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TWO-LAYER GRAPH INVARIANT FOR PATTERN RECOGNITION

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ABSTRACT

Context. The relevance of the article is driven by the need for further development of object recognition (classification) algorithms, reducing computational complexity, and increasing the functional capabilities of such algorithms. The graph invariant proposed in the article can be applied in machine vision systems for recognizing physical objects, which is essential during rescue and monitoring operations in crisis areas of various origins, as well as in delivering firepower to the enemy using swarms of unmanned aerial vehicles.

Objective is to develop a graph invariant with low computational complexity that enables the classification of physical objects with a certain level of confidence in the presence of external interference.

Method. The physical object to be recognized (identified) is modeled by a connected undirected weighted graph. To identify the constant characteristics of different model graphs, the idea of selecting the minimum and maximum weighted spanning trees in the structure of these graphs is applied. For this purpose, the classical and modified Boruvka-Sollin's method are used (modified – for constructing the maximum weighted spanning tree). Such a stratification of the structure of the initial graph into two layers provides a larger information base during image analysis regarding the belonging of a certain implementation to a certain class of objects.

Next, for each of the resulting spanning trees, two numerical characteristics are calculated: the weight of the spanning tree and the Randić index. The first characteristic contains indirect information about the linear dimensions of the object, while the second conveys its structural features. These characteristics are independent of vertex labeling and the graphical representation of the graph, which is a necessary condition for graph isomorphism verification. From these four obtained characteristics, an invariant is formed, which describes the corresponding physical object present in a single scene.

To fully describe one class or subclass of objects in four scenes (top view; front and rear hemispherical views; side view), the pattern recognition system must have four corresponding invariants.

Results. 1) A two-layer invariant of a weighted undirected graph has been developed, enabling the recognition of physical objects with a certain level of confidence; 2) A method for recognizing physical objects has been formalized in graph theory terms, based on hashing the object structure using the weights of the minimum and maximum spanning trees of the model graph, as well as the Randić index of these trees; 3) The two-layer invariant of the weighted undirected graph has been verified on test tasks for graph isomorphism checking.

Conclusions. The conducted theoretical studies and a series of experiments confirm the feasibility of using the proposed graph invariant for real-time pattern recognition and classification tasks. The estimates obtained using the developed method are probabilistic, allowing the system operator to flexibly approach the classification of physical objects within the machine vision system's field of view, depending on the technological process requirements or the operational situation in the system's deployment area.

KEYWORDS: physical object, weighted undirected graph, isomorphism, minimal (maximal) spanning tree of a model graph, graph invariant, pattern recognition, algorithm, method.

ABBREVIATIONS

UMS is an unmanned systems;

EW is an electronic warfare.

NOMENCLATURE

G is an undirected weighted graph modeling an object;

V is a set of vertices of the model graph G;

v is a number of vertices of the graph G;

E is a set of edges of a model graph G;

q is a number of edges of a graph G;

G_* is a model graph of object implementation;

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G' is a minimum spanning tree of a model graph G;

G'' is a maximum spanning tree of a model graph G;

E' is a set of edges that make up a tree G';

E'' is a set of edges that make up a tree G'';

$W(G')$ is a tree weight G';

$W(G'')$ is a tree weight G'';

$r(G')$ is a Randić index for the tree G';

$r(G'')$ is a Randić index for the tree G'';

$B(G)$ is a graph invariant G;

n is a dimension of the invariant;

e_{ij} is an edge of a graph;

$w(e_{ij})$ is a weighting edge e_{ij};

$d(v_i)$, $d(v_j)$ is a degrees of vertices between which there is an edge e_{ij};

X^m is a training sample;

m is a sample size;

Y is a set of object class names;

N is a number of implementations of objects subject to classification;

O is an estimation of the computational complexity of the algorithm;

P is a probability of correct classification of objects.

INTRODUCTION

The experience of combat operations conducted on the territory of Ukraine since 2014 and Russia's large-scale war against Ukraine, which began on February 24, 2022, indicate that increasing the combat effectiveness of means of defeating enemy manpower and equipment, developing new and improving existing types of weapons equipped with elements of artificial intelligence, remains a promising direction of development in the field of developing new models of weapons and military equipment.

The experience gained during the war years confirms that the massive use of both air, surface (underwater), and ground-based unmanned systems (UMS) on the battlefield clearly creates advantages over the enemy [1]. Such advantages can lead to positive changes in favor of their troops even at the operational level. In confirmation of these words, it is enough to recall the change in the operational situation in the Black Sea basin, which occurred as a result of the use of unmanned surface vehicles of the Sea Baby and Magura types by the Ukrainian Defense Forces against the Black Sea Fleet of the Russian Federation over a certain period of time.This influence forced the Russian Federation to relocate the main combat fleet to Novorossiysk.

At the same time, it should be noted that UMSs (ground, air, surface) are equipped with video cameras, which allows the operator of such a complex to control the drone in real time and direct it to the target. One of the effective ways to combat unmanned aerial vehicles is the use of electronic warfare (EW) means.The general principle of operation of EW means is to introduce artificial interference into the radio communication channels of the drone control, which allows such a device to be diverted from the target, and even to disable it at a considerable distance from the target.

Recently, to increase the effectiveness of the use of UMS in conditions of EW, manufacturers have begun to use elements of artificial intelligence, equipping their products with target "capture" systems, the so-called automatic UMS targeting systems.After the operator fixes the target, the drone attacks it in autonomous mode, which makes it invulnerable to enemy EW at the final section of the trajectory.

In the case of using a swarm (large group) of unmanned aerial vehicles, control according to the "operator-drone" scheme becomes ineffective, since it requires an appropriate number of operators, and most importantly, their coordinated work in a group in a rapidly changing environment, which is practically impossible to achieve in practice (this especially applies to aerial drones and is due to their relatively high flight speeds and the complexity of controlling the drone in the air).In order to eliminate this problematic situation, the control of the drone swarm is carried out under the control of artificial intelligence, according to the "launch it and forget it" principle.For this purpose, the next stage in the development of artificial intelligence was the introduction of so-called machine vision systems, equipped with methods and appropriate special mathematical and software for following a set route, recognizing, identifying images (targets) and distributing them between individual swarm agents (its subgroups) for effective destruction of enemy military equipment and other targets (inflicting maximum possible losses).

Upon arrival in the designated area, such a swarm of drones "independently" solves the task of inflicting the most effective fire damage on the enemy [2]. In the face of enemy EW and other obstacles on the battlefield (weather, natural, time of day, smoke, dust, camouflage elements, etc.), the problem arises of detecting enemy targets and correctly recognizing them for further distribution among the group's agents for effective destruction.The essence of distributing targets is to determine their number by category, to determine the degree of importance of each category, to screen out unimportant targets and to directly distribute the group's agents to important (defined) targets for their destruction.

The problematic issue of machine vision in this process remains obtaining a clear image of targets in conditions of external interference, their (targets) correct recognition and classification into certain categories in conditions of image noise.Another problematic issue is the time characteristics of pattern recognition algorithms (their computational complexity), which is caused by the high speed of aerial drones. Of course, these characteristics in machine vision systems tend to be reduced.

Endowing multi-agent systems with certain behavior (intelligence), in particular in matters of pattern recognition in conditions of external battlefield interference, requires the development of certain methods of working with images.

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In general, the task of pattern recognition lies in the plane of checking images for isomorphism of their corresponding model graphs. For this purpose, the calculation of constant characteristics of graphs, the so-called invariants, is carried out. In completely identical images, the invariants of the corresponding model graphs selected for evaluation are the same, which allows us to draw certain conclusions. If the invariants differ, the corresponding images are considered different. This approach is classical and is used in cases where there are no interferences in the process of obtaining the initial image of a physical object (or in conditions where such influence is insignificant), for example: passenger flow at airports and border checkpoints; quality control of products in production; identification of a citizen within the framework of performing various police functions; electronic processing of texts and documents, etc.

In battlefield conditions, the influence of extraneous conditions on the quality of the image of objects is usually significant, which significantly affects the efficiency of machine vision systems. However, even in such conditions, the task of recognizing and classifying images must be performed, even with some loss of recognition reliability. Therefore, the decision rules for classifying images can be probabilistic in nature.

The object of the study is a pattern recognition process.

The subject of the study is the stability of graph invariants for solving pattern recognition problems.

The purpose of the study is to develop a graph invariant with low computational complexity, which will allow classifying physical objects with a certain level of confidence in the presence of external interference.

1 PROBLEM STATEMENT

Pattern recognition problems, among others, can be formalized and solved using graph theory models and methods [3], since graphs best model the structure of a physical object. That is why we will model the implementation of images by some weighted undirected graph $G=(V,E)$, where $V=\{v_1,...,v_v\}$ – a set of graph vertices that model key points in the structure of an object; $E=\{e_1,...,e_q\}$ – the set of graph edges that model the linear elements of an object. Each edge from the set E will have a certain weight coefficient $w(e_{ij})$, which will quantitatively characterize the length of the edge e_{ij}.

Taking into account the features of pattern recognition in conditions of external interference, the formulation of the corresponding problem in general terms will have the following form.

Let X be a set of descriptions of physical objects, and each object $x \in X$ is modeled by a weighted undirected graph G. Let Y is the set of object class names. There is an unknown target dependency – mapping $y^*: X \to Y$, whose values are known only on the objects of the training sample $X^m=\{(x_1,y_1),...,(x_m,y_m)\}$. It is necessary to develop an invariant $B(G)=\{\omega_1,\omega_2,...,\omega_n\}$, and based on such an invariant, construct an algorithm $a: X \to Y$, capable of classifying arbitrary objects $x \in X$.

2 LITERATUREREVIEW

One of the theoretical bases of pattern recognition is graph theory. Within the framework of this theory, pattern recognition usually involves testing different model graphs for isomorphism. It is believed that if the model graphs are isomorphic, then the objects corresponding to them are identical, and vice versa, in the absence of isomorphism, the test objects differ from each other to some extent. Such a simple rule allows for the classification of objects. A sign of graph isomorphism is the identity of their invariants – constant characteristics of the graphs selected for comparison (numerical, structural, geometric, etc. characteristics). In the case of an ordered set of several such characteristics, we speak of a hash function. Typically, hash functions are used to increase the reliability of recognition.

A fairly large number of works are devoted to the problem of detecting isomorphisms of graphs, one of the most characteristic of which is [4–6]. A number of generalizations are also given in such works as [7–10]. In [5] it is shown that such problems are combinatorial and difficult to solve. Algorithms for solving them in asymptotics have factorial computational complexity. In this regard, only heuristic methods remain acceptable for solving such problems [3, 6].

Therefore, neither the branch and bound method nor mathematical programming methods will be effective here, which at best reduce the complexity of the problem from factorial dependence to polynomial (as a rule, relative to the number of vertices of the graph), and this is unacceptable for solving problems of practical dimension. At the same time, existing heuristic methods for solving such problems (or rather, attempts to solve them) have, as a rule, computational complexity $O(|V|^c)$, where $4 \le c \le 6$ [5, 6, 11], which also sharply limits the dimensionality of the problems solved in practice. For real-time operation, it is desirable to have the computational complexity of the corresponding methods (algorithms) at the level $c \le 3$ [6].

In [6], the authors presented an interesting approach to determining graph isomorphism, which is based on an invariant in the form of binary trees obtained as a result of the convolution (reduction) of model graphs. The authors of the article claim that the computational complexity of the corresponding algorithm does not exceed the estimate $O(|V|^3)$.

In [12], the author proposed a graph invariant based on calculating the shortest path matrix between all pairs of vertices of the model graph. The corresponding algorithm also has a computational complexity of $O(|V|^3)$, since it is based on the Warshall-Floyd or Bellman algorithm with the appropriate complexity [13, 14, 15].

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However, the problem of reducing the computational complexity of pattern recognition algorithms does not lose its relevance today, and in the military sphere it acquires new, more stringent requirements. Our article is devoted to solving this problem.

3 MATERIALS AND METHODS

Unfortunately, no graph invariant has been discovered yet that would unambiguously indicate graph isomorphism. Attempts to find such an invariant were made by scientists in the 1960s–1980s, but were unsuccessful [16].

However, it is necessary to solve specific problems of pattern recognition, and this forces us to return to the development and study of invariants that would allow us to solve the problem of the existence or absence of isomorphism with a high degree of reliability.

The invariant proposed in this article is based on the definition in the structure of the initial model graph G its minimum and maximum weight of spanning trees G' and G'' respectively, Fig. 1.

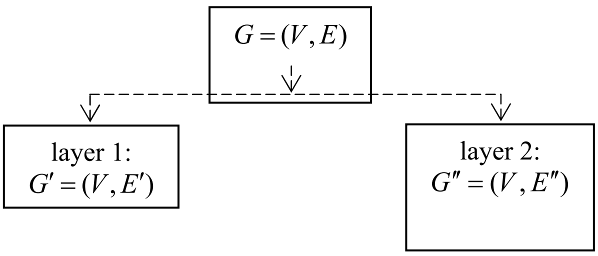

Figure 1 – Model graph layering G

Assertion. Any arbitrary undirected weighted graph $G=(V,E)$ with cycles, if only $\forall w(e_{ij})$ are not the same, has at least two spanning trees G' and G'', moreover $w(G')<w(G'')$, $E' \bigcap E'' \neq \varnothing$.

This splitting of the structure of the initial graph into two layers (hence the title of the article) provides a larger information base during further analysis of the image for the purpose of its reliable classification, since both trees characterize the same physical object. Thus, the spanning tree G' will contain a certain set of minimal, in a certain sense, linear dimensions of a physical object, and the spanning tree G'' – respectively maximum. Therefore, it becomes possible to obtain two numerical characteristics $w(G')$ and $w(G'')$, which will characterize the model graph G:

$$w(G') = \sum_{\forall e_{ij} \in G'} w(e_{ij}), \tag{1}$$

$$w(G'') = \sum_{\forall e_{ij} \in G''} w(e_{ij}) \cdot \tag{2}$$

We will also determine the structural features of a physical object by two layers. For this, we will use the Randić index [17], which characterizes the degrees of vertices between which there is an edge:

$$r = \sum_{\forall e_{ij} \in E} \frac{1}{\sqrt{d(v_i) \cdot d(v_j)}} \cdot \tag{3}$$

Thus, for G' and G'' we will obtain the corresponding numerical characteristics $r(G')$ and $r(G'')$.

Therefore, the invariant proposed in this article is a four-dimensional object (hash function) and will be described by the following expression:

$$B(G) = \{w(G'); w(G''); r(G'); r(G'')\}. \tag{4}$$

The general algorithm for pattern recognition will consist of the following steps:

1. Obtain an image of a physical object from video and photo recording devices.
2. Using the Haar feature method, find the key points of an object in its image [18, 19, 20].
3. Based on the resolution of video or photo recording devices, determine the linear dimensions of the object, then proceed to the normalized values of these dimensions. To this end, determine the largest linear dimension in the composition of the physical object and list all the others relative to it. Therefore, all dimensions must be within the interval $(0,1]$. Form a model weighted graph G.
4. Based on the model graph G, construct the trees G' and G''. For this purpose, use one of the well-known algorithms: Boruvka-Sollin's [21] or Kruskal's [22]. To construct a tree G'', modify the specified algorithms in terms of the order of selection of edges of the model graph G – the selection should be made from edges with a larger value of $w(e_{ij})$ in the direction of edges with a smaller value of $w(e_{ij})$.
5. Behind the built trees G' and G'' by expressions (1) and (2) calculate numerical characteristics $w(G')$ and $w(G'')$ in accordance. By expression (3) – characteristics $r(G')$ and $r(G'')$. Using expression (4), construct an invariant of the model graph G – prototype of a physical object.
6. For different classes of objects that are potentially subject to recognition, form appropriate training samples with the most probable characteristics of invariants. One of the known methods is to perform recognition of physical objects on the ground.

4 EXPERIMENTS

In the course of the experiment, we will first of all investigate the stability of the invariant to image distortions (noise). To do this, we will calculate the corresponding characteristics of the invariant for the reference image, then for the image of the same physical

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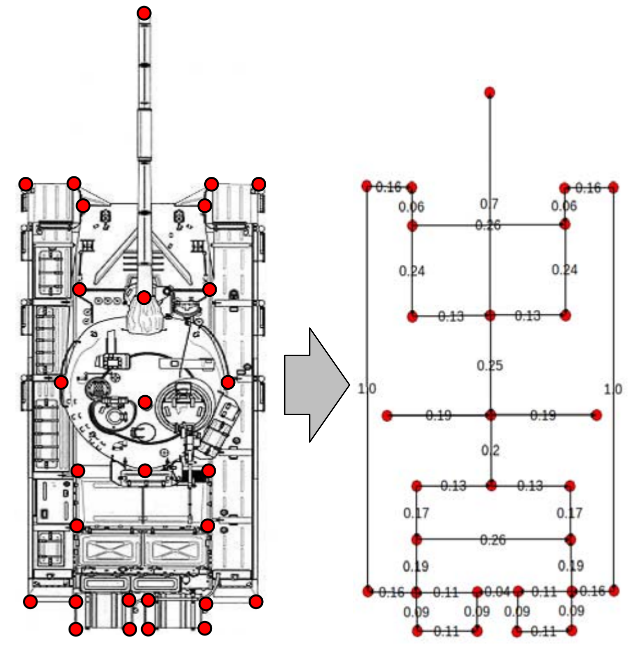
object, but in a different scene. Then we will determine the degree of deviation of the second invariant from the invariant for the reference image. A tank is chosen as the physical object T-72M, Fig. 2, *a*.

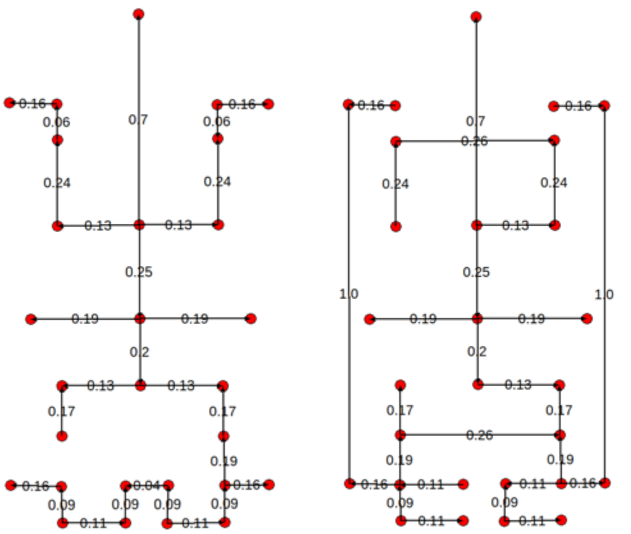


a b

Figure 2 – T-72M tank (top view):
a – key points of the object; b – weighted model graph G

The model graph G and the normalized weights of its edges are presented in Fig. 2, *b*. The length of the object's chassis was chosen as the standard linear dimension.

The constructed trees G' and G'' are presented in Fig. 3.

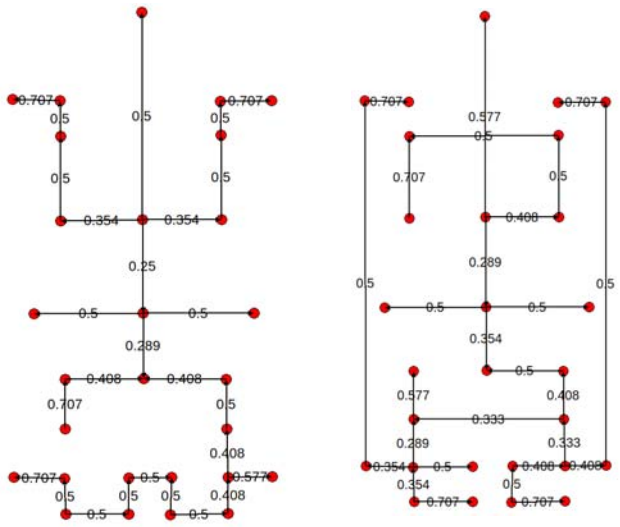


a b

Figure 3 – Weighted Span Trees of a Model Graph G:
a – tree G'; b – tree G''

Based on the edge weights shown in Fig. 3 according to expressions (1) and (2), we obtain $w(G')$=4,44, $w(G'')$=6,77.

The calculated Randić indices of the edges of the trees G' and G'' are shown in Fig. 4.



a b

Figure 4 – Randić indices:
a – tree G'; b – tree G''

According to expression (3), we obtain $r(G')$ =13.285 and $r(G'')$=13.128. Therefore, the invariant for the reference image of a physical object (see Fig. 2, *a*) will have the form $B(G) = \{4.44; 6.44; 13.285; 13.128\}$.

Let us imagine that the observer is in a different position relative to the object (upper hemisphere, side view). The object is represented by a weighted model graph G_*, Fig. 5. As can be seen from the figure, in this position additional elements of the object were opened for viewing, which were not visible in Fig. 2. Such elements are: the front part of the object; its left caterpillar. In addition, thanks to such a scene, some linear dimensions of the object changed, but insignificantly.

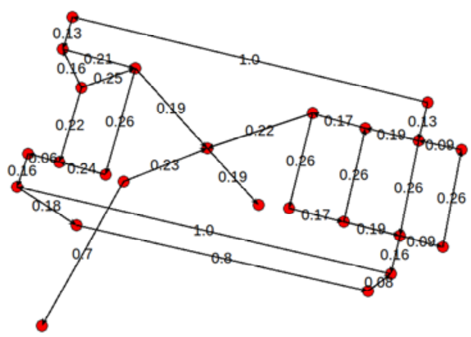


Figure 5 – Weighted Model Graph G_* of the T-72M tank (upper hemisphere, side view)

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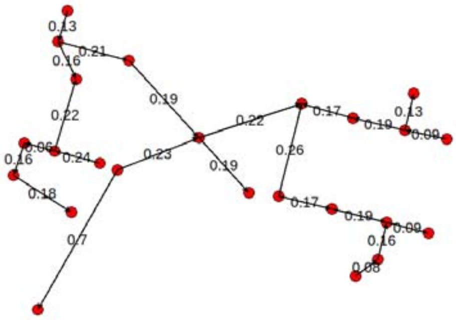
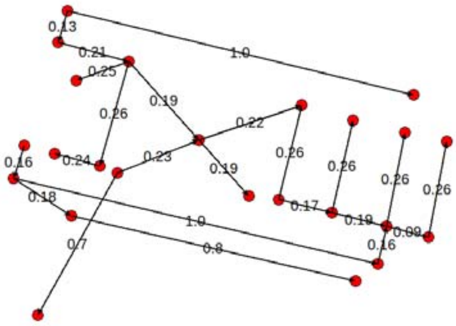
Let's build the appropriate trees G'_* and G''_*, see Fig. 6.



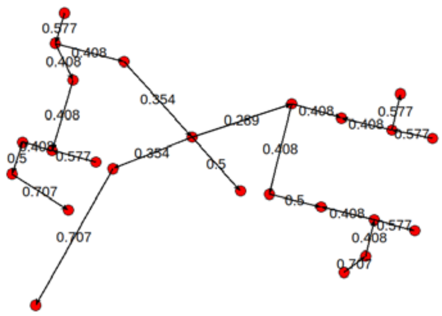
a



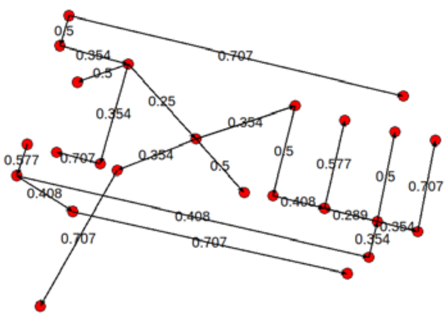
b

Figure 6 – Weighted Span Trees of a Model Graph G_*:
a – tree G'_*; b – tree G''_*

Using expressions (1) and (2) for the graph G_*, we obtain its numerical characteristics: $w(G'_*)$ =4.42 and $w(G''_*)$ =7.41. The calculated Randić indices for the edges of the trees G'_* and G''_* are shown in Fig. 7.



a



b

Figure 7 – Randić indices:
a – tree G'_*; b – tree G''_*

According to expression (3), we obtain the following numerical characteristics: $r(G'_*)$=13.178 and $r(G''_*)$=11.075.

Thus, the invariant for an alternative representation of a physical object (see Fig. 5) will be represented by the expression $B(G_*) = \{4{,}42; 7{,}41; 11{,}178; 11{,}075\}$.

Next, we will evaluate the recognition capabilities of the algorithm based on the proposed invariant. To do this, we will determine the degree of deviation of the images shown in Fig. 2 and Fig. 5. For this purpose, according to expression (5), in four-dimensional space (by the number of elements of the invariant), we calculate the distance (d) between two points corresponding to the invariants $B(G)$ and $B(G_*)$. The expression looks like:

$$d = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}\,, \tag{5}$$

where d – distance between points; x_i and y_i – elements of invariants $B(G)$ and $B(G_*)$ in accordance.

For this, with respect to $r(G')$, which is the maximum numerical characteristic in tuples $B(G)$ and $B(G_*)$, their elements are normalized. As a result of normalization, we obtained:

$\overline{B(G)} = \{0.334; 0.485; 1; 0.988\}$

$\overline{B(G_*)} = \{0.333; 0.557; 0.841; 0.833\}$.

The results of the calculations show that $d = 0.23$. Since Fig. 2 shows a "reference" image of a physical object of the "tank" type, with a probability of $P \approx 0{,}77$ the image presented in Fig. 5 can also be attributed to the class of physical objects of the "tank" type.

The obtained probability value is quite high. However, it should be noted that this probability estimate was obtained based on the results of a single comparison of two images of the same object located in different scenes (only to demonstrate the essence of the approach to pattern recognition using the developed invariant). It is clear that this estimate cannot be extended to all other possible cases and other physical objects. Under other conditions, the probability of assigning a particular physical object to one or another class of objects will be somewhat different.

In order to reveal the actual recognition ability of pattern recognition methods using the developed type of invariant, a well-known method was used – the method of standards [27].

The experiment was built according to the following scheme. The input training matrix had three classes of possible enemy targets: "tank"; "plane"; "truck", (see Fig. 8). The number of subclasses was from two to three. The corresponding training matrices mainly consisted of 4 records of invariants (tuples) characterizing the corresponding objects in four scenes: top view; view of the posterior and anterior hemispheres; side view.

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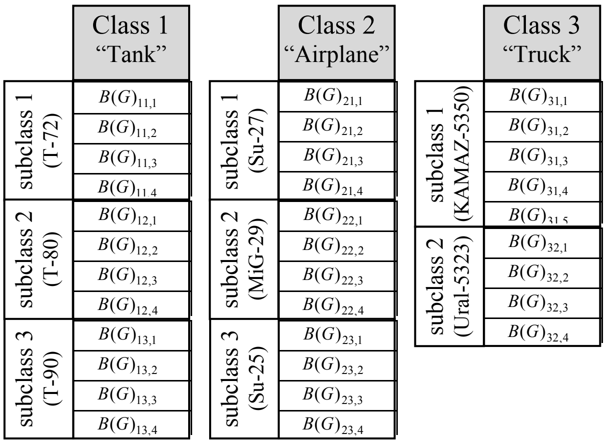

Figure 8 – Structure of the training matrix

For each subclass, 40 different implementations were tested – images of the corresponding physical objects. Therefore, the total number of implementations that were subject to classification according to the training matrix was equal to $N = 320$.

The following results were obtained from the experiment. At the class level, all images were correctly classified. At the subclass level, there were isolated cases of misclassification within one class. Thus, some implementations were classified as belonging to other subclasses. The number of such errors and their nature are given in Table 1.

Table 1 – Number and nature of cases of image misclassification during the experiment

	«Tank»	«Airplane»	«Truck»
subclass 1	(2)	(5)	(3)
subclass 2	(4)	(4)	(6)
subclass 3	(2)		--
Total images	120	120	80
Error, (%)	~6.6	~7.5	~11.2

Therefore, taking into account the results of the experiment, it can be stated that the proposed invariant provides the correct classification of images: at the class level with probability $P = 1$; at the subclass level with probability up to $0.8 \le P \le 0.9$.

5 RESULTS

The two-layer invariant of the weighted undirected model graph proposed in the article allows:

– due to the individual properties of the two spanning trees into which the initial model graph is stratified, expand the information base for analyzing physical objects during their classification.

– using the numerical characteristics of two spanning trees used in its composition, describe both the linear dimensions of a certain physical object and its structural features, which are the main aspects for classifying objects.

– with a high probability of performing correct pattern recognition.

6 DISCUSSION

The stratification of the object structure into two substructures and the set of numerical characteristics of the model graph proposed in the article allowed us to develop and propose an invariant that allows us to check graphs for isomorphism, and thus, to use this approach for the classification of real physical objects.The numerical characteristics used in the invariant characterize both the linear properties of the object and its structural properties, which is an important point in the process of recognizing enemy military equipment on the battlefield.

The stratification of the initial structure of the model graph into two conditional layers (minimum and maximum weight spanning trees) allowed to increase the recognition capabilities of methods based on graph theory tools. Thus, the probability of correct classification of objects belonging to different classes is equal to $P = 1$, the accuracy of classification of objects of different subclasses approaches $0.8 \le P \le 0.9$.

An important characteristic of various algorithms, including pattern recognition algorithms, is their computational complexity. It is clear that on real structures, and therefore on structures with a larger number of nodes and denser ones, the number of operations for searching for spanning trees will be much higher. Therefore, the computational complexity of the combinatorial algorithm that determines the characteristics of the proposed invariant will be determined by the computational complexity of its “basic element” – the algorithm for finding the spanning tree, which is estimated by $O(E \log V)$ [24]. Since two trees need to be found, the total computational complexity of the combinatorial algorithm can be estimated as $O(2[E \log V])$.

The obtained logarithmic estimate of the computational complexity of the algorithm is quite acceptable for its use in real time.

CONCLUSIONS

The article solves a relevant scientific and applied problem – the development of a graph invariant, using which it is possible, with a high level of probability, to correctly recognize various physical objects.

The scientific novelty of the developed graph invariant is as follows:

1) in the stratification of the initial structure of the model weighted undirected graph into two spanning trees – minimal and maximal in weight, which allows doubling the degree (depth) of verification of linear and structural properties of the same physical object;

2) in proposing an invariant structure containing two linear and two structural properties (characteristics) of objects, which allows checking model graphs for isomorphism and, on this basis, classifying objects.

The practical value of the proposed invariant is due to the fact that its application ensures the probability of

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correct classification of objects $P = 1$ (at the class level), and $0.8 \le P \le 0.9$ (at the subclass level).

The computational complexity of the algorithm for calculating the invariant proposed in the article has a logarithmic dependence on the dimension and density of the model graph, which allows using such an algorithm in real time.

A promising direction for further researchis the development of a complete invariant of a model graph with polynomial computational complexity.

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REFERENCES

1. Dotsenko V. Drone Armies. How Drones Are Replacing Artillery, Aviation, and Boats in the War in Ukraine [Electronic resource]. Access mode: https://forbes.ua/war-in-ukraine/armii-droniv-yak-u-viyni-v-ukraini-bpla-zaminyuyut-artileriyu-aviatsiyu-i-kateri-06122022-10273.
2. Girnyk E. Ukraine creates “drone swarm” capable of attacking without human intervention – The Times [Electronic resource]. Access mode: https://unian.net/weapons/voyna-dronov-ukraina-sozdaet-bespilotniki-kotorye-ubivayut-roem-the-times-12674718.html.
3. Christofides N. Theory of graphs. Algorithmic approach. Moscow, Mir, 1978, 432 p.
4. Berge C. Isomorphism problems forhypergraphs, Mathematical centre tracts, 1974, pp. 3–12. DOI: 10.1007/BFb0066174.
5. Corneil D. G., Gotlieb C. C. An efficient algorithm for Graph isomorphism, *J. of A.C.M.,*1970, Vol. 17, No. 1, pp. 51–64. DOI: 10.1145/321556.321562.
6. A’Ggel Al-Zabi Bilal Rady, Kernytsky A. B., Lobur M. V., et al. New invariants for establishing graph isomorphisms, *Academic Journals and Conferences*, 2008, Vol. 626, pp. 90–93.
7. McCarthy M. K., Davida G. J. Invariant features and the graph isomorphism problem, *IEEE Syst. Man and Cybern. Soc. Proc. Int. Conf. Cybern and Soc. Boston, Mass,* 1973, pp. 81–85.
8. Foggia P., Sansone C., Vento M. A performance comparison of five algorithms for graph isomorphism, *In Proc. 3rd IAPR-TC15 Workshop Graph-Based Representations in Pattern Recognition,* 2001, pp. 188–199.
9. Messmer B. T., Bunke H. Efficient subgraphisomorp-hismdetection: a decomposition approach, *IEEE Trans. Knowl. Data Eng,* 2000, Vol. 12, pp. 307–323. DOI: 10.1109/69.842269.
10. Conte D., Foggia P., Sansone C. et al. Thirtyyears of Graph Matching in pattern Recognition, *International Journal of Pattern Recognition and Artificial Intelligence*, 2004, Vol. 18, No. 3, pp. 265–298. DOI:10.1142/S0218001404003228.
11. Automatic generation of graphs and graphs identification problem, *Technical Report, april,* 1974, No. 16.
12. Dunaev A. A. Using Graph Theory for Face Identification, *International Research Journal*, 2017, Vol. 09(63), Issue 3. pp. 31–35. DOI: 10.23670/IRJ.2017.63.089.
13. Floyd R. Algorithm 97: Shortest Path, *Communications of the ACM*, 1961, Vol. 5 (6), 345 p. DOI: 10.1145/367766.368168.
14. Warshall S. Algorithm on Boolean matrices, *Journal of the ACM,* 1962, Vol. 9 (1), pp. 11–12. DOI: 10.1145/321105.321107.
15. Batsamut V., Manzura S., Kosiak O. et al. Fast Algorithm for Calculating Transitive Closures of Binary Relations in the Structure of a Network Object, *International Journal of Computing,* 2021, Vol. 20(4), pp. 560–566. DOI: 10.47839/ijc.20.4.2444.
16. Graph invariant[Electronic resource]. Access mode: https://uk.wikipedia.org/wiki/Інваріант_графа.
17. Delorme C., Favaron O., Rautenbach D. On the Randić index, *Discrete Mathematics, 6 November,* 2002, Vol. 257, Issue 1, pp. 29–38. DOI: 10.1016/S0012-365X(02)00256-X.
18. Messom C. H., Barczak A.L.C. Fast and Efficient Rotated Haar-like Features Using Rotated Integral Images, *Australian Conference on Robotics and Automation (ACRA 2006)*, 2006, pp. 1–6.
19. Lienhart R., Maydt J. An Extended Set of Haar-like Features for Rapid Object Detection, *IEEE ICIP Sep.,* 2002, Vol. 1, pp. 900–903. DOI: 10.1109/ICIP.2002.1038171.
20. Bernardin K., Camp F. V. D., Stiefelhagen R. Automatic person detection and tracking using fuzzy controlled active cameras, *In: Proc. IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 2007, pp. 1–8. DOI: 10.1109/CVPR.2007.383502.
21. Faridawaty Marpaung, Arnita, Wirdatull Jannah Idris Comperative of Prim’s, Kruskal’s and Boruvka’s algoritma to solve minimum spanning tree problems, *Jurnal Handayani,* 2019, Vol. 10, No. 2, pp. 80–89. DOI:10.24114/jh.v10i2.16053.
22. Kruskal J. On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem, *Proc. AMS*, 1956, Vol. 7, No. 1, pp. 48–50. DOI: 10.1090/S0002-9939-1956-0078686-7.
23. Dovbysh A. S., Shelekhov I. V. Fundamentals of Pattern Recognition Theory. Sumy, Sumy State University, 2015, Vol. 1, 109 p.
24. Boruvka’s algorithm [Electronic resource]. Access mode: https://uk.wikipedia.org/wiki/Алгоритм_Борувки.

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ДВОШАРОВИЙ ІНВАРІАНТ ГРАФА ДЛЯ РОЗПІЗНАВАННЯ ОБРАЗІВ

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АНОТАЦІЯ

Актуальність. Актуальність статті обумовлюється потребою у подальшому розвитку алгоритмів розпізнавання (класифікації) об’єктів, у зменшенні обчислювальної складності і збільшенні функціональних можливостей таких алгоритмів. Запропонований у статті інваріант графа може бути застосований у системах машинного зору для розпізнавання фізичних об’єктів, що є важливим у ході виконання рятувальних, моніторингових завдань у кризових районах різного характеру походження, а також у ході нанесення противнику вогневого ураження із застосуванням рою безпілотних апаратів.

Мета роботи полягає в розробленні інваріанту графа з низькою обчислювальною складністю, який дозволятиме з певним рівнем довірчої ймовірності класифікувати фізичні об’єкти в умовах зовнішніх завад.

Метод. Фізичний об’єкт, що підлягає розпізнаванню (ідентифікації) моделюється зв’язним неорієнтованим зваженим графом. Для виявлення сталих характеристик різних модельних графів застосовано ідею виділення в структурі цих графів мінімального і максимального за вагою каркасних дерев. З цією метою застосовується класичний і модифікований методи Борувки-Солліна (модифікований – для побудови максимального зваженого каркасного дерева). Таке розшарування структури початкового графа на два шари надає більшої інформаційної бази у ході аналізу зображення щодо приналежності певної реалізації до деякого класу об’єктів.

Далі, для кожного з отриманих таким чином каркасних дерев, відшукуються значення двох числових характеристик: ваги каркасного дерева та індексу Рандіча. Перша характеристика несе в собі опосередковану інформацію про лінійні розміри об’єкту, а друга – про його структурні особливості. Ці характеристики не залежать від способу позначення вершин та графічного зображення графа, що є необхідною умовою для перевірки графів на ізоморфізм.З отриманих таким чином чотирьох характеристик складається інваріант, яким описується відповідний фізичний об’єкт, що перебуває в одній сцені.

Для повного опису одного класу або підкласу об’єктів в чотирьох сценах (вид зверху; вид передньої та задньої полусфер; вид збоку) система розпізнавання образів повинна мати чотири відповідні інваріанти.

Результати. 1) Розроблено двошаровий інваріант зваженого неорієнтованого графу, який дозволяє з певним рівнем довірчої ймовірності розпізнавати фізичні об’єкти; 2) В термінах теорії графів формалізовано метод розпізнавання фізичних об’єктів, що заснований на хешуванні структури об’єкту вагою мінімального і максимального каркасних дерев модельного графу, а також індексом Рандіча цих дерев; 3) Виконано верифікацію двошарового інваріанту зваженого неорієнтованого графу на тестових задачах з перевірки графів на ізоморфізм.

Висновки. Проведені теоретичні дослідження та низка проведених експериментів підтверджують можливість використання пропонованого інваріанту графів в задачах розпізнавання та класифікації образів в масштабі реального часу. Оцінки, що виробляються із використанням розробленого методу, носять ймовірнісний характер, що дозволяє особі, яка налаштовує систему машинного зору, гнучко підходити до класифікації фізичних об’єктів в полі зору такої системи, виходячи з вимог до технологічного процесу або з умов оперативної обстановки в районі застосування системи.

КЛЮЧОВІ СЛОВА: фізичний об’єкт, зважений неорієнтований граф, ізоморфізм, мінімальне (максимальне) каркасне дерево модельного графа, інваріант графа, розпізнавання образів, алгоритм, метод.

ЛІТЕРАТУРА

1. Доценко В. Армії дронів. Як безпілотники замінюють артилерію, авіацію і катери на війні в Україні[Електрон. ресурс]. – Режим доступу: https://forbes.ua/war-in-ukraine/armii-droniv-yak-u-viyni-v-ukraini-bpla-zaminyuyut-artileriyu-aviatsiyu-i-kateri-06122022-10273.
2. Гирнык Є. Украина создает «рой дронов», способный атаковать без вмешательства человека – The Times [Електрон. ресурс]. – Режим доступу: https://unian.net/weapons/voyna-dronov-ukraina-sozdaet-bespilotniki-kotorye-ubivayut-roem-the-times-12674718.html.
3. Кристофидес Н. Теория графов. Алгоритмический поход / Н. Кристофидес. – М. : Мир, 1978. – 432 с.

DOI 10.15588/1607-3274-2025-2-7

4. Berge C. Isomorphism problems for hypergraphs / C. Berge // Mathematical centre tracts. – 1974. – P. 3–12. DOI: 10.1007/BFb0066174.
5. Corneil D. G. An efficient algorithm for Graph isomorphism / D. G. Corneil, C. C. Gotlieb // J. of A.C.M. – 1970. – Vol. 17. – No. 1. – P. 51–64. DOI: 10.1145/321556.321562.
6. New invariants for establishing graph isomorphisms / [Bilal Rady A'Ggel Al-Zabi, A. B. Kernytsky, M. V. Lobur, et al.] // Academic Journals and Conferences. – 2008. – Vol. 626. – P. 90–93.
7. McCarthy M. K. Invariant features and the graph isomorphism problem / M. K. McCarthy, G. J. Davida // IEEE Syst. Man and Cybern. Soc. Proc. Int. Conf. Cybern and Soc. Boston, Mass. – 1973. – P. 81–85.
8. Foggia P. A performance comparison of five algorithms for graph isomorphism / P. Foggia, C. Sansone, M. Vento // In Proc. 3rd IAPR-TC15 Workshop Graph-Based Representations in Pattern Recognition, 2001. – P. 188–199.
9. Messmer B. T. Efficient subgraph isomorphism detection: a decomposition approach / B. T. Messmer, H. Bunke // IEEE Trans. Knowl. Data Eng. – 2000. – Vol. 12. – P. 307–323. DOI: 10.1109/69.842269.
10. Thirty years of Graph Matching in pattern Recognition / [D. Conte, P. Foggia, C. Sansone et al.] // International Journal of Pattern Recognition and Artificial Intelligence. – 2004. – Vol. 18. – No. 3. – P. 265–298. DOI:10.1142/S0218001404003228.
11. Proskurowski A. Automatic generation of graphs and graphs identification problem / A. Proskurowski // Technical Report, april. – 1974. – No. 16.
12. Дунаев А. А. Использование теории графов для идентификации лиц / А. А. Дунаев // Международный научно-исследовательский журнал, 2017. – № 09(63). – ч. 3. – С. 31–35. DOI: 10.23670/IRJ.2017.63.089.
13. Floyd R. Algorithm 97: Shortest Path / R. Floyd // Communications of the ACM. – 1961. – Vol. 5 (6). – 345 p. DOI: 10.1145/367766.368168.
14. Warshall S. Algorithm on Boolean matrices / S. Warshall // Journal of the ACM. – 1962. – Vol. 9 (1). – P. 11–12. DOI: 10.1145/321105.321107.
15. Fast Algorithm for Calculating Transitive Closures of Binary Relations in the Structure of a Network Object / [V. Batsamut, S. Manzura, O. Kosiak et al.] // International Journal of Computing. – 2021. – Vol. 20(4). – P. 560–566. DOI: 10.47839/ijc.20.4.2444.
16. Інваріант графа [Електрон. ресурс]. – Режим доступу: https://uk.wikipedia.org/wiki/Інваріант_графа.
17. Delorme C. On the Randić index / C. Delorme, O. Favaron, D. Rautenbach // Discrete Mathematics, 6 November. – 2002. – Vol. 257, Issue 1. – P. 29–38. DOI: 10.1016/S0012-365X(02)00256-X.
18. Messom C. H. Fast and Efficient Rotated Haar-like Features Using Rotated Integral Images / C. H. Messom, A.L.C. Barczak // Australian Conference on Robotics and Automation (ACRA2006). – 2006. – P. 1–6.
19. Lienhart R. An Extended Set of Haar-like Features for Rapid Object Detection / R. Lienhart, J. Maydt // IEEE ICIP Sep., 2002. – Vol. 1. – P. 900–903. DOI: 10.1109/ICIP.2002.1038171.
20. Bernardin K. Automatic person detection and tracking using fuzzy controlled active cameras / K. Bernardin, V. D. Camp F., R. Stiefelhagen // In: Proc. IEEE Computer Society Conference on Computer Vision and Pattern Recognition. – 2007. – P. 1–8. DOI: 10.1109/CVPR.2007.383502.
21. Faridawaty Marpaung. Comperative of Prim's, Kruskal's and Boruvka's algoritma to solve minimum spanning tree problems / Faridawaty Marpaung, Arnita, Wirdatull Jannah Idris // Jurnal Handayani. – 2019. – Vol. 10, No. 2. – P. 80–89. DOI:10.24114/jh.v10i2.16053.
22. Kruskal J. On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem / J. Kruskal // Proc. AMS. – 1956. – Vol. 7, No. 1. – P. 48–50. DOI: 10.1090/S0002-9939-1956-0078686-7.
23. Довбиш А. С. Основи теорії розпізнавання образів: навчальний посібник / А. С. Довбиш, І. В. Шелехов. –Суми : Сумський державний університет, 2015. – ч. 1. – 109 с.
24. Алгоритм Борувки [Електрон. ресурс]. – Режим доступу: https://uk.wikipedia.org/wiki/Алгоритм_Борувки.

DOI 10.15588/1607-3274-2025-2-7