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SEGMENTATION OF LOW-CONTRAST IMAGES IN THE BASIS OF EIGEN SUBSPACES OF TYPE-2 FUZZY MEMBERSHIP FUNCTIONS

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ABSTRACT

Context. The study addresses the current task of automating a sensitive image segmentation algorithm based on the Type-2 fuzzy clustering method. The research object is low-contrast greyscale images which are outcomes of standard research methods across various fields of human activity.

Objective. The aim of the work is to create a new set of informative features based on the input data, perform sensitive fuzzy segmentation using a clustering method that employs Type-2 fuzziness, and implement automatic defuzzification in eigen subspace of membership functions.

Method. A method for segmenting low-contrast images is proposed. It consists of the following steps: expanding the feature space of the input data, applying singular value decomposition (SVD) to the extended dataset with subsequent automatic selection of the most significant components, which serve as input for fuzzy clustering using Type-2 fuzzy sets. Clustering is performed using the T2FCM method, which allows the automatic selection of the number of fuzzy clusters based on an initially larger guaranteed number, followed by the merging of close clusters (proximity was defined in the study using a weighted Euclidean distance). After fuzzy clustering, the proposed method integrates its results (fuzzy membership functions) with the input data for clustering, preprocessed using fuzzy transformations. The resulting matrix undergoes another fuzzy transformation, followed by SVD and the automatic selection of the most significant components. A grayscale image is formed based on the weighted sum of these selected components, to which the adaptive histogram equalization method is applied, resulting in the final segmentation output. The proposed segmentation method involves a small number of control parameters: the initial number of fuzzy clusters, the error of the T2FCM method, the maximum number of iterations, and the coefficient of applied fuzzy transformations. Adjusting these parameters to the processed images does not require significant effort.

Results. The developed algorithm has been implemented as software, and experiments have been conducted on real images of different physical nature.

Conclusions. The experiments confirmed the efficiency of the proposed algorithm and recommend its practical application for visual analysis of low-contrast grayscale images. Future research prospects may include analyzing the informative potential of the algorithm when using other types of transformations of fuzzy membership functions and modifying the proposed algorithm for segmenting images of various types.

KEYWORDS: Image Segmentation, Fuzzy Clustering, Type-2 Fuzzy Clustering, orthogonal transformation, singular value decomposition, singular subspaces.

ABBREVIATIONS

2DPCA is a Two-Dimensional Principal Component Analysis;

FCM is a fuzzy clustering algorithm;

T2 refers to Type-2 fuzziness;

T2FCM is a fuzzy clustering algorithm based on Type-2 fuzziness;

MRI is Magnetic Resonance Imaging;

PCA is a Principal Component Analysis method.

NOMENCLATURE

N is a number if pixels of an image;

M is a width of an image;

L is a height of an image;

C is a vector of the coefficients of algorithm;

dC is a vector of differences of neighbouring elements of vector C of algorithm;

$dC_{\min}$ is a minimum element of vector dC;

$dC_{\max}$ is a maximum element of vector dC;

c is a number of fuzzy clusters;

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dC_a is a coefficient of algorithm;

K is a coefficient of algorithm;

I is an input image;

I^{out} is a segmented image;

$I^l_{\max}$ is a maximal value of processing window;

$I^l_{\min}$ is a minimal value of processing window;

$I^g_{\max}$ is a maximal value of image;

$I^g_{\min}$ is a minimal value of image;

I_0 is a center pixel of processing window;

I^i is image at various stages of algorithm, $i \in \{svd, tr, u, s\}$;

u_{ji} is a degree of membership of the object i to cluster j;

m is a weight coefficient that characterizes a measure of fuzziness;

v_j is a center of cluster j;

d_{ij} is an Euclidean distance between a center of cluster v_j and an instance of original data x_i;

U is a membership function;

w_j is a singular value from singular value decomposition, $j \in \{i, 1–6\}$;

x *is* a pixel coordinate;

y *is* a pixel coordinate;

z *is* a pixel coordinate.

INTRODUCTION

Image segmentation refers to a high level of processing and it is a mandatory stage in there most image analysis technologies. Currently, there is no universal algorithm for its implementation and the result largely depends on the quality of the initial data [1]. Low-contrast images are often used in practice, and the insufficient quality of which is due to both the features of the equipment (heterogeneity of fields, nonlinearity of sensor's characteristics, noise) and the process of their formation (anatomical features of the analysis objects, dynamic distortions, etc.). In addition to randomness, which can be controlled in accordance with the theory of probability, the objective property of images is the presence of uncertainty and ambiguity, which must be taken into account during processing.

The object of study is the process of grayscale low-contrast images' segmentation that are the result of standard research methods.

The segmentation process often involves the synthesis of the new information parameters formed on the basis of initial data which make it possible to increase the values of the initial brightness characteristics' variations and to separate the information from different sources. This is an ambiguous task related to the field of artificial intelligence. The reliability of the result depends significantly on the type and characteristics of the initial data and, as a rule, there is no a priori information about the system of their formation and noise component.

To increase the validity and sensitivity of segmentation it is necessary to take into account the ambiguity and uncertainty that are always present in digital images.

The subject of study is the technology of grayscale low-contrast images' segmentation based on the use of the T2FCM method (fuzzy clustering type_2) with automatic determination of the number of clusters with further conversion to the singular subspaces of the obtained membership functions using singular value decomposition and synthesis of the resulting image based on its eigen images of orthogonal components.

The known fuzzy clustering algorithms [2–13] are usually characterized by uncertainty in determining the number of clusters and initializing initial values, ambiguity of the defuzzification process and also, they do not take into account the spatial information. Besides, they are sensitive to the noise component.

The purpose of the work is to increase the sensitivity and reliability of grayscale low-contrast images' segmentation and, to a certain extent, to neutralize the sensitivity to the influence of noise factors (due to the synthesis of a segmented image based on its eigen images of fuzzy membership functions).

1 PROBLEM STATEMENT

It is supposed that the low-contrast grayscale image I is given as a set of brightness values I_i for i=1, 2, ..., N pixels with coordinates (x, y), x = 1, 2, ... , M, y = 1, 2, ..., L.

The synthesis problem of a segmented image can be represented as the problem of visualizing the result of its clustering on the basis of a fuzzy membership functions' matrix $U{:}\langle u_{ki}, I_i\rangle)$, where u_{ki} is a fuzzy membership function of an instance of initial data I_i with coordinates (x, y) to the k-th cluster, k=1, 2, ..., c, and $\forall k(\forall i(\text{sum}(u_{ki})=1))$.

The IU transformation: $f(\langle u_{ki}{}^m\rangle, \langle d_{ki}\rangle) \rightarrow min$, where d_{ki} is the Euclidean distance between the centre of the cluster v_j and the object I_i; m is the weight coefficient that characterizes the measure of fuzziness. The clustering parameters c, m are chosen experimentally.

Creation of a segmented image $I^{out} : f(\forall k(\forall i(u_{ki})))$.

2 REVIEW OF THE LITERATURE

Image segmentation is a complex analysis procedure. It belongs to the tasks of unsupervised learning. Modern algorithms can be used in conditions of almost complete lack of information about the data distribution laws. They use the measure of object parameters' proximity in a multidimensional space and rely solely on heuristic considerations about the nature and characteristics of the investigated set.

Inaccuracy and uncertainty are present in all digital images and can lead to errors while forming data for analysis based on brightness characteristics [2]. The concept of "ambiguity of grey" reflects the fact that the accuracy of brightness values is primarily limited by the dig-

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itization process (quantization in amplitude and spatial discretization), as well as, by the precision of the hardware systems. For example, ambiguity in medical images is caused by the forming fields' heterogeneity or movement of a patient. It's the weather conditions in satellite images. It's the inaccuracy and unevenness of the measurement grid in geological field images. It's the uneven background, significant noise, aberration artifacts, etc. in microscopic images. Geometric fuzziness manifests when determining object boundaries because of deformations from motion capture, insufficient resolution, low contrast and noise influence. It should be noted separately the need to consider the peculiarities of a human eye as an instrument of visual analysis which adheres to Weber's laws. In particular, it cannot detect changes in the grey level below the threshold of visual perception [3, 4].

The procedure of image segmentation to enhance analysis reliability must correspond to the contradictory requirements – noise removal while preserving fine details, as well as the delineation of object boundaries without excessive detailing and emerging artifacts. One of the ways to solve the problem of improving images' quality and the reliability of their analysis is based on the formation of the new informative features on the assumption of the initial data, which change the relationship (the space structure) of the analyzed features [5]. The use of fuzzy logic due to nonlinearity allows to increase the influence of variations in the brightness properties and eliminate ambiguity of the initial data [6]. The processing medical images using a fuzzy approach allows to increase resolution, highlight the features of the individual areas' structure and improve the accuracy of segmentation [7]. It is necessary to carry out preliminary conversion (improving) for noisy low contrast images before the targeted processing is performed. Most often, the local characteristics of each pixel are converted.

The key problem for the implementation of sensitive segmentation is the formation of a relevant set of inputs that provide a solution to the problem. The synthesis of information parameters is considered as the transforming process of the images' initial brightness characteristics into a new virtual space where there is a redistribution of brightness characteristics, which allowing to increase the analyzed brightness range and (or) the metric distance between the object of interest and the "background". Choosing different types of source image transformations and segmentation methods leads to different results.

Fuzzy clustering methods provide a set-theoretic partitioning into subsets and associate each object with a fuzzy set with a membership function that varies in the interval [0, 1] [8]. The first developed fuzzy clustering method was the FCM (Fuzzy C-Means) algorithm. In 1980 J. Bezdek proved its convergence and in 1981 he generalized the algorithm in the case of arbitrary fuzzy sets [9]. Today there are a significant number of FCM method modifications aimed at solving the specific problems with maximum consideration of images` specifics [10–13].

The FCM algorithm problems can be formulated as follows:

1. There is no theoretical justification for the importance of the choice of the clusters` number and the fuzziness degree;
2. The use of pseudorandom initial values of centroids leads to different results;
3. Noise sensitivity;
4. The spatial information which is essential for image processing is not used [14, 15];
5. The process of defuzzification is ambiguous.

The membership functions obtained within the framework of FCM method can be interpreted as an ensemble of multiparameter data with the possibility of using multivariate analysis algorithms for its processing, for example, orthogonalization methods. The ideas of projection methods into eigen subspaces as one of the tools for mathematical processing of experimental data, were presented in the works [16, 17].

In particular, methods called principal component analysis (PCA) involve the use of statistical principles to reduce the number of inputs in order to extract the most significant factors from the input data. This approach in the problems of processing of images which are two-dimensional structures was applied in practice only in the 2000s [18]. Two-dimensional principal component analysis (2DPCA) and other orthogonalization techniques are currently used to solve such important problems as compression of visual information, feature extraction in object recognition and search for video images, reduction of calculations in image processing, etc. [19, 20]. The image covariance matrix is constructed directly with using the original image matrices and its eigenvectors are computed to highlight the features of the image that determine the internal structure of the experimental data.

This article deals with the information possibilities of using the method of multivariate information analyzing based on fuzzy clustering and singular decomposition in the analysis of low-contrast greyscale images.

3 MATERIALS AND METHODS

The formalization of the term "fuzzy set" involves generalizing the concept of membership reflecting the idea that elements of a set can have a common property to varying degrees. Unlike the probability which is connected with the uncertainty regarding an object's membership in a crisp set, fuzzy logic provides a foundation for developing a more flexible approach to data analysis and allows to make a decision among a set of alternative options. The general scheme of fuzzy image processing includes the following stages: input parameter formation, fuzzification, processing/interpretation of membership function values and defuzzification on the basis of which the final result is formed [21, 22].

The mutual mapping of grey levels and fuzzy membership function surfaces can be interpreted as a specific type of nonlinear coding-decoding, in terms of fuzzy logic is data fuzzification-defuzzification. The complexity of the fuzzy approach is the uncertainty in the

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formation of the final result in a multidimensional space. The methods for its implementation are determined mainly by the purpose of processing and differ in the peculiarities of taking into account the topology and properties of the analyzed image, for example, the dynamic range of brightness, the used color model, the number of channels, the size of the ensemble, etc.

Image I of size $N = M \times L$ can be represented as an array of fuzzy sets regarding the analyzed property, in particular, brightness, with the value of the membership function $u_{x,y}$ varying in the interval $[0,1]$, for each pixel:

$$I = \bigcup_{x=1}^{M}\bigcup_{y=1}^{L}\frac{u_{x,y}}{I_{x,y}}. \qquad (1)$$

The fuzzification process can be carried out in various ways. For example, fuzzification based on histogram analysis refers to global methods that take into account the grey level of each pixel on the basis of which the membership function to one or more classes, such as very dark, slightly bright, medium, etc., is determined according to the given requirements. Its use requires prior knowledge about the analyzed image; for example, the minimum and maximum of grey levels` frequencies. However, the accuracy of these points` detecting on the histogram does not need to be very high because the concept of fuzziness is used.

Neighbourhood-based fuzzification takes into account a defined neighbourhood of pixels and typically requires more computational time compared to the histogram-based approach.

This approach requires additional data analysis because noise and outliers (anomalous values) can lead to false values of the membership functions for a subset of pixels. For example, for a window of size $r \times r$ the membership function of the central pixel can be described as follows:

$$u = 1 - \left[1 + \frac{1}{\max(I_i)}\sum_{i=1}^{r-1}\left\|I_0 - I_i\right\|\right]^{-1} \qquad (2)$$

or

$$u = 1 - \left[1 - \frac{I_{\max}^{l} - I_{\min}^{l}}{I_{\max}^{g} - I_{\min}^{g}}\right]^{-1}, \qquad (3)$$

where, $I_{\max}^{l}$, $I_{\min}^{l}$, $I_{\max}^{g}$, $I_{\min}^{g}$, are the local and global extremes of the window and the image as a whole, respectively; I_0 is the central pixel.

To solve high-level problems, it is necessary to extract properties of the analyzed image (such as the object length, the region homogeneity, the entropy, the average value, etc.), which are then subjected to fuzzification.

Fuzzy clustering techniques that iteratively determine the matrix U – the degree of pixels` belonging to fuzzy clusters can also be interpreted as a fuzzification stage.

The simplest way to perform the defuzzification procedure of the clustering result is to select the value corresponding to the maximum of the membership function. When using this approach, a new parameter whose visualization provides the image segmentation is synthesized as follows:

$$I_{x,y}^{out} = \max(u_{k,x,y}), \forall k \in [1,c], \qquad (4)$$

where $u_{k,x,y}$ is the membership function of a pixel in the output image with coordinates x, y to the k-th fuzzy cluster. The resulting image will always be greyscale.

Another method of visualization is to form an output image I^{out} based on the centres of the clusters (v):

$$I_{x,y}^{out} = \sqrt{\sum_{j=1}^{q} v_{I_{x,y,j}^{in}}^{2}}. \qquad (5)$$

However, the values of the membership function can have comparable or even equal values for different clusters, which leads to ambiguity and therefore, unreliability of defuzzification. Fig.1 c shows the type of two membership functions` graphs for clustering into 6 clusters for the model image in Fig. 1 a whose histogram is presented in Fig. 1 b where the two membership functions have a substantial number of identical values.

On the other hand, each class, by definition, also contains the information suitable for the analysis that can be lost when performing expression-based defuzzification (4, 5). If the membership function of each class is interpreted as an image where each one carries specific information, then during segmentation, it is necessary to form a "composite" image based on the image merging of all membership functions.

Fuzzy logic of type_2 (T2) makes it possible to consider the problems with a higher degree of uncertainty, in particular, in the methods of image representation and in the algorithms for their processing [23].

The theoretical basis of our approach to the defuzzification process is the fact that by specifying a specific number of c classes for fuzzy clustering, a three-dimensional matrix U containing a set of c values of membership functions for each field pixel is gotten. The dimension of the third z coordinate is equal to the specified number of classes. The latter means that within the framework of FCM method it is possible to obtain automatically an ensemble of multidimensional data and apply methods for multidimensional information processing. Each membership function can be interpreted as an image, and the U array can be interpreted as a multidimensional image with c channels.

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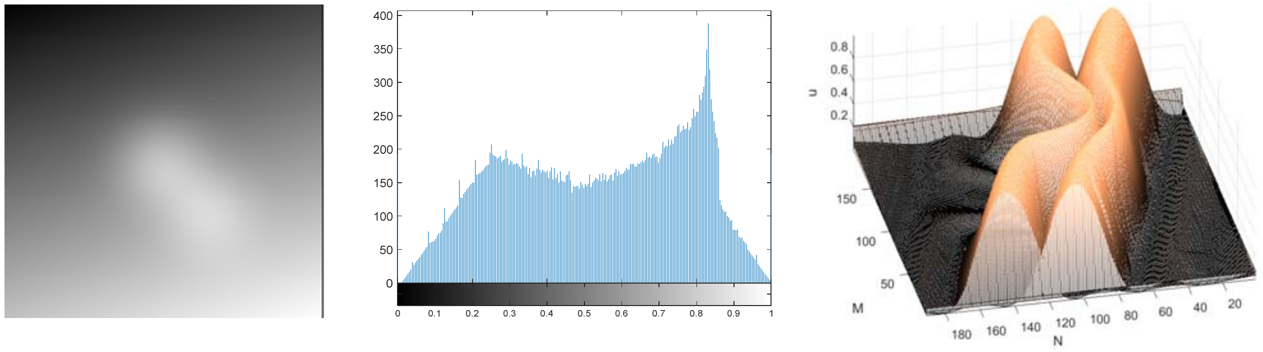

a b c

Figure 1 – View of the Fuzzy Membership Functions: a – model image; b – histogram; c – membership functions of two classes

In particular, the application of singular value decomposition (SVD) for multidimensional data enables a transition to a new (eigen) orthogonal subspace, where each new component results from a linear composition of all original parameters. This allows for a comprehensive analysis. The informational contribution of each new component can be determined by analyzing the spectrum of normalized singular values $w_i, i \in [1, n]$, and $w_1 \geq w_2 \geq \ldots \geq w_n \geq 0$.

Applying the singular transformation to the matrix of fuzzy cluster membership functions forms an orthonormal "eigen images" basis of these membership functions. Each of the "eigen images" contains H_i % of all the information contained in the original ensemble of the matrix U. This circumstance allows defuzzification to be performed based on the information contained in all membership functions. By additionally imposing, for example, a constraint:

$$\sum_{i=1}^{p} \hat{w} \geq H, \tag{6}$$

where H is a specified boundary value, such as 95% or 99%, $p \leq n$, it becomes possible to influence the sensitivity of segmentation. The physical meaning of the expression (6) is a filtration of non-essential information components (noise).

Based on the above, an image segmentation algorithm is proposed that, firstly, synthesizes new informative parameters based on the original brightness characteristics, secondly, applies a type-2 fuzzy clustering algorithm with automatic cluster number determination, and thirdly, performs automatic defuzzification using the "eigen images" of membership functions.

The algorithm includes the following steps:

1. Transforming the original greyscale image brightness values using a window transformation (window size 3x3), forming a 9-dimensional ensemble including the brightness of neighbouring pixels to take into account for spatial characteristics.

2. Performing an orthogonal transformation (SVD) on the expanded original data, followed by automatic selection of the most significant components [24] based on calculating the coefficient vector C by the formula:

$$C_i = \frac{\left|\sum_{j=1}^{1+c} (V_s)_{i,j}\right| + \left|\sum_{j=1}^{1+c} (V_s)_{j,i}\right|}{2}, i \in [1, 1+c], \tag{7}$$

where V_s contains the right singular vectors for the SVD. This vector C is sorted in descending order, and a difference vector dC is created, containing the differences for each neighbouring pair in the sorted vector C. The value dC_a is then calculated as:

$$dC_a = \frac{\sum_{j=1}^{c} \frac{dC_j}{c} + \frac{dC_{\min} + dC_{\max}}{2}}{2}, \tag{8}$$

where $dC_{\min}$ and $dC_{\max}$ are the minimum and maximum elements of vector dC, respectively. This threshold value dC_a is used to determine the number of the most significant elements of the left singular vector matrix. The selected indices correspond to the original indices in vector C prior to sorting I^{svd}.

3. Scaling each component of matrix I^{svd} to the interval $[0,1]$, forming a multidimensional input matrix for fuzzy data clustering.

4. Performing fuzzy clustering on the scaled matrix I^{svd} using the T2FCM [25] method with dynamic compression of the fuzzy membership function [9]. This involves initially setting a larger number of fuzzy clusters, which dynamically decreases during training by merging close clusters. The weighted Euclidean distance is used to determine closeness, calculated (for the distance between the centers of the *k*-th and *l*-th clusters) as follows:

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$$d_{k,l}^{0} = \sqrt{\sum_{j=1}^{q} \left(S_{u_k} \cdot v_{k,j}^{t} - S_{u_l} \cdot v_{l,j}^{t} \right)^2}, \tag{9}$$

where values S_{u_k} and S_{u_l} are calculated as:

$$S_{u_j} = \frac{\overline{u_j}}{\overline{u_{\min}}}, \left(\forall j \in \{1, \ldots, c\} \right), \tag{10}$$

where $\overline{u_j}$ is the average membership value for the j-th fuzzy cluster, and $\overline{u_{\min}}$ is the minimum value of the vector $\bar{u}$ of average membership values for each cluster.

During each training iteration t, the MFT2 matrix a^t is determined based on the difference between the "upper" u_h^t and "lower" u_l^t membership functions as follows:

$$\left(u_{h1}^{t} \right)_{i,k} = \left(\left(u^{t} \right)_{i,k} \right)^{1 - \left(\left(u^{t} \right)_{i,k} \right)^{1 + \left(\left(u^{t} \right)_{i,k} \right)^{1 - \left(u^{t} \right)_{i,k} + \overline{\left(u^{t} \right)}_{k}^{1} - 0.5}}}, \tag{11}$$

$$\left(u_{l1}^{t} \right)_{i,k} = \left(\left(u^{t} \right)_{i,k} \right)^{1 + \left(\left(u^{t} \right)_{i,k} \right)^{1 - \left(\left(u^{t} \right)_{i,k} \right)^{1 - \left(u^{t} \right)_{i,k} - \overline{\left(u^{t} \right)}_{k}^{1} + 0.5}}}, \tag{12}$$

$$\left(u_{h2}^{t} \right)_{i,k} = \left(\left(u^{t} \right)_{i,k} \right)^{K + 0.75 + \overline{\left(u^{t} \right)}_{k}^{1} / 2}, \tag{13}$$

$$\left(u_{l2}^{t} \right)_{i,k} = \left(\left(u^{t} \right)_{i,k} \right)^{K + 1.25 + \overline{\left(u^{t} \right)}_{k}^{1} / 2}, \tag{14}$$

$$\overline{\left(u^{t} \right)}_{k}^{1} = \left(\overline{\left(u^{t} \right)}_{k} \right)^{1 - \max\left(\overline{\left(u^{t} \right)}_{k}, 1 - \overline{\left(u^{t} \right)}_{k} \right)}, \tag{15}$$

$$\left(u_{h}^{t} \right)_{i,k} = \left(\left(u_{h1}^{t} \right)_{i,k} + \left(u_{h2}^{t} \right)_{i,k} \right) / 2, \tag{16}$$

$$\left(u_{l}^{t} \right)_{i,k} = \left(\left(u_{l1}^{t} \right)_{i,k} + \left(u_{l2}^{t} \right)_{i,k} \right) / 2, \tag{17}$$

where $\overline{u^t}_k$ is the average membership for the k-th cluster, and $i \in [1, N]$, K is the coefficient, values of which are recommended to be chosen within the interval $[0, 0.2]$. This coefficient has a significantly impact the transformation results. The final membership function matrix a^t is then interpreted as a multidimensional image.

5. Transforming the scaled matrix I^{svd} by the formula:

$$I_{i,j}^{tr} = \left(I_h^{svd} \right)_{i,j} - \left(I_l^{svd} \right)_{i,j}, \tag{18}$$

where I_h^{svd} and I_l^{svd} are calculated as per formulas (11) and (12), respectively, forming matrix I^{tr}.

6. Constructing matrix I^u as the union of the columns of matrices I^{tr} and a^t.

7. Applying the transformation described in step 4 to matrix I^u to compute matrix a^t (formulas (11)–(17)), resulting in matrix I^{T2}.

8. An orthogonal transformation (SVD) is applied to the matrix I^{T2}. Following this, the most significant components are selected from the left singular vectors, as described in step 2. A grayscale image I^s is formed using a weighted sum of selected significant components based on the values of the vector C, corresponding to the selected significant components of the left singular vectors, scaled so that their sum equals 1.

9. Applying the adaptive histogram equalization method (using a uniform transformation function) to the image I^s, forming the final grayscale image I^{out}.

4 EXPERIMENTS

Traditionally, the number of clusters for fuzzy segmentation is determined empirically, and defuzzification is based on the maximum of the membership function. Our algorithm employs the T2FCM method with automatic determination of the final number of clusters. The following control parameters were used: fuzziness coefficient $m = 2$, exit condition $\varepsilon \le 10^{-5}$, maximum iteration number set at 100, and an initial number of fuzzy clusters $c = 9$ (the experiments were conducted on medical images, and therefore, this amount is more than enough). The starting initialization of the cluster centers for the T2FCM method was carried out by filling them with values from the original data vectors. Coefficient $K = 0.075$ (for formulas (13) and (14)) was used in all experiments.

It was found that for some images, the best results could be obtained without applying step 7 in the proposed algorithm (using matrix I^u in step 8 instead of I^{T2}).

The presented experiments were conducted on images of various physical nature, termed "low-contrast". Examples of images and histograms are shown in Figure 2: MRI (a, b) and X-ray (c, d), respectively. Unlike typical low-contrast images, characterized by narrow histograms in low brightness regions, these images have features such as multimodal histograms, a full or nearly full intensity range with significant dark and light regions, smooth brightness transitions in areas of interest, and blurred boundaries, lack of a priori information about the presence and location of anomalies that can be compared with the noise level making visual analysis challenging.

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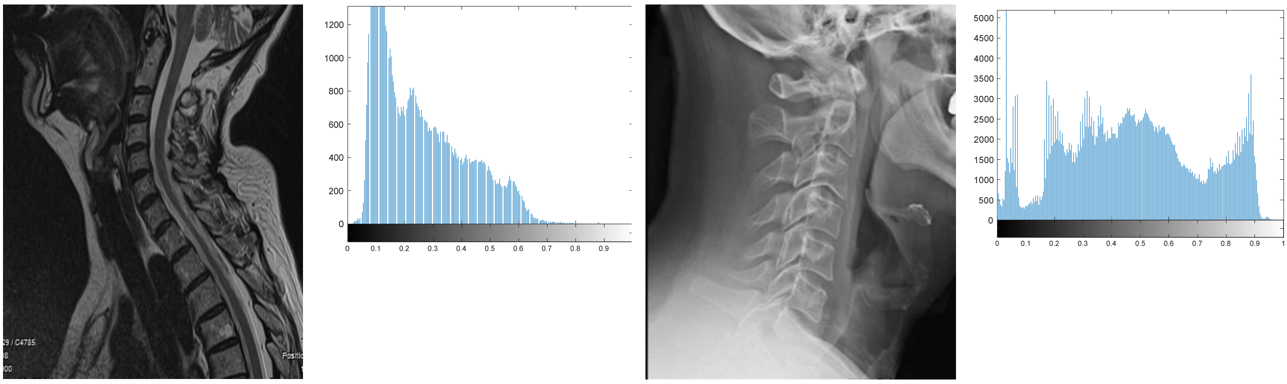


a b c d

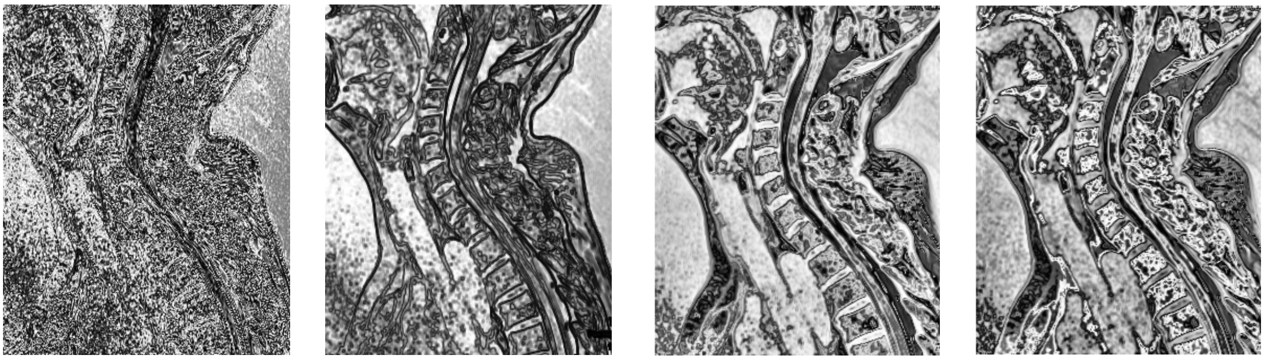
Figure 2 – Low-contrast images and their histograms: a, b – MRI; c, d – X-ray

5 RESULTS

Figure 3 illustrates various segmentation methods applied to the MRI image shown in Figure 2 a: Figure 3 a depicts the FCM method with $c = 6$ (since medical images typically contain 5–7 different tissues) based solely on the intensity characteristics using expression (4); Figure 3 b shows segmentation with an extended feature space as described in Section 1; Figure 3 c presents the proposed algorithm without Step 7; and Figure 3 d shows the result of the full proposed algorithm.

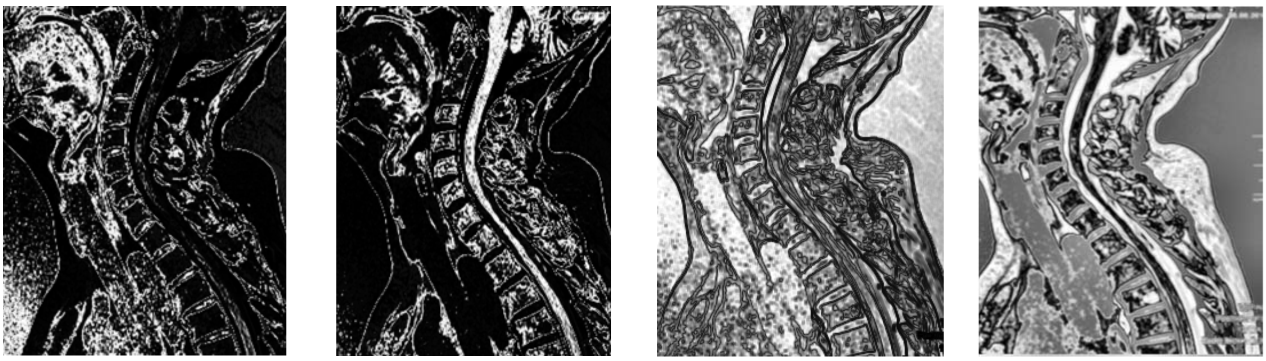
Figures 4 a and 4 b present the membership functions of two different classes of the membership function U, while Figures 4 c and 4 d show the first and second eigen images of the membership functions.

Figure 5 a shows the segmentation result of the X-ray image shown in Figure 2 c, and Figure 5 c presents the result for the optoelectronic metallographic image shown in Figure 5 b.



a b c d

Figure 3 – Visualization of fuzzy clustering results for the MRI image (6 classes): a – original image; b – maximum membership function based on the original data; c – with an extended feature space (step 1); d – proposed method



a b c d

Figure 4 – Visualization of fuzzy clustering results for the MRI image into 6 classes: a, b – two arbitrary fuzzy classes; c, d – first and second eigen images of membership functions

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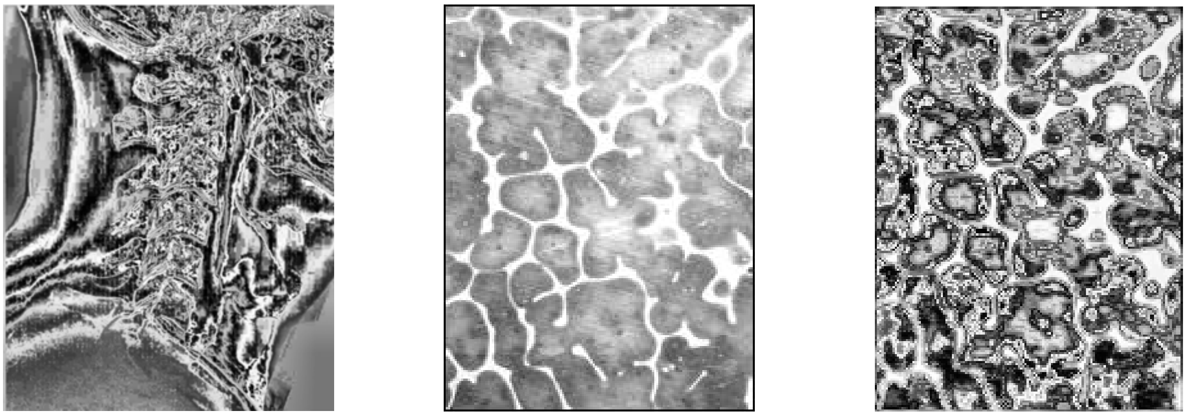

a b c

Figure 5 – Image segmentation: a – segmentation of the X-ray image from Figure 2 c; c – segmentation of the optoelectronic metallographic image from Figure 5 b

6 DISCUSSION

Fuzzy clustering is a sensitive tool, particularly responsive to the quality and composition of the original dataset. Visual analysis in Figure 3a shows that defuzzification by maximum membership function based solely on brightness forms a "noisy" image. Expanding the original brightness space per steps 1 and 2 significantly improves the result (Figure 3 b).

For example, for Figure 2 a, Table 1 presents the percentage of information H contained in each of the eigen images from the singular value decomposition of the membership functions. The first principal component carries the most information among all the obtained membership functions, while the second one contains the highest amount of information among the remaining components, and so on.

Table 1 – Eigenvalues of singular value decomposition (%)

	w_1	w_2	w_3	w_4	w_5	w_6
H_0	45.858	22.102	13.881	9.553	4.751	3.853
H_1	56.533	21.511	11.140	6.348	3.516	0.952

The analysis of eigenvalues shows increased informativeness for the first three principal components, from 81.841% to 89.184%, for cases where only the brightness of pixels (H_0) is used as the initial data and its conversion in accordance with step 1 (H_1).

In addition to the above, the information contained in the membership function of each class is largely lost during defuzzification based on expressions (4), (5) (Fig. 4 a, b).

The process of orthogonalizing membership functions enables conversion from peer cluster space to non-peer eigen image space. As seen in Figure 3 c, the use of the eigen space of membership functions from the singular value decomposition during defuzzification eliminates excessive detail and artifacts, enhances the clarity of object delineation, and simplifies the procedure for visually analyzing the results.

The automatic defuzzification method for the fuzzy clustering process, implemented by the proposed algorithm, demonstrates a significant improvement in the sensitivity and informativeness of the synthesized image. It ensures clear delineation of the objects of interest and reveals the structure of various tissues compared to the original image.

CONCLUSIONS

A relevant problem has been solved: a sensitive algorithm for automatic segmentation of low-contrast grayscale images was developed based on type-2 fuzzy clustering.

The scientific novelty lies in the first-time proposal of a method for automatically forming segmented images in the eigen space of type-2 fuzzy clustering membership functions, enhancing accuracy and sensitivity.

The algorithm incorporates mechanisms for accounting for spatial components (expansion of the feature space based on window transformation), a clustering method using type-2 fuzziness, and result formation in the orthogonal space of membership functions. The singular value decomposition method was applied to the input data for fuzzy clustering, which were preliminarily subjected to fuzzy transformations and to the membership functions of the T2FCM method, with automatic determination of the final number of clusters, followed by the selection of the most significant components. The final result is formed based on the weighted sum of these selected components.

The practical significance of the results obtained lies in the fact that the proposed algorithm ensures improved accuracy and sensitivity in the segmentation of low-contrast grayscale images of various physical origins.

The prospects for further research involve studying the proposed algorithm's application to segmenting various types of images, analyzing its informational potential when using alternative types of functional dependencies for forming type-2 fuzzy membership functions and other transformation's types of fuzzy membership functions for result generation.

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СЕГМЕНТАЦІЯ СЛАБКОКОНТРАСТНИХ ЗОБРАЖЕНЬ У БАЗИСІ ВЛАСНИХ ПІДПРОСТОРІВ НЕЧІТКИХ ФУНКЦІЙ НАЛЕЖНОСТІ ТИПУ-2

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АНОТАЦІЯ

Актуальність. Розглянуто актуальне завдання автоматизації чутливого алгоритму сегментації зображень на основі методу нечіткої кластеризації типу-2. Об'єктом дослідження є слабкоконтрастні напівтонові зображення, які є результатом стандартних методів дослідження в різних галузях діяльності людини.

Мета роботи – створення нового набору інформативних ознак на основі вихідних даних, виконання чутливої нечіткої сегментації на основі метода кластеризації з використанням нечіткості 2-го порядку, реалізація автоматичної дефаззифікації у власному підпросторі функцій приналежності.

Метод. Запропоновано метод сегментації слабкоконтрастних зображень, який складається з наступних кроків: розширення простору ознак вхідних даних, застосування сингулярного розкладу до розширеного набору даних з наступним автоматичним відбором найбільш значущих компонентів, що є вхідними даними для нечіткої кластеризації з використанням нечітких множин типу-2. Ця кластеризація відбувається за допомогою методу T2FCM, який дозволяє автоматично підбирати кількість нечітких кластерів на основі початкового задання гарантовано більшої кількості з наступним злиттям близьких кластерів (в роботі близькість визначалась на основі зваженої Евклідової відстані). Після виконання нечіткої кластеризації в запропонованому методі здійснюється об'єднання її результатів (нечіткої функції належності) з вхідними для нечіткої кластеризації даними, які попередньо оброблюються нечітким перетворенням. Результуюча матриця знов підлягає нечіткому перетворенню, після чого до отриманих результатів застосовується сингулярний розклад з наступним автоматичним відбором найбільш значущих компонентів. На основі зваженої суми цих відібраних компонентів формується напівтонове зображення, до якого застосовується метод адаптивної еквалізації гістограми, в результаті чого і отримується кінцевий результат сегментації. Запропонований метод сегментації має невелику кількість керуючих параметрів: початкову кількість нечітких кластерів, помилку методу T2FCM та максимальну кількість ітерацій, а також коефіцієнт використаних нечітких перетворень, причому їх налаштування на зображення, що оброблюються, не вимагають значних зусиль.

Результати. Розроблений алгоритм реалізовано програмно, проведено експерименти на реальних зображеннях різної фізичної природи.

Висновки. Проведені експерименти підтвердили працездатність запропонованого алгоритму та дозволяють рекомендувати його для використання на практиці при вирішенні задач візуального аналізу слабкоконтрастних напівтонових зображень. Перспективи подальших досліджень можуть полягати в аналізі інформативних можливостей алгоритму при використанні інших типів перетворень нечітких функцій належності та в модифікації запропонованого алгоритму для сегментації зображень різного типу.

КЛЮЧОВІ СЛОВА: сегментація зображень, нечітка кластеризація, нечітка кластеризація типу_2, ортогональні перетворення, сингулярне розкладання, власні підпростори.

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