical experiment, were obtained results (Table 1, Fig. 6–Fig. 9) that testify in favor of this statement. Thus, the noted types of QS turn out to be special cases of the analytical description of the final probabilities of QS $M_L/M/n$ with refusals, with partially accessible service devices and with an input flow of requirements groups with random composition obtained in this research.

CONCLUSIONS

In the course of the research, the analytical formulas for calculating numerical values of final probabilities of states in the $M_L/M/n$ QS with incomplete approachability of service devices for the input flow of requirements groups were received. The results of numerical experiment testify in favor of correctness the analytical formulas for calculating numerical values of final probabilities and in favor of possibility of their practical application in real QSs when solving problems of forecasting efficiency, as well as analyzing and synthesizing the parameters of real queuing systems.

The scientific novelty of the results obtained in research lies in the creation of possibilities for forecasting the effectiveness of known type of Markov queuing systems with refusals, with incomplete approachability of service devices and with an input flow of requirements groups with random composition. The obtained description (24), (27)–(29), (33) of a queuing system is a general for known QS with refusals, with not full approachability of service devices and with the simplest input flow of requirements ($M/M/n$), for known QS of $M_L/M/n$) type and for known Erlang model $M/M/n$.

The practical significance of the research results consists in obtaining analytical calculation formulas for performing rapid quantitative assessments of final probabilities, as well as all indicators of the efficiency of QS $M_L/M/n$ with incomplete approachability of service devices and its less complex QS variants in the course of solving practical problems of analysis, synthesis and management of real objects, for which such models can serve as a formal representation. The recurrent formulas obtained for calculating the values of the deformation functions of probabilities for states groups in QS are convenient for performing quick practical calculations.

Prospects for further research may be a research and development of an analytical description for final probabilities in Markov models with incomplete accessibility of service devices, with an input flow of groups of requirements and with waiting.

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АНАЛІТИЧНИЙ ОПИС ФІНАЛЬНИХ ІМОВІРНОСТЕЙ СТАНІВ У НЕПОВНО ДОСТУПНІЙ СИСТЕМІ ОБСЛУГОВУВАННЯ З ВІДМОВИМИ І З ВХІДНИМ ПОТОКОМ ГРУП ВИМОГ

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АНОТАЦІЯ

Актуальність. Основою створення та управління реальними системами масового обслуговування є можливість прогнозу їхньої ефективності. Для загального випадку таких систем з відмовами, з неповною доступністю приладів обслуговування та з випадковим складом груп вимог у вхідному потоці прогноз ефективності їх роботи залишається не вирішеною проблемою.

Метод. Для вищевказаних типів СМО ймовірності станів можна описати системою диференціальних рівнянь Колмогорова, яка для стаціонарного випадку перетворюється в однорідну систему лінійно залежних алгебраїчних рівнянь. Для реальних СМО в системах зв'язку кількість рівнянь може бути оцінена множиною-ступенем і досягати кількох тисяч, що породжує проблему їх запису та чисельного розв'язання для конкретного набору значень параметрів умов роботи. Прогностична цінність такого рішення не перевищує ймовірність вгадування числових значень параметрів умов роботи СМО, а для параметрів з безперервними значеннями, наприклад, для випадкових інтервалів часу між вимогами, дорівнює нулю.

Використаний метод заснований на аналітичному переході до опису груп станів СМО з однаковою кількістю зайнятих пристроїв. При цьому прагнення отримати кінцеві ймовірності станів у формі, наближеній до формул Ерланга, залишається. Вплив згаданих вище властивостей СМО можна локалізувати в окремих рекурентних функціях, які мультиплікативно спотворюють формули Ерланга.

Результати. Для вищезазначених типів СМО вперше знайдено аналітичні розрахункові формули для оцінки фінальних ймовірностей станів СМО, що дає змогу прогнозувати значення всіх відомих показників ефективності системи. У цьому випадку функції деформації розподілу ймовірностей груп станів у СМО мають рекурентний вигляд, що зручно як для знаходження їх аналітичних виразів, так і для чисельних розрахунків.

Коли параметри умов роботи СМО вироджуються, результуючий опис автоматично перетворюється на опис однієї з відомих СМО з відмовами, аж до СМО Ерланга.

Висновки. Знайдені аналітичні розрахункові вирази для фінальних ймовірностей вищезгаданої СМО виявилися застосовними до всіх типів Марківської СМО з відмовами, що підтверджено результатами чисельного експерименту. У результаті стало можливим практично застосовувати отриманий аналітичний опис розглянутої СМО для оперативних оцінок ефективності розробленої та існуючої СМО в можливому діапазоні умов їх функціонування.

КЛЮЧОВІ СЛОВА: Марківські системи масового обслуговування, групи вимог.

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