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METHOD FOR DETERMINING THE STRUCTURE OF NONLINEAR MODELS FOR TIME SERIES PROCESSING

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ABSTRACT

Context. The practice of today's problems actualizes the increase in requirements for the accuracy, reliability and completeness of the results of time series processing in many applied areas. One of the methods that provides high-precision processing of time series with the introduction of a stochastic model of measured parameters is statistical learning methods. However, modern approaches to statistical learning are limited, for the most part, to simplified polynomial models. Practice proves that real data most often have a complex form of a trend component, which cannot be reproduced by polynomials of even a high degree. Smoothing of nonlinear models can be implemented by various approaches, for example, by the method of determining the parameters of nonlinear models using the differential spectra balance (DSB) in the scheme of differential-non-Taylor transformations (DNT). The studies proved the need for its modification in the direction of developing a conditional approach to determining the structure of nonlinear mathematical models for processing time series with complex trend dynamics.

Objective. The development of a method for determining the structure of nonlinear by mathematical models for processing time series using DSB in DNT transformations.

Method. The paper develops a method for constructing nonlinear mathematical models in the DNT transformation scheme. The modification of the method consists in controlling the conditions for the formation of a certain system of equations in the DSB scheme to search for the parameters of a nonlinear model with its analytical solutions. If the system is indeterminate, the nonlinear model is supplemented by linear components. In the case of an overdetermined system, its solution is carried out using the least squares norm. A defined system is solved by classical approaches. These processes are implemented with the control of stochastic and dynamic accuracy of models in the areas of observation and extrapolation. If the results of statistical learning are unsatisfactory in accuracy, the obtained values of the nonlinear model are used as initial approximations of numerical methods.

Result. Based on carried-out research, a method for determining the structure of nonlinear models for processing time series using BDS in the scheme of DNT transformations is proposed. Its application provides a conditional approach to determining the structure of models for processing time series and increasing the accuracy of estimation at the interval of observation and extrapolation.

Conclusions. The application of the proposed method for determining the structure of nonlinear models for processing time series allows obtaining models with the best predictive properties in terms of accuracy.

KEYWORDS: data science, statistical learning, time series, nonlinear models, numerical methods, least square method.

ABBREVIATIONS

ARMA – autoregressive moving average;
ARIMA – autoregressive integrated moving average;
DNT – differential-non-Taylor transformations;
DS – differential spectrum;
DSB – is a balance of differential spectra;
EMA – exponential moving average;
MA – moving average;
OLS – ordinary least squares.

NOMENCLATURE

Δ – linear deviation of the model;
Φ –OLS matrix algorithm;
$\varphi_i(t)$ – basis model functions.
a_i – parameters of the nonlinear model;
c_i – free polynomial coefficients e.g. in the format of a step basis;
d – distance control;
$f(t,a)$ – nonlinear model;
H – segment of the argument on which the function is considered;
k – ordinal number of the discrete spectrum;
n – time series size (sample size);
$P\{\ldots\}_{t^*}$ – direct differential transformation;
R^2 – determination coefficient of the model;
t – function argument;
t^* – specific value of the argument at which the conversion is performed;
y_n – n-th dimension (element of the time series – dimension);
$z(t)$ – polynomial model;
$Z(t)$ – discrete argument function $k = 0,1,2,\ldots$;
$\hat{z}(t)$ – trained polynomial model;
$\hat{Z}(k), F(k)$ – images of models.

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INTRODUCTION

One of the fastest growing areas in the field of modern information technology is undoubtedly Data Science. Currently, methods and technologies of data research play a key role in e-commerce software systems (trading, retailing, aggregation), computer systems of automated and automatic control (unmanned systems, traffic control systems), etc. [1, 4]. These applied fields often operate with data in the time series format. Their processing is quite illustratively presented: approximation methods (Moving Average (MA) algorithms, Exponential Moving Average (EMA), autocorrelation algorithms such as ARIMA); statistical learning methods (smoothing) – Statistical Learning (Ordinary Least Square Method (OLS), Kalman filtering, etc.); methods of deep learning using artificial neural networks [1–7].

Real data in the form of time series, as a rule, depends on many factors that are difficult to describe. This introduces errors into discrete measurements that are attributed to the model randomness. Therefore, despite the successful application of approximation and deep learning methods in practice, statistical learning methods are quite effective from the point of view of accuracy, reliability of the result and productivity of calculations.

Traditionally, Statistical Learning methods use linear models in the form of power polynomials. But the reality is that most of the studied processes are non-linear in nature, thus a priori limiting the possibility of using this approach. That is why approximation and deep learning methods are sometimes preferred, which, in fact, reproduce the stochastic process. There are quite a few approaches that allow us to partially solve the problem of nonlinear smoothing (linearization, numerical methods) [1–3]. Most of them consist in simplifying a nonlinear model to a set of linear components or require initial conditions for starting iterative processes of "fitting" the parameters of nonlinear models to the data. However, in practice, such models lose both most of their nonlinear information and their own predictive value.

Therefore, the task of developing an approach to determining the parameters of a nonlinear model based on the data of the time series format using statistical learning methods is relevant.

The object of study is the process of determining the structure of nonlinear mathematical models for processing time series.

The subject of study is methods of processing time series with nonlinear models in terms of parameters

The purpose of the work is to develop a methodology for determining the structure of nonlinear mathematical models for processing time series using DSB in DNT transformations.

1 PROBLEM STATEMENT

Suppose while observing a certain process, a time series is y obtained that is described as:

$$y = \{y_0, y_1, y_2, \dots, y_n\}\,. \tag{1}$$

This data may be heterogeneous, contaminated, and have anomalous measurements, but all this must be addressed at the data preparation stage. Therefore, we further assume that the time series (1) is homogeneous, and its values are normally distributed with standard deviation σ (base random error).

The general form of the nonlinear approximating function is known:

$$f(t,a), a = a_0, a_1, \dots, a_i, \dots, a_m\,, \tag{2}$$

which with a high degree of adequacy describes the process under study, represented by a set of discrete dimensions (1). The task of processing the time series (1) by statistical learning methods is to determine the parameters a of the nonlinear model (2) with the requirement to reduce the random measurement error (1). The result is a nonlinear mathematical model, consistent – "trained" from measurements (1), which describes the law of trend change of the process under study with reduction – smoothing of the random error of the input data [12].

However, classical methods of statistical learning are based on the use of linear models – polynomials. The nonlinearity of such approaches is reproduced by adding high-stage components to the structure of the model. This approach shows good results within the measurement sample (1), but the predictive properties in terms of accuracy (point and interval) and the forecast interval are not the best. Nonlinear in terms of parameters models (2) overcome these disadvantages of polynomial forms [7, 10, 11].

To determine the parameters of a nonlinear model, it is proposed, first, to use OLS to train a polynomial model $\hat{z}(t)$:

$$\hat{z}(t) = \Phi(y)\,, \tag{3}$$

$$z(t) = \sum_{i=0}^{m} c_i \varphi_i(t)\,, \tag{4}$$

So, we have two models $f(t,a)$ and $\hat{z}(t)$ are, respectively, theoretical and experimental, and the parameters of the second are known. To determine the parameters of the theoretical nonlinear model, it is necessary to perform its approximation to the experimental one. That is, to transfer the properties of one to the other: the certainty of the parameters $\hat{z}(t)$, which are obtained from sample (1), on a nonlinear model that is uncertain in terms of parameters $f(t,a)$. One method to accomplish this is DSB.

The balance of differential spectra is based on the differential transformations described in [7]. In general, they can be presented as:

$$Z(k) = P\{z(t)\}_{t^*} = \frac{H^k}{k!}\left[\frac{d^k z(t)}{dt^k}\right]_{t^*}, \tag{5}$$

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$$z(t) = \sum_{k=0}^{\infty} (\frac{t}{H})^k Z(k) . \quad (6)$$

Expression (5) provides the ability to get an image $Z(k)$ from its original $z(t)$ (direct transformation). Inverse conversion (6) provides the ability to restore the original $z(t)$. Differential image $Z(k)$ is called the differential spectrum (DS) or a P-spectrum, and the value of the $Z(k)$ for specific values of the argument – DS discretes (P-discretes). The reconstruction of the original is reduced to the summation of the discrete P-spectrum in the form of a segment of the Taylor series. ex. (6) (basic or differential-Taylor (DT) transformations). If the restoration of DS is carried out on an arbitrary (non-Taylor) basis, such transformations are called differential-non-Taylor (DNT). It is possible to implement DNT by equating the discrete of the same name to the original function $Z(k)$ and functions from the selected basis, e.g. $f(t,a)$. This is the essence of the DSB method.

Combining the properties of an experimental model $\hat{z}(t)$ and a nonlinear model $f(t,a)$ according to the DSB in the DNT scheme is carried out according to the model:

$$\begin{aligned} &[P\{\hat{z}(t)\}_{t^*} \Rightarrow \hat{Z}(k)] = \\ &= [P\{f(t,a)\}_{t^*} \Rightarrow F(k,a)] \\ &\to \hat{Z}(k) = F(k,a). \end{aligned} \quad (7)$$

Determination of the parameters a of a nonlinear model $f(t,a)$ is implemented by forming and solving a system of equations:

$$\begin{cases} \hat{Z}(0) - F(0,a_1) = 0, \\ \hat{Z}(1) - F(1,a_2) = 0, \\ \hat{Z}(2) - F(2,a_3) = 0, \\ \quad\dots \\ \hat{Z}(k) - F(k,a_m) = 0. \end{cases} \quad (8)$$

The processes described represent the essence of the method of constructing nonlinear mathematical models in the DNT transformation scheme [7].

The problem of the practical application of the DSB is the need to comply with a number of requirements: DS $\hat{z}(t)$ and $f(t,a)$ must give a definite system of equations (8); model $\hat{z}(t)$ must be adequate to the dynamics of data changes (1); model $f(t,a)$ should not have zero discrete for at the interval of existence of the DS model $\hat{z}(t)$. Such requirements can only be met in partial cases.

Therefore, the paper proposes the development of a method for constructing nonlinear mathematical models in the DNT transformation scheme by developing a conditional approach to determining the structure of nonlinear models for processing time series.

2 REVIEW OF THE LITERATURE

The focus of the analysis of existing approaches to the processing of time series with nonlinear models is placed on the methods of statistical learning. It should be noted that we will consider nonlinear models where the reproduction of the nonlinearity (seasonality, fluctuations, etc.) of the trend is ensured by introducing a nonlinear function into the structure of the model – exponential, logarithmic, trigonometric, differential components, etc. In this case, nonlinear operations are implemented on the parameters of the model, and the models are nonlinear in terms of parameters. Otherwise, the models will be considered linear in parameters, although capable of reproducing nonlinear time series. The advantage of nonlinear models over linear ones is the smaller number of structural components of the former. That is, the reproduction of the nonlinearity of data is assumed by a nonlinear function, and not a significant number, for example, of high-degree polynomial elements.

Currently, there are quite a few approaches to reproducing nonlinear models, but they can be divided into three classes: linearization; calculation of parameters by iterative numerical methods; operator methods. The best efficiency was proved by methods based on operator transformations. In this direction, [4, 6, 10, 11] proposes an approach to determining the parameters of nonlinear models using the method of differential transformations [7]. However, its shortcomings lead to the possibility of effective application to a rather narrow range of partial cases.

This determines the formed goal of research on the development of an approach that will unify the use of operator methods to determine the parameters of nonlinear models consistent with the time series of the format (1).

3 MATERIALS AND METHODS

The use of DSB to determine the parameters of a nonlinear model implements the transfer of properties of a simplified polynomial model to a complex nonlinear model. This means a priori that in the observation area, or in the provided time series, this polynomial must adequately describe the process itself sufficiently, otherwise the transfer of low-quality characteristics will be performed. That is, within the time series (1), the characteristics of the curvature of the trend must be comparable with the radius of convergence of the polynomial of the selected order, otherwise it simply cannot cope with the nonlinearity of the process under study. Or vice versa, for a time series with significant nonlinearity of trend and volume, one should have a polynomial of comparable radius of convergence, which is not always possible. Because in this case, the number of calculation operations for statistical training will increase and random errors of polynomial coefficients will accumulate. It should be noted that the indicators of the radius of convergence will

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have a wide range of variation depending on the shape and physical essence of the process under study. For asymptotic processes, the convergence of the model will be supported by small degrees of the polynomial (up to the third and fourth orders) trained on relatively small samples. For processes with seasonal repetitions, periodic polynomials and polynomials with significant nonlinearity that are acceptable due to computational complexity will not give adequate processing results.

Such a limitation is critical for the DSB (7) and a priori makes it impossible to use some models that do not have enough non-zero components in the differential spectrum to form a certain system of equations (8). An example of this is nonlinear models:

$$f(t.a) = a_0 \sin(a_1 t) + a_2 \cos(a_1 t)\,, \tag{9}$$

$$f(t,a) = a_0 + a_1 t + a_2 e^{a_3 t}\,. \tag{10}$$

For models (9), (10) we have only three unknown parameters $\{a_0, a_1, a_2\}$, i.e., for a defined system of type (8), three discrete DSs are sufficient and, accordingly, it is sufficient to have a third-order polynomial. But such a polynomial is effective only for a limited time interval and trend curvature of a time series characterized by models (8), (7).

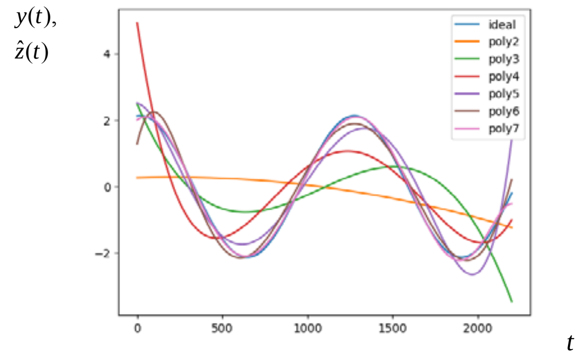

Figure 1 – Periodic function charts: ideal $y(t)$ and with polynomial models $\hat{z}(t)$, trained according to the OLS algorithm

Thus, the DSB in the DNT scheme (7) limits the requirements for structural optimization of the approximation model with control of the concordance coefficient, linear deviation, and high-order derivatives [1]. It is generally characteristic of periodic processes/functions to have many cycles, and they can be represented within an empirically assembled time series (1) as a superposition of many processes (factors).

Fig. 1 shows graphs of reproduction of periodic time series [8] by polynomial models trained by the OLS algorithm. The periodic series was modeled as a perfect trend.

The presented results clearly demonstrate the obvious consequences. Increasing the order of the polynomial model to a certain value provides an increase in its adequacy (according to absolute deviation Δ and concordance coefficient R^2 – see Table 1).

Table 1 – Results of model adequacy assessment

n	2	3	4	5	6	7
Δ	1.439	1.169	0.835	0.394	0.198	0.064
R^2	9.050	0.500	0.559	0.927	0.983	0.998

At the same time, a significant increase in the structure of the polynomial model leads to a decrease in the rate of improvement in the results of the use of OLS. This is due to a decrease in the absolute value of the high-degree coefficients of the polynomial, which gives "sensitive" solutions from the standpoint of the accumulation of calculation errors, the number of calculation operations and the influence of random errors of input data. Therefore, for a time series with any trend complexity, including a periodic structure, it is potentially possible to determine the optimal (with a minimum Δ and maximum R^2 and with acceptable estimates of accuracy) structure of a polynomial model. For example, the optimal order of the model $m = 6$.

However, such a decision on the order of a polynomial on the example of a nonlinear model with transcendental functions (9) that reproduce the periodic properties of the time series contradicts with the requirements of the DBS. That is, the number of significant (all non-zero – seven) discrete DSs of a polynomial model is greater than the number of necessary and sufficient (three) discrete models (9). The loss of discrete DSs of a polynomial optimal in terms of structure will lead to the negligence of the positive properties of the nonlinear model in terms of parameters. Thus, we have a redefined system of equations of the form (7).

These statements are explained by the following calculations.

$$P\{\hat{z}(t)\}_{t^*} \Rightarrow \hat{Z}(k) = \{\hat{Z}(0), \hat{Z}(1), \ldots, \hat{Z}(7)\}\,, \tag{11}$$

$$P\{f(t,a)\}_{t^*} \Rightarrow F(k,a) = \\ = \{F(0,a), F(1,a), \ldots, F(7,a)\}\,, \tag{12}$$

$$\{\hat{Z}(0) = F(0,a)\,. \tag{13}$$

To solve the redefined system of equations (11), it is proposed to use an OLS with a numerical approximation algorithm, which in general has a norm for minimizing the quadratic residual:

$$\sum_{i=1}^{m} [\hat{Z}_i(i) - F_i(i,a)]^2 \to \min\,. \tag{14}$$

Minimization of the quadratic form (14) with respect to the parameters of the nonlinear model (see general form of (2) and specific examples (9), (10) considering the known parameters of the polynomial model (see (3),

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(4)) proposed to be carried out using the Leverberg-Marhardt algorithm [2]. This algorithm is presented in most software libraries with an interface for a wide range of modern high-level programming languages. Importantly, the Leverberg-Marhardt algorithm gives stable solutions of minimization forms (14) for approximation models of any complexity. To establish initial approximations, it is proposed to choose the solution of the system of equations (12), (13) with the constraints of the DSB (7), (8). That is, as a solution for models (9), (10) of a definite system of form (7) from three equations.

The use of numerical methods gives an approximate and partial solution – the accuracy of which depends on the computational costs of iterative calculation processes. Therefore, such a solution is not productive, especially for high-stage models and for large information arrays.

Consequently, the determination of the optimal polynomial structure by the properties of the time series with the control of the indicators in Table 1 is not always acceptable from the standpoint of the computational complexity of the solution of the system of equations (12), (13). Moreover, the goal of processing the time series in the presented studies is a high-precision nonlinear model (2), where the polynomial (3) is an intermediate result and a means to achieve the goal [11]. At the same time, it is possible to assume that the shortcomings of the polynomial model (stochastic and dynamic errors) in transferring its properties to a nonlinear model in the DSB scheme (7), (8) can be partially compensated by the minimizing form of OLS (14). Therefore, it is important to control not so much the adequacy indicators of the polynomial model Δ and R^2, how many indicators of adequacy of a nonlinear model in terms of parameters [12]. However, to implement such an operation, it is necessary to investigate, identify and prove the dependence of the indicators of adequacy of models $f(t,a)$, $\hat{z}(t)$ – Δ and R^2:

$$\Delta = \frac{1}{n-1}\sum_{i=0}^{n}(y_i - \hat{y}_i)^2 , \qquad (15)$$

$$R^2 = 1 - \sum_{i=0}^{n}\frac{(y_i - \hat{y}_i)^2}{(y_i - \bar{y}_i)^2} . \qquad (16)$$

Research conditions. To obtain the time series, an additive mixture of an ideal trend with random noise is implemented [9]. To model an ideal trend, nonlinear models with the following parameters were used:

– Trigonometric:
  – model: $f(t) = 2.2\sin(0.01t)$;
  – sample size: 1000 times;
  – random noise parameters: normal law of distribution, standard deviation $\sigma = 0.2 c.u.$
– Exponential:
  – model: $f(t) = 0.001t + e^{0.0001t}$;
  – sample size: 10000 times;
  – random noise parameters: normal law of distribution, standard deviation $\sigma = 5 c.u.$

The processing of time series was carried out by constructing nonlinear mathematical models in the DNT scheme of transformations according to the DSB [6].

The dependencies of indicators (15), (16) for polynomial models of form (4) and nonlinear models (9), (10) were studied. The latter have an infinite DC, which makes it possible to test polynomial models of any degree.

The research results are presented in Figs. 2–4 [8].

Fig. 2. The curves and realizations of dependencies between indicators (15) and (16) for the polynomial model (4) and the nonlinear transcendental model (9) are depicted. On the axis y metrics are given for the nonlinear, and on the axis x for a polynomial model. For research by the Monte Carlo method, 100 implementations of the method for determining nonlinear by mathematical models for using DSB in the DNT scheme were carried out. The result is the dispersion of the dependence of indicators (15), (16) with a clear correlation.

Analogous dependencies for the nonlinear exponential model (10) are shown in Fig. 3.

The above graphs clearly demonstrate the presence of a correlation between the structural properties of the polynomial and nonlinear models. The variation that is present in the results is due to random errors in the data. The graphs also show: the horizontal line is the median of the scattering, and the arc corresponds to the quadratic approximation of the resulting correlation dependence. The latter visualizes the nature of the dependence of indicators (15), (16), polynomial (4) and nonlinear models (9), (10).

Analysis of the graphs of Figs. 2, 3 proves that the structure of the nonlinear model under study affects the stochastic properties of scattering, i.e. the size of the confidence domain.

Compared to the transcendental models (9), the exponential model (10) demonstrates a high correlation between parameters (15), (16) (see Fig. 3 for a linear trend with a slope) and a relatively small confidence interval of scattering (concentration of realizations relative to the linear trend from the comparison of Figs. 2, 3). In practice, this gives and explains the stability of the solution of the DSB system in the DNT, including in the presence of random measurement errors.

At the same time, the transcendental model (10) is relatively unstable in terms of R^2 (see Fig. 2b).

As a result, we have the final confirmation of the informativeness and the conditioned properties of indicators (15), (16), which are suitable for the formation of the structure of models for DBS in the DNT, as a requirement for minimization Δ and maximization of R^2.

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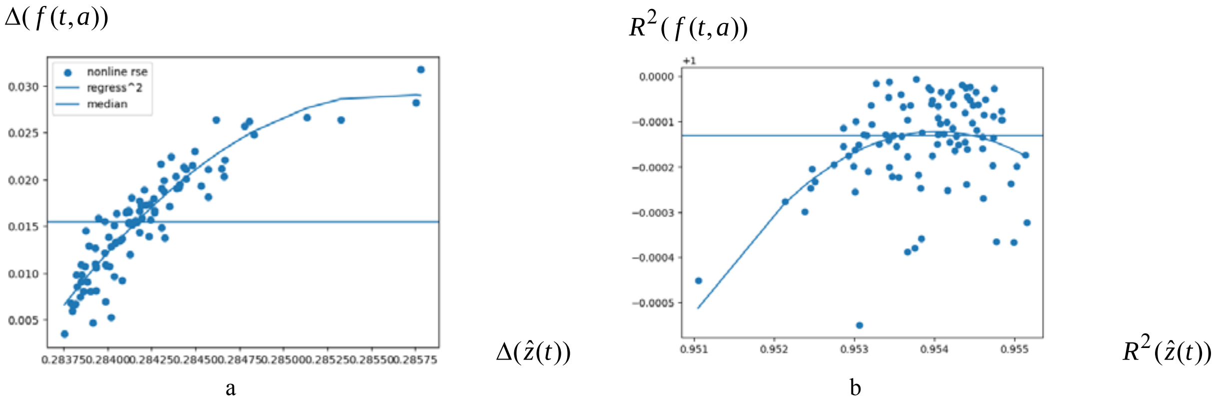

Figure 2 – Depiction of the dependence of model parameters from DSB in DNT: a – indicator (15) for models (4) and (9), b – indicator (16) for models (4) and (9)

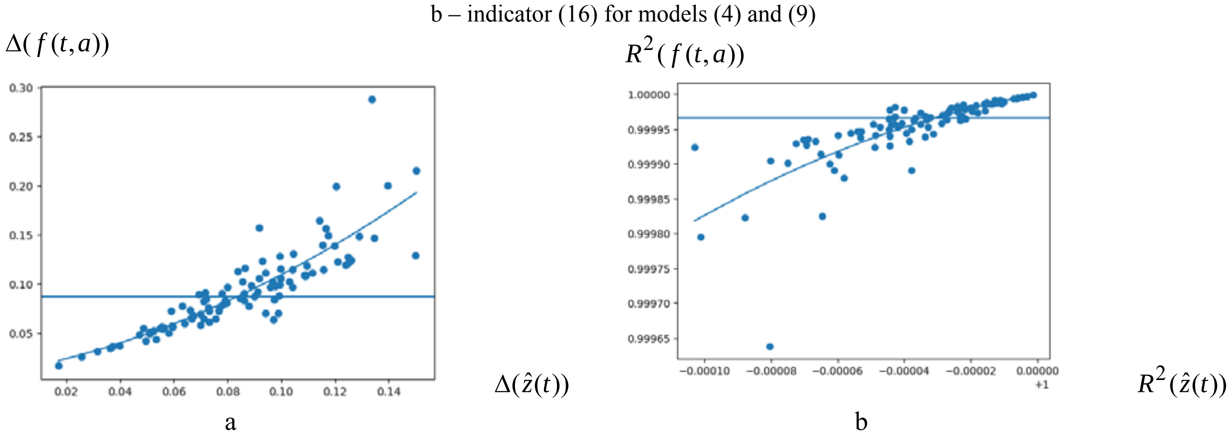

Figure 3 – Depiction of the dependence of model parameters from DSB in DNT: a – indicator (15) for models (4) and (10), b – indicator (16) for models (4) and (10)

Thus, optimization of the structure of the polynomial model with the control of indicators (15), (16) improves the properties of the nonlinear model determined from the DSB in the DNT scheme. However, optimization of a polynomial model is possible, albeit with big data, but on a limited time series (1). That is, the volume of the time series through the composition of nonlinear oscillations (seasonality) directly determines the order of the polynomial. Therefore, it is worth investigating the influence of the sample size (1) – the time series interval on the properties of the polynomial model.

Fig. 4 shows the results of studies of the dependence of the indicator (15) – control of the dynamic error of the model on the time series interval n for a polynomial of the 6th (Fig. 4a) and 10th (Fig. 4b) orders. A perfect (without random errors) sample generated by the following model was subject to modeling: $f(t) = 2.2\sin(0.00t)$. The choice of indicator (15) is due to its sensitivity to the relationship between the structure of the model and the sample size, while indicator (16) is responsible for the probabilistic ratio – relative to the stochastic component of accuracy.

We have the following local results. For a 6th-order polynomial, the minimum $\Delta(\hat{z}(t))$ is achieved for $n = 780$, and for a polynomial of degree 10 – for $n = 2100$. Therefore, the values of indicators (15) and (14) should be correlated with the size of the time series.

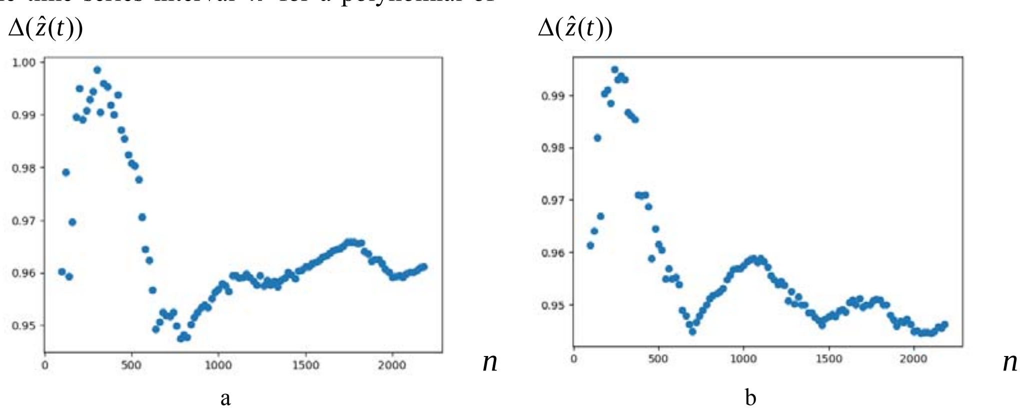

Figure 4 – Graph of the indicator Δ dependence on the size of the model the polynomial rank: a – 6th, b – 10th

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Based on the results of the research, it is possible to formalize *the methodology for determining the structure of nonlinear models for processing time series using DSB in DNT transformations.*

1. Form a polynomial model (4), the order of which is equal to the number of unknown parameters of the nonlinear model (2).

2. Based on the results of step 1, using the classical OLS, determine the parameters of the polynomial model (3) from the input time series (1) and evaluate its indicators (15), (16).

3. Repeat step 2 of the methodology for the polynomial model above in the order specified in step 1, with the control of indicators (15), (16). If the distance d (see (17)) between its indicators and the previous ones (obtained in step 2 of the methodology) decreases – continue to increase the order of the model until the distance begins to increase.

$$d = |\Delta(\hat{z}_{i-1}(t) - \hat{z}_i(t))| \times \times |R^2{}_{i-1}(t) - R^2{}_i(t)|. \tag{17}$$

Distance control (17) – indirectly allows you to consider the volume of the time series.

4. If the polynomial model obtained under the conditions of step 3 has an order equal to the number of parameters of the nonlinear model (3), then a system of equations of the form (7) should be formed, which is solved analytically with respect to the desired parameters of the nonlinear model.

5. If the polynomial model obtained under the conditions of step 3 is of an order of magnitude higher than the number of parameters of the nonlinear model (3), then a system of equations of the form (12), (13) is formed, which is solved by the chosen numerical method. To establish initial approximations, a system of equations (7) with DSB constraints is formed.

6. To implement the formation of a nonlinear model (3) from the parameters specified in step 4 or 5.

7. To apply the nonlinear model (3) (because of step 6 of the methodology) to estimate the parameters of the stochastic time series (1) in the observation area and in the forecasting interval – the formation of the current and forecast trend.

8. Evaluate indicators (15), (16) of the nonlinear model.

9. If the results of step 8 do not meet the established efficiency requirements – modify the structure of nonlinear models (3) by adding polynomial and / or nonlinear components with control of the fulfillment of one of the conditions of steps.4 and 5. Repeat steps 1–8 of the methodology for determining the parameters of the nonlinear model.

10. If the results of step 9 are unsatisfactory, use the results of steps 4 and 5 as initial approximations for any numerical methods for solving nonlinear problems of time series approximation.

4 EXPERIMENTS

Evaluation of the effectiveness of the proposed approach was carried out by methods of mathematical modeling. The time series was generated according to model (9). Samples of different sizes were studied, which ensures the variability of the composition of seasonal changes (fluctuations) [9]. Simulation configuration:

– model: $f(t) = 2.2\sin(0.01t) + \cos(0.03t)$;

– random noise parameters: normal law, $\sigma = 0.2$;

– training sample sizes: 500, 1000, 1500;

– the time series is generated from the training (observation area) and test (forecasting area) parts with a volume of $[0, n]$ and $[n, 2n]$ – respectively.

– nonlinear model for processing the form (9), polynomial structure was used in accordance with the results of the proposed technique.

5 RESULTS

The results of the research are presented in Fig. 5 and Table 2.

The graphs of Fig. 5 contain: training data (blue); ideal trend, relative to which the indicators given in Table 2 are calculated (orange); Trend restored using the proposed technique (green color).

Table 2 contains data on model quality indicators (15), (16) and standard deviations of the results of processing the input time series with random noise. Characteristics of the degree of the polynomial – record the result of the proposed technique.

The results of the studies demonstrate the obvious properties of statistical learning results – an increase in seasonality in the data gives an increase in dynamic and stochastic error. However, the discrepancy between the estimation results, compared to the ideal trend, is commensurate with the confidence interval of the forecasting error. This confirms the compliance of the proposed solutions with the basic provisions of the methods of processing time series.

In favor of the proposed solutions should be attributed the useful prognostic properties of nonlinear models, which were obtained entirely by the methods of analytical synthesis of the structure of nonlinear models in terms of parameters and determination of their parameters. This is ensured using positive properties of differential transformations in the proposed technique [4, 5].

6 DISCUSSION

In this paper the method of constructing nonlinear mathematical models in the DNT transformation scheme was further developed. The modification of the method consists in controlling the conditions for the formation of a certain system of equations in the DSB scheme to find the parameters of a nonlinear model with its analytical solutions. If the system is uncertain, the nonlinear model is supplemented with components that are linear in parameters. In the case of an overdetermined system, its solution is carried out using the least Squares. A defined system is solved by classical approaches. These processes

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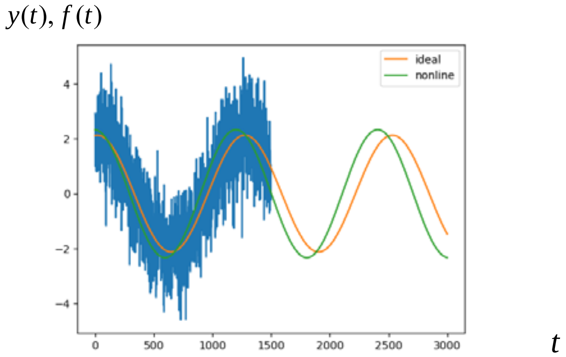
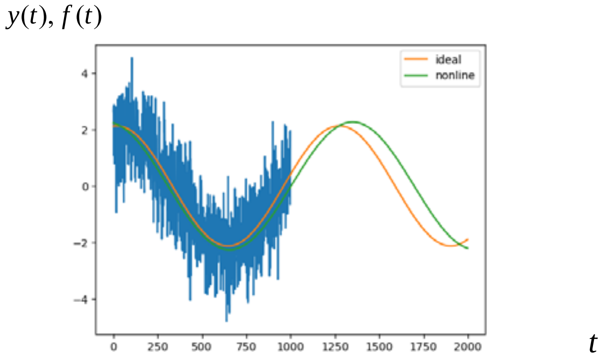
are implemented with the control of stochastic and dynamic accuracy of models in the areas of observation and extrapolation. If the results of statistical learning are unsatisfactory in terms of accuracy, the obtained parameters of the nonlinear model are used as initial approximations of numerical methods.

All these modifications are based on empirically proven interdependencies of quality indicators of polynomial models at the level of linear deviation and probability of approximation with the quality of the results of synthesis and application of nonlinear models in terms of parameters.

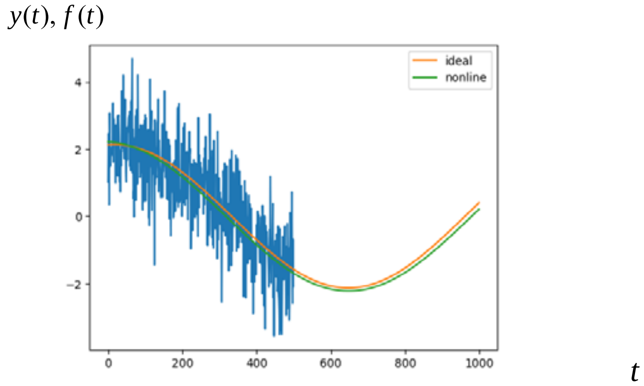
The proposed solutions are based on the analytical synthesis of the structure of nonlinear models and the determination of their parameters due to the positive properties of the method of differential transformations.



a



b



c

Figure 5 – Research results of the proposed methodology for samples: a – 1500, b – 1000, c – 500

Table 2 – Results of the model experiment

	Sample size	**Polynomial degree**	Δ	R^2	σ
train	500	4	0.12747	0.98981	1.26074
test	500	4	0.13555	0.9643	0.71604
train	1000	5	0.23514	0.97291	1.42798
test	1000	5	0.71622	0.73303	1.3848
train	1500	7	0.46893	0.91643	1.62107
test	1500	7	0.9894	0.62647	1.61779

Practical processing of the synthesized data proved the high predictive properties of the proposed approaches to the construction of nonlinear models.

The practice of applying the proposed approaches is possible in e-commerce software systems (trading, retailing, aggregation), computer systems of automated and automatic control (unmanned systems, traffic control systems) in the medical field and in other industries where there are large amounts of data in the time series format.

CONCLUSIONS

In this paper a conditional approach to analytical synthesis and determination of parameters of nonlinear models is proposed. The obtained solutions are an alternative and complement to the methods of numerical solution of nonlinear problems for processing time series and belong to the class of statistical machine learning methods.

The scientific novelty. The method of constructing nonlinear mathematical models in the DNT transforma-

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tion scheme was further developed. The modification of the method consists in controlling the conditions for the formation of a certain system of equations in the DSB scheme to search for the parameters of a nonlinear model with its analytical solutions.

The practical significance of the proposed methodology is to obtain an analytical solution to the problem of determining the parameters of nonlinear models, which increases the predictive accuracy and productivity of statistical training methods on time series.

Prospects for further research. Extension of the proposed solutions to approximate methods into an alternative to DBS, creation of a program script in the format of a specialized library of nonlinear statistical learning.

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МЕТОДИКА ВИЗНАЧЕННЯ СТРУКТУРИ НЕЛІНІЙНИХ ЗА ПАРАМЕТРАМИ МОДЕЛЕЙ ДЛЯ ОБРОБКИ ЧАСОВИХ РЯДІВ

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АНОТАЦІЯ

Актуальність. Практика задач сьогодення актуалізує підвищення вимог до точності, достовірності і повноти результатів обробки часових рядів в багатьох прикладних сферах. Одним із методів, що забезпечує високоточну обробку часових рядів із впровадженням стохастичної моделі виміряних параметрів є методи статистичного навчання. Однак, сучасні підходи до статистичного навчання обмежуються, здебільшого, спрощеними – лінійними за параметрами поліноміальними моделями. Практика доводить, що реальні дані найчастіше мають складну форму трендової складової, яка не може бути відтворена поліномами навіть високого ступеня. Згладжування нелінійних за параметрами моделей можливо реалізувати різними підходами, наприклад методом визначення параметрів нелінійних моделей з використанням балансу диференціальних спектрів (БДС) в схемі диференціально-нетейлорівських перетворень (ДНТ). Дослідження довели необхідність його модифікації в напрямку розробки обумовленого підходу до визначення структури нелінійних за параметрами математичних моделей для обробки часових рядів із складною динамікою тренду.

Метою роботи є розробка методики визначення структури нелінійних за математичних моделей для обробки часових рядів з використанням БДС в ДНТ перетвореннях.

Метод. В статті отримав розвиток метод побудови нелінійних за параметрами математичних моделей в схемі ДНТ перетворень. Модифікація методу полягає у контролі умов формування визначеної системи рівнянь в схемі БДС для пошуку параметрів нелінійної моделі з її аналітичним розв'язком. Якщо система невизначена – нелінійна модель доповнюється лінійними за параметрами компонентами. У випадку перевизначеної системи – її розв'язок здійснюється з використанням

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норми найменших квадратів. Визначена система – розв'язується класичними підходами. Зазначені процеси реалізуються із контролем стохастичної та динамічної точності моделей да ділянках спостереження та екстраполяції. Якщо результати статистичного навчання є незадовільними за точністю – отримані значення нелінійної моделі використовуються як початкові наближення чисельних методів.

Результат. На підстава проведених досліджень запропоновано методику визначення структури нелінійних за параметрами моделей для обробки часових рядів з використанням БДС в схемі ДНТ перетворень. Її застосування забезпечує обумовлений підхід до визначення структури моделей для обробки часових рядів та підвищення точності оцінювання на інтервалі спостереження та екстраполяції.

Висновки. Застосування запропонованої в статті методики визначення структури нелінійних за параметрами моделей для обробки часових рядів дозволяє отримати моделі із кращими, за показником точності, прогностичними властивостями.

КЛЮЧОВІ СЛОВА: наука про дані, статистичне навчання, часові послідовності, нелінійні моделі, чисельні методи, метод найменших квадратів.

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