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CONTROL IN TECHNICAL SYSTEMS

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TERMINAL CONTROL OF QUADCOPTER SPATIAL MOTION

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ABSTRACT

Context. Constructing quadcopter control algorithms is an area of keen interest because controlling them is fundamentally complex despite the quadcopter's mechanical simplicity. The key problem of quadcopter control systems is to effectively couple three translational and three rotational freedom degrees of motion to perform unique target manoeuvres. In addition, these tasks are relevant due to the high demand for quadcopter in various human activities, such as cadastral aerial photography for monitoring hard-to-reach areas and delivering cargo over short distances. They are also widely used in military affairs.

Objective. This work objective is to develop and substantiate novel methods for algorithms constructing the high-precision control of a quadcopter spatial motion, allowing for its autonomous operation in all main flight modes: stabilization mode, position holding mode, automatic point-to-point flight mode, automatic takeoff and landing mode.

Method. The given objective determined the use of the following research methods. Pontryagin's maximum principle was applied to develop algorithms for calculating program trajectories for transferring a quadcopter from its current state to the given one. Lyapunov functions and modal control methods were used to synthesise and analyse quadcopter angular position control algorithms. Numerical modelling methods were used to verify and confirm the obtained theoretical results.

Results. An approach for constructing algorithms for controlling the spatial quadcopter motion is proposed. It consists of two parts. The first part solves the problem of transferring a quadcopter from its current position to a given one. The second part proposes an original method to construct algorithms for quadcopter attitude control based on a dynamic equation for a quaternion.

Conclusions. The proposed quadcopter motion mathematical model and methods for constructing control algorithms are verified by numerical modelling and can be applied to develop quadcopter control systems.

KEYWORDS: quadcopter, quaternion, Pontryagin's maximum principle, Hamiltonian, Lyapunov functions.

ABBREVIATIONS

BFF is a body fixed frame;
EFF is an Earth fixed frame;
UAV is a Unmanned Aerial Vehicle.

NOMENCLATURE

D is a propeller diameter;
$\boldsymbol{F}_E$ is a vector of the sum all forces acting on quadcopter in the EFF;
$\boldsymbol{G}_E$ is a force of gravity;
H is a Hamiltonian;
$\boldsymbol{J}$ is a moment of inertia with respect to the BFF;
J_r is an inertia of the rotors;
$\boldsymbol{L}_B$ is an angular momentum in the BFF;
$\boldsymbol{M}_B$ is a moment of external forces given by projections on the BFF axes;
$\boldsymbol{M}_g$ is a gyroscopic moment;
$\boldsymbol{M}_i$ is a reactive torque from the rotation of the *i*-th propeller;
$\boldsymbol{M}_u$ is a control moment;
$\boldsymbol{P}_{Bi}$, is a lift force created by the *i*-th propeller, given by projections on BFF axes;
$\boldsymbol{P}_{Ei}$ is a lift force created by the *i*-th propeller, given by projections on EFF axes;
$\boldsymbol{X}_E, \boldsymbol{Y}_E, \boldsymbol{Z}_E$ are the EFF axes;
$\boldsymbol{X}_B, \boldsymbol{Y}_B, \boldsymbol{Z}_B$ are the BFF axes;
$\boldsymbol{e}$ is a pointing error;
g is a gravitational acceleration on Earth;
$\boldsymbol{h}_{Bi}$ is an angular momentum of the *i*-th rotor given by projections on the BFF axes;
k_m is a proportionality constants;
k_p is a proportionality constants;
ℓ is a distance from the motor rotation shaft axis to the quadcopter centre of mass;

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m is a quadcopter mass;

$\boldsymbol{n}_B$ is a unit vector of $\boldsymbol{Y}_B$ axis;

$\boldsymbol{n}_E$ is a unit vector of the propellers' total thrust force given by the projections on the EFF axes;

$\boldsymbol{r}_E$ is a vector of the quadcopter centre mass position in the EFF;

$\boldsymbol{u}$ is a control vector;

$\boldsymbol{\Lambda}_{EB}$ is a quaternion of transition from EFF to BFF;

$\boldsymbol{\Phi}(t_0, t)$ is a transition matrix of the extended system;

α is a lift coefficient;

β_ρ is a propeller power factor;

λ_0^{EB} is a scalar part of the quaternion $\boldsymbol{\Lambda}_{EB}$;

$\boldsymbol{\lambda}_{EB}$ is a vector part of the quaternion $\boldsymbol{\Lambda}_{EB}$;

μ_i is a reactive moment on the shaft of the i-th rotor;

ρ is an air density;

φ is an Euler roll angle;

ϑ is an Euler pitch angle;

ψ is an Euler yaw angle;

$\boldsymbol{\omega}_B^{BE}$ is an angular velocity vector of BFF rotation relative to EFF, given by the projections onto the BFF axes;

ω_i is an angular speed of the i-th rotor;

$\sim$ is a conjugate quaternion notation;

$\circ$ is a quaternion multiplication operation symbol.

INTRODUCTION

Currently, work is actively underway in the field of developing new and improving existing UAV control systems. Miniature UAV multi-rotor type is an area of significant interest because of their unique features such as overall dimensions, the ability to fly in limited space and at very low speeds, vertical takeoff and landing and so on. These capabilities enable their use in various fields of activity, and for some tasks, they are indispensable. A diverse array of UAV applications can be found, for instance, in specialized literature reviews [1–3].

One of the main directions in the field of UAVs is related to increasing their flight autonomy. This, in turn, places increased demands on equipment reliability, control system intelligence, and the efficient use of power sources. On the other hand, these features ensure ease of use and reduce the cost of target tasks performed by UAVs.

This work is devoted to the development of the quadcopter spatial motion controlling algorithms, which allow autonomous implementation of its flight main modes: the stabilization mode in which the aircraft automatically maintains the zero values of roll and pitch angles and stabilizes the yaw angle; mode of keep a given position in which the UAV automatically hovers over a given point on the earth's surface; mode of automatic flight by points; automatic take-off and landing mode.

The object of the study is the process of controlling the spatial motion of a quadcopter.

The subject of the study is the synthesis of the quadcopter spatial motion control laws in the form of stationary feedback by state.

The purpose of the work is to develop algorithms for controlling the spatial motion of a quadcopter.

1 PROBLEM STATEMENT

The spatial movement of the quadcopter, a mechanical system with four propeller rotation engines and a supporting rigid frame is considered. The kinematic diagram of the quadcopter is shown in Fig. 1. On the kinematic diagram, the axes $\boldsymbol{X}_B$, $\boldsymbol{Y}_B$ and $\boldsymbol{Z}_B$ form a BFF, which is rigidly connected to the quadcopter, and the axes $\boldsymbol{X}_E$, $\boldsymbol{Y}_E$ and $\boldsymbol{Z}_E$ form the EFF, in which the observer is located. The propeller rotation engines are rigidly fixed to the quadcopter frame, and the BFF's $\boldsymbol{X}_B$ and $\boldsymbol{Y}_B$ axes intersect the centres of these engines.

The quadcopter motion control is carried out by applying control voltages to the propellers' engines. As a result of the propellers' rotation angle speeds ω_1, ω_2, ω_3 and ω_4, the lifting forces $\boldsymbol{P}_{B1}$, $\boldsymbol{P}_{B2}$, $\boldsymbol{P}_{B3}$, $\boldsymbol{P}_{B4}$ and the corresponding reaction moments $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, $\boldsymbol{M}_3$ and $\boldsymbol{M}_4$ arise.

It is necessary to develop a quadcopter spatial motion mathematical model and based on it synthesize the algorithms for controlling the propeller angular velocities ω_i to transfer the quadcopter from a current position to a given one (hover point) and ensure the quadcopter's angular stabilization in this position.

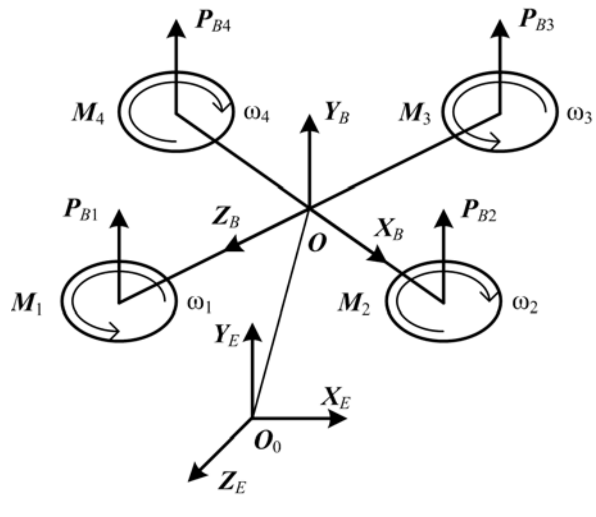

Figure 1 – The quadcopter kinematic scheme

2 REVIEW OF THE LITERATURE

The research and development of quadcopter spatial motion control have been the focus of many publications for several decades. The advances in microcontrollers, telecommunications, and quadcopter applications have

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significantly increased the number of publications on this topic in recent years.

In studies [4–6], the control laws based on the Lyapunov function are introduced. These laws establish sufficient conditions for the asymptotic stability of a closed-loop system. and specific methodologies for determining the desired Lyapunov function are discussed. The works [7–8] discussed the use of a sliding mode control, which is simple and reliable, but requires adaptation of the switching logic to the flight modes of the quadcopter. In the work of [9], an approach that combines the method of a nonlinear observer and sliding mode control is proposed. In this approach, a nonlinear observer predicts the impact of engine failures on the quadcopter dynamics and ensures the stability of the sliding mode to uncertainties and disturbances. In work [10], a nested double-loop control scheme based on the adaptive backstepping approach concerning uncertain parameters is proposed. To avoid the analytic derivative calculation of the virtual command, a command filter is introduced into the designing procedure with a compensated signal employed in the attitude error. The backstepping-based formation control using the state transformation technique and asymptotic stability analysis based on Lyapunov's theorem is presented in the paper [11]. In [12], a highly complex controller is proposed that combines an optimal H_∞ controller with an integral predictive controller supplemented by a Kalman filter implementation. In the paper [13] a quadcopter model is developed using the Hamiltonian approach is considered and a nonlinear orientation controller for this model is proposed. In paper [14] a new nonlinear robust control algorithm with output feedback based on quaternions is presented.The work [15] is devoted to modelling a quadcopter based on certain physical parameters to build a desired model before designing a specific control system. According to the authors, this study should help save time and costs on possible errors in designing quadcopter control systems. Systematic literature reviews such as [16–18] comprehensively analyse the main modern control strategies for quadcopter UAVs.

3 MATERIALS AND METHODS

Let's assume that the origin of the coordinate system EFF coincides with the centre of mass of the quadcopter at the initial moment of movement, the EFF $\boldsymbol{Y}_E$ axis coincides with the direction of the local vertical at this point, the $\boldsymbol{X}_E$ axis is directed along the line of the given flight course, and the $\boldsymbol{Z}_E$ axis is defined as $\boldsymbol{Z}_E = \boldsymbol{X}_E \times \boldsymbol{Y}_E$. Considering the small durations of the time intervals of the quadcopter's autonomous flight, the rotation of the Earth can be neglected and the EFF coordinate system will be considered inertial (stationary) in the first approximation. The BFF axes are rigidly associated with the quadcopter body and the BFF origin coincides with the quadcopter centre of mass. Let's assume that the BFF axes coincide with the quadcopter's main central axes of inertia. In this case, the dynamic control characteristics are significantly improved and the equations of rotational motion of the quadcopter are simplified.

The relative orientation of EFF and BFF is defined as follows. The position of the coordinate system BFF relative to the coordinate system EFF is determined by the quaternion $\boldsymbol{\Lambda}_{EB}$. The BFF angular orientation relative EFF is given by three rotations: the first rotation is performed around the $\boldsymbol{X}_E$ axis by an angle φ, the second rotation is performed around the $\boldsymbol{Z}'$ axis by an angle ϑ, the third rotation is performed around the $\boldsymbol{Y}_B$ axis by an angle ψ (Fig. 2).

In this case, the quaternion $\boldsymbol{\Lambda}_{EB}$ is defined by the expression

$$\boldsymbol{\Lambda}_{EB} = \lambda_0^{EB} + \boldsymbol{\lambda}_{EB}, \tag{1}$$

where the scalar part is

$$\lambda_0^{EB} = \cos\frac{\psi}{2}\cos\frac{\vartheta}{2}\cos\frac{\varphi}{2} + \sin\frac{\psi}{2}\sin\frac{\vartheta}{2}\sin\frac{\varphi}{2}, \tag{2}$$

and the vector part is

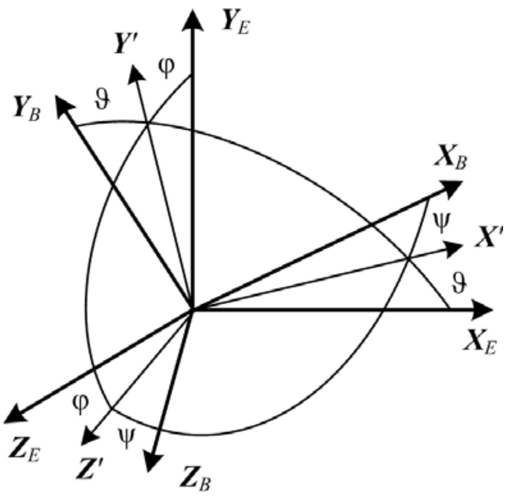

Figure 2 – Sequence of rotations for transforming from BFF to EFF

$$\boldsymbol{\lambda}_{EB} = \begin{pmatrix} \sin\frac{\varphi}{2}\cos\frac{\vartheta}{2}\cos\frac{\psi}{2} - \cos\frac{\varphi}{2}\sin\frac{\vartheta}{2}\sin\frac{\psi}{2} \\ -\sin\frac{\varphi}{2}\sin\frac{\vartheta}{2}\cos\frac{\psi}{2} + \cos\frac{\varphi}{2}\cos\frac{\vartheta}{2}\sin\frac{\psi}{2} \\ \cos\frac{\varphi}{2}\sin\frac{\vartheta}{2}\cos\frac{\psi}{2} + \sin\frac{\varphi}{2}\cos\frac{\vartheta}{2}\sin\frac{\psi}{2} \end{pmatrix}. \tag{3}$$

The system of equations for the quadcopter centre of mass motion in EFF by Newton's second law has the following form

$$m\ddot{\boldsymbol{r}}_E = \boldsymbol{F}_E. \tag{4}$$

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Let's assume that only the thrust forces $\boldsymbol{P}_i$ of the aerodynamic propellers ($i = 1, 2, 3, 4$) and the force of gravity act on the quadcopter. In this case,

$$\boldsymbol{F}_E = \boldsymbol{P}_{E1} + \boldsymbol{P}_{E2} + \boldsymbol{P}_{E3} + \boldsymbol{P}_{E4} + \boldsymbol{G}_E = \boldsymbol{P}_E + \boldsymbol{G}_E , \quad (5)$$

where

$$\boldsymbol{G}_E = mg \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} . \quad (6)$$

The direction of the force $\boldsymbol{P}_i$ coincides with the positive direction of the $\boldsymbol{Y}_B$ axis of BFF

$$\boldsymbol{P}_{Bi} = p_i \boldsymbol{n}_B . \quad (7)$$

The unit vector $\boldsymbol{n}_B$ of $\boldsymbol{Y}_B$ axis and modulus p_i of the forces $\boldsymbol{P}_i$ are determined by expressions

$$\boldsymbol{n}_B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} , \quad (8)$$

$$p_i = |\boldsymbol{P}_i| = k_p \omega_i^2 , \quad k_p = \alpha \rho D^4 . \quad (9)$$

Projecting the vector $\boldsymbol{P}_{Bi}$ (7) onto the EFF axes gives

$$\boldsymbol{P}_{Ei} = p_i \boldsymbol{n}_E , \quad (10)$$

where

$$\boldsymbol{n}_E = \begin{pmatrix} -\sin\vartheta \\ \cos\vartheta\cos\varphi \\ \cos\vartheta\sin\varphi \end{pmatrix} . \quad (11)$$

Thus, the following equation describes the motion of the quadcopter centre of mass

$$m\ddot{\boldsymbol{r}}_E = k_p(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)\boldsymbol{n}_E + \boldsymbol{G}_E . \quad (12)$$

To obtain the equation of the rotational motion of the quadcopter relative to the centre of mass, consider the equation for the angular momentum $\boldsymbol{L}$ of the "carrying body + rotors" system. This equation in BFF has the form

$$\boldsymbol{L}_B = \boldsymbol{J}\boldsymbol{\omega}_B^{BE} + J_r \boldsymbol{n}_B(\omega_1 + \omega_2 - \omega_3 - \omega_4) . \quad (13)$$

According to the theorem on change of angular momentum [19], the change of the vector $\boldsymbol{L}$ in time is described by the equation

$$\dot{\boldsymbol{L}}_B = -\boldsymbol{\omega}_B^{BE} \times \boldsymbol{L}_B + \boldsymbol{M}_B , \quad (14)$$

where the moment $\boldsymbol{M}_B$ is the sum of the moments $\boldsymbol{M}_i$ created by the thrust forces $\boldsymbol{P}_{Bi}$ of the propellers. According to Fig. 1, it can be written

$$\boldsymbol{M}_B = \ell \begin{pmatrix} p_3 - p_1 \\ 0 \\ p_2 - p_4 \end{pmatrix} = k_p \ell \begin{pmatrix} \omega_3^2 - \omega_1^2 \\ 0 \\ \omega_2^2 - \omega_4^2 \end{pmatrix} . \quad (15)$$

Let us write equation (14) as follows

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{BE} + J_r \boldsymbol{n}_B(\dot{\omega}_1 + \dot{\omega}_2 - \dot{\omega}_3 - \dot{\omega}_4) = \boldsymbol{M}_g + \boldsymbol{M}_B . \quad (16)$$

In equation (16), the gyroscopic moment $\boldsymbol{M}_g$ caused by the rotation of the quadcopter body and rotors is determined by the expression

$$\boldsymbol{M}_g = -\boldsymbol{\omega}_B^{BE} \times \boldsymbol{L}_B =$$
$$= -\boldsymbol{\omega}_B^{BE} \times (\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{BE} + J_r \boldsymbol{n}_B(\omega_1 + \omega_2 - \omega_3 - \omega_4)) . \quad (17)$$

For the angular momentum of the *i*-th rotor, the following expression is valid

$$\boldsymbol{h}_{Bi} = J_r(\boldsymbol{\omega}_B^{BE} + \boldsymbol{n}_B \omega_i) . \quad (18)$$

Therefore, taking into account (14), it can be obtained

$$J_r(\dot{\boldsymbol{\omega}}_B^{BE} + \boldsymbol{n}_B \dot{\omega}_i) + \boldsymbol{\omega}_B^{BE} \times (J_r(\boldsymbol{\omega}_B^{BE} + \boldsymbol{n}_B \omega_i)) = \boldsymbol{n}_B \mu_i , \quad (19)$$

where the reactive torque on the shaft of the *i*-th rotor is determined by the formulas

$$\mu_i = \beta_\rho D^5 \omega_i^2 = k_m \omega_i^2 , \; k_m = \beta_\rho D^5 / k_p . \quad (20)$$

The presence of the torque μ_i in equation (19) reflects the fact that when the propeller rotates, due to air resistance, a moment arises that prevents this rotation. To overcome this moment, the same torque is required on the engine shaft in the opposite direction. From equation (19) it follows

$$J_r \boldsymbol{n}_B \dot{\omega}_i = \boldsymbol{n}_B \mu_i - J_r \dot{\boldsymbol{\omega}}_B^{BE} - \boldsymbol{\omega}_B^{BE} \times (J_r(\boldsymbol{\omega}_B^{BE} + \boldsymbol{n}_B \omega_i)) . \quad (21)$$

Multiplying equation (21) by the vector $\boldsymbol{n}_B^T$ gives

$$\boldsymbol{n}_B^T J_r \boldsymbol{n}_B \dot{\omega}_i = \mu_i - \boldsymbol{n}_B^T J_r \dot{\boldsymbol{\omega}}_B^{BE} . \quad (22)$$

Whence, given that $\boldsymbol{n}_B^T J_r \boldsymbol{n}_B = J_r$, it follows

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$$\dot{\omega}_i = \frac{\mu_i}{J_r} - \boldsymbol{n}_B^T \dot{\boldsymbol{\omega}}_B^{BE} . \quad (23)$$

Substituting expression (23) into equation (16) and the necessary transformations yield

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{BE} = -\boldsymbol{n}_B(\mu_1 + \mu_2 - \mu_3 - \mu_4) + \boldsymbol{M}_g + \boldsymbol{M}_B . \quad (24)$$

or taking into account formula (20) it follows

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{BE} = -\boldsymbol{n}_B k_m(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) + \boldsymbol{M}_g + \boldsymbol{M}_B . \quad (25)$$

Let us introduce the following matrix into consideration

$$\boldsymbol{F} = \begin{pmatrix} -k_p\ell & 0 & k_p\ell & 0 \\ -k_m & k_m & -k_m & k_m \\ 0 & k_p\ell & 0 & -k_p\ell \end{pmatrix} . \quad (26)$$

Then equation (25) can be written in the following form

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{BE} = \boldsymbol{M}_g + \boldsymbol{F}\begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} . \quad (27)$$

An equation describing the quadcopter angular orientation is needed to fully describe its spatial motion. This equation has the following form when a quaternion is used [19]:

$$\dot{\lambda}_0^{EB} = -\left(\boldsymbol{\omega}_B^{BE}\right)^T \boldsymbol{\lambda}_{EB}, \ \dot{\boldsymbol{\lambda}}_{EB} = \lambda_0^{EB}\boldsymbol{\omega}_B^{BE} + \boldsymbol{\lambda}_{EB} \times \boldsymbol{\omega}_B^{BE} . \quad (28)$$

Thus, the spatial motion of the quadcopter is described by the following system of differential equations

$$\begin{cases} m\ddot{\boldsymbol{r}}_E = k_p(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2)\boldsymbol{n}_E + \boldsymbol{G}_E; \\ \boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{BE} = \boldsymbol{M}_g + \boldsymbol{M}_u; \\ \dot{\lambda}_0^{EB} = -\left(\boldsymbol{\omega}_B^{BE}\right)^T \boldsymbol{\lambda}_{EB}; \\ \dot{\boldsymbol{\lambda}}_{EB} = \lambda_0^{EB}\boldsymbol{\omega}_B^{BE} + \boldsymbol{\lambda}_{EB} \times \boldsymbol{\omega}_B^{BE}, \end{cases} \quad (29)$$

where

$$\boldsymbol{M}_u = \boldsymbol{F}\begin{pmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{pmatrix} . \quad (30)$$

The algorithms for controlling the spatial motion of a quadcopter were built on the assumption that on board the quadcopter there is information about the vector $\boldsymbol{r}_E$, the angular velocity vector $\boldsymbol{\omega}_B^{BE}$ and the orientation of the frame BFF relative to the frame EFF in the form of a quaternion $\boldsymbol{\Lambda}_{EB}$. The main tasks of spatial motion control are solved using the following algorithms:

– algorithm for determining the required orientation of the propellers thrust vector in the EFFs, which ensures the transfer of the quadcopter from the current position to the desired one;

– algorithm for calculating the torque, which ensures that the real direction of the thrust force coincides with the calculated one;

– algorithm for calculating the torque, which ensures stabilization of the yaw angle.

Let us introduce the following variables $\boldsymbol{x}_1 = \boldsymbol{r}_E$, $\boldsymbol{x}_2 = \dot{\boldsymbol{r}}_E$, $\boldsymbol{F}_E = \boldsymbol{u}$ and write equation (4) in Cauchy form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} , \quad (31)$$

where

$$\boldsymbol{A} = \begin{pmatrix} 0 & \boldsymbol{I}_3 \\ 0 & 0 \end{pmatrix}, \ \boldsymbol{B} = \begin{pmatrix} 0 \\ \boldsymbol{I}_3 \end{pmatrix}, \boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{pmatrix}, \boldsymbol{I}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

For system (31), the following terminal control problem is formulated: *find a control* $\boldsymbol{u}$ *that transfers system* (31) *from the current state* $\boldsymbol{x}(t_0)$ *at time* t_0 *to the given state* $\boldsymbol{x}(t_1)$ *at time* t_1 *and provides the minimum of the functional* $V(\boldsymbol{u}) = \frac{1}{2}\int_{t_0}^{t_1} \|\boldsymbol{u}\|^2 dt$. *The times* t_0 *and* t_1 *are given*.

This problem can be formulated as a two-point boundary value problem, represented as a Hamiltonian system with a maximum condition for the control Hamiltonian (Pontryagin's maximum principle [20]).

To solve this problem, let us set the boundary conditions for system (31) in the form

$$\boldsymbol{x}_1(t_0) = \boldsymbol{r}_E(t_0), \boldsymbol{x}_2(t_0) = \dot{\boldsymbol{r}}_E(t_0), \quad (32)$$

$$\boldsymbol{x}_1(t_1) = \boldsymbol{r}_E(t_1), \boldsymbol{x}_2(t_1) = \dot{\boldsymbol{r}}_E(t_1) , \quad (33)$$

and the Hamiltonian has the form

$$H = \frac{1}{2}\|\boldsymbol{u}\|^2 + \boldsymbol{\mu}^T(\boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}) . \quad (34)$$

In expression (34) $\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}$, is the related variable and $\boldsymbol{\mu}_1, \boldsymbol{\mu}_2$ are the three dimensional vectors. The optimality conditions are of the form

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$$\frac{\partial H}{\partial \boldsymbol{x}} = -\dot{\boldsymbol{\mu}} = \boldsymbol{A}^T \boldsymbol{\mu}\,, \tag{35}$$

$$\frac{\partial H}{\partial \boldsymbol{\mu}} = \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}\,, \tag{36}$$

$$\frac{\partial H}{\partial \boldsymbol{u}} = \boldsymbol{u} + \boldsymbol{B}^T \boldsymbol{\mu} = 0\,. \tag{37}$$

From condition (37) it follows

$$\boldsymbol{u} = -\boldsymbol{B}^T \boldsymbol{\mu}\,. \tag{38}$$

Substituting condition (38) into equation (36) yields the equation of a two-point boundary value problem in the form of an extended system

$$\boldsymbol{Y} = \boldsymbol{Q}\boldsymbol{Y} + \begin{pmatrix} \boldsymbol{B} \\ 0 \end{pmatrix} \boldsymbol{G}_E\,, \boldsymbol{Y}(t) = \begin{pmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\mu}(t) \end{pmatrix}, \boldsymbol{Q} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{B}\boldsymbol{B}^T \\ 0 & -\boldsymbol{A}^T \end{pmatrix}. \tag{39}$$

The solution to system (39) can be written as

$$\boldsymbol{x}(t) = \boldsymbol{\Phi}_{11}(t,t_0)\boldsymbol{x}(t_0) + \boldsymbol{\Phi}_{12}(t,t_0)\boldsymbol{\mu}(t_0), \tag{40}$$

$$\boldsymbol{\mu}(t) = \boldsymbol{\Phi}_{22}(t,t_0)\boldsymbol{\mu}(t_0)\,, \tag{41}$$

where $\boldsymbol{\Phi}(t_0,t) = \begin{pmatrix} \boldsymbol{\Phi}_{11}(t,t_0) & \boldsymbol{\Phi}_{12}(t,t_0) \\ 0 & \boldsymbol{\Phi}_{22}(t,t_0) \end{pmatrix}$ is the transition matrix of the extended system. The initial value of the vector $\boldsymbol{\mu}(t)$ is found by the formula

$$\boldsymbol{\mu}(t_0) = \boldsymbol{\Phi}_{12}^{-1}(t_1,t_0)[\boldsymbol{x}(t_1) - \boldsymbol{\Phi}_{11}(t_1,t_0)\boldsymbol{x}(t_0)]\,. \tag{42}$$

The calculated control values $\boldsymbol{u}^*$, trajectory $\boldsymbol{r}_E^*(t)$ and velocity $\dot{\boldsymbol{r}}_E^*(t)$ when moving the quadcopter from the current position to the desired one can be found as follows

$$\boldsymbol{u}^* = -\boldsymbol{B}^T \boldsymbol{\mu} = -\boldsymbol{\mu}_2\,, \tag{43}$$

$$\boldsymbol{r}_E^*(t) = \boldsymbol{x}_1(t)\,, \tag{44}$$

$$\dot{\boldsymbol{r}}_E^*(t) = \boldsymbol{x}_2(t)\,. \tag{45}$$

In this case, according to equation (4), the calculated force is determined by the expression

$$\boldsymbol{F}_E^*(t) = m\ddot{\boldsymbol{r}}_E^*(t) = m\boldsymbol{u}^*\,. \tag{46}$$

The force $\boldsymbol{F}_E^*(t)$ is the calculated total force. The real total force $\boldsymbol{F}_E(t)$ will differ from the calculated one due to the presence of disturbing forces. The difference in forces will lead to the quadcopter moving along a certain trajectory $\boldsymbol{r}_E(t)$ different from the calculated trajectory $\boldsymbol{r}_E^*(t)$. To eliminate this phenomenon, it is necessary to add a stabilizing component in the form of feedback on the state through the thrust force of the propellers. To find this component, consider the equation of the pointing error

$$\boldsymbol{e} = \boldsymbol{r}_E(t) - \boldsymbol{r}_E^*(t)\,. \tag{47}$$

This error can be found by subtracting equation (46) from equation (4):

$$m\ddot{\boldsymbol{e}}(t) = m(\ddot{\boldsymbol{r}}_E(t) - \ddot{\boldsymbol{r}}_E^*(t)) = \boldsymbol{F}_E(t) - \boldsymbol{F}_E^*(t) = \Delta\boldsymbol{F}\,, \tag{48}$$

which means

$$\boldsymbol{F}_E = \boldsymbol{F}_E^* + \Delta\boldsymbol{F}\,. \tag{49}$$

Let us choose $\Delta\boldsymbol{F}$ such that

$$\Delta\boldsymbol{F} = -m(\boldsymbol{K_1}\boldsymbol{e} + \boldsymbol{K_2}\dot{\boldsymbol{e}})\,, \tag{50}$$

where

$$\boldsymbol{K}_1 = diag(k_{1i}), \boldsymbol{K}_2 = diag(k_{2i}), \quad i = 1, 2, 3. \tag{51}$$

Then equation (48) can be written as

$$\ddot{\boldsymbol{e}}(t) = -(\boldsymbol{K_1}\boldsymbol{e} + \boldsymbol{K_2}\dot{\boldsymbol{e}})\,. \tag{52}$$

According to the main theorem on the asymptotic stability of a linear system, equation (52) for $k_{1i} > 0$ and $k_{2i} > 0$ is asymptotically stable and the quadcopter state vector will tend to the calculated one. In this case, the required direction of the force $\boldsymbol{P}_E(t)$ in the basis EFF will be determined by the expression

$$\boldsymbol{n}_E^*(t) = \frac{\boldsymbol{P}_E(t)}{\|\boldsymbol{P}_E(t)\|} = \frac{\boldsymbol{F}_E(t) - \boldsymbol{G}_E}{\|\boldsymbol{F}_E(t) - \boldsymbol{G}_E\|}\,. \tag{53}$$

Comparing (53) with (11), the equation for determining the required pitch ϑ^* and roll φ^* angles can be obtained:

$$\boldsymbol{n}_E^*(t) = \frac{\boldsymbol{P}_E(t)}{\|\boldsymbol{P}_E(t)\|} = \begin{pmatrix} -\sin\vartheta^* \\ \cos\vartheta^*\cos\varphi^* \\ \cos\vartheta^*\sin\varphi^* \end{pmatrix}, \tag{54}$$

$$\vartheta^*(t) = -\arcsin n_{1E}^*(t)\,, \tag{55}$$

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$$\varphi^*(t) = -\arctan \frac{n_{3E}^*(t)}{n_{2E}^*(t)}. \quad (56)$$

In the hover mode $\ddot{\boldsymbol{r}}_E(t_1) = 0$. Then, according to expression (6), $\boldsymbol{P}_E(t_1) = -\boldsymbol{G}_E$ and

$$\boldsymbol{n}_E^*(t_1) = \frac{\boldsymbol{P}_E(t_1)}{\|\boldsymbol{P}_E(t)\|} = -\frac{\boldsymbol{G}_E(t_1)}{\|\boldsymbol{G}_E(t)\|} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \quad (57)$$

Thus, when hovering, the required direction of the total thrust force of the propellers coincides with the direction of the local vertical. In this case

$$\varphi^*(t_1) = 0, \quad (58)$$

$$\vartheta^*(t_1) = 0, \quad (59)$$

and the program yaw angle $\psi^*(t)$ is a free parameter selected based on the quadcopter yaw angle requirements.

Now we will perform the synthesis of control of the angular motion of the quadcopter, ensuring the coincidence of the real direction of the thrust force of the propellers with the calculated one. Let us consider the quaternion mapping $\boldsymbol{N}_B^*$ of the vector $\boldsymbol{n}_B^*$. Its scalar part $scal(\boldsymbol{N}_B^*) = 0$, and vector part $vect(\boldsymbol{N}_B^*) = \boldsymbol{n}_B^*$. Since the quaternion $\boldsymbol{N}_B^*$ is a normalized quaternion, the quaternion equation is valid for it [21]

$$\ddot{\boldsymbol{N}}_B^* = \boldsymbol{U}_n - \left\|\dot{\boldsymbol{N}}_B^*\right\|^2 \boldsymbol{N}_B^*, \quad (60)$$

where $\boldsymbol{U}_n$ is an arbitrary quaternion with a zero scalar part, specifying which can form the required character of the change in the vector $\boldsymbol{n}_B^*$ projections on the BFF axes. In this case, the quaternion $\boldsymbol{U}_n$ satisfies the constraint

$$scal(\tilde{\boldsymbol{N}}_B^* \circ \boldsymbol{U}_n) = 0. \quad (61)$$

From constraint (61) it follows

$$(\boldsymbol{n}_B^*)^T \boldsymbol{U}_n = 0. \quad (62)$$

The vector form of equation (63) is

$$\ddot{\boldsymbol{n}}_B^* = \boldsymbol{u}_n - \left\|\dot{\boldsymbol{n}}_B^*\right\|^2 \boldsymbol{n}_B^*. \quad (63)$$

Let us represent the control $\boldsymbol{u}_n$ as follows

$$\boldsymbol{u}_n = -\boldsymbol{n}_B^* \times \boldsymbol{n}_B^* \times \boldsymbol{\tau}. \quad (64)$$

With this choice of $\boldsymbol{u}_n$, the relation (62) will hold for any $\boldsymbol{\tau}$. Taking into account (64), equation (63) can be written as

$$\ddot{\boldsymbol{n}}_B^* = -\boldsymbol{n}_B^* \times \boldsymbol{n}_B^* \times \boldsymbol{\tau} - \left\|\dot{\boldsymbol{n}}_B^*\right\|^2 \boldsymbol{n}_B^*. \quad (65)$$

Let us decompose the left-hand side of equation (65) into two components: perpendicular $\boldsymbol{n}_B^*$ and parallel $\boldsymbol{n}_B^*$:

$$-\boldsymbol{n}_B^* \times \boldsymbol{n}_B^* \times \ddot{\boldsymbol{n}}_B^* + \boldsymbol{n}_B^* \boldsymbol{n}_B^{*T} \ddot{\boldsymbol{n}}_B^* = \\ = -\boldsymbol{n}_B^* \times \boldsymbol{n}_B^* \times \boldsymbol{\tau} - \left\|\dot{\boldsymbol{n}}_B^*\right\|^2 \boldsymbol{n}_B^*. \quad (66)$$

Taking into account (64) and (65) transforming (66) gives

$$-\boldsymbol{n}_B^* \times \boldsymbol{n}_B^* \times (\ddot{\boldsymbol{n}}_B^* - \boldsymbol{\tau}) = \left(\boldsymbol{n}_B^{*T} \ddot{\boldsymbol{n}}_B^{*T} + \left\|\dot{\boldsymbol{n}}_B^*\right\|^2\right) \boldsymbol{n}_B^* = \\ = \left(-\left\|\dot{\boldsymbol{n}}_B^*\right\|^2 + \left\|\dot{\boldsymbol{n}}_B^*\right\|^2\right) \boldsymbol{n}_B^*. \quad (67)$$

From relation (67) it follows

$$\ddot{\boldsymbol{n}}_B^* = \boldsymbol{\tau} + \alpha \boldsymbol{n}_B^*, \quad (68)$$

where α is an arbitrary parameter. Since α is an arbitrary parameter, when solving various problems of controlling the motion of the vector $\boldsymbol{n}_B^*$ it can be set equal to zero ($\alpha = 0$). In this case, the dynamic model for synthesizing the control $\boldsymbol{\tau}$ takes on a simple form

$$\ddot{\boldsymbol{n}}_B^* = \boldsymbol{\tau}. \quad (69)$$

This equation is a linear equation with constant coefficients and allows the application of well-developed methods of the theory of linear systems with constant coefficients in the synthesis of control laws.

The control $\boldsymbol{u}_n$ is virtual, the real control is the control moment $\boldsymbol{M}_u$. Therefore, when using equation (69) to solve the problem of controlling the orientation of the quadcopter, it is necessary to know the dependence of the rotational moment $\boldsymbol{M}_u$ on the elements of the control vector $\boldsymbol{u}_n$. According to the work [22], this dependence has the form

$$\boldsymbol{M}_u = -\boldsymbol{M}_g - \boldsymbol{J}(\boldsymbol{n}_B^* \times (\boldsymbol{u}_n - \boldsymbol{p})), \quad (70)$$

where

$$\boldsymbol{p} = -\boldsymbol{\omega}_B^{BE} \times \left(\dot{\boldsymbol{n}}_B^* + \dot{\tilde{\boldsymbol{n}}}_B^*\right) + \ddot{\tilde{\boldsymbol{n}}}_B^*, \quad (71)$$

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$$\dot{\boldsymbol{n}}_B^* = -\boldsymbol{\omega}_B^{BE} \times \boldsymbol{n}_B^* + \dot{\tilde{\boldsymbol{n}}}_B^*, \quad (72)$$

$$\dot{\tilde{\boldsymbol{n}}}_B^* = \tilde{\boldsymbol{\Lambda}}_{EB} \circ \dot{\boldsymbol{n}}_E^* \circ \boldsymbol{\Lambda}_{EB}, \quad (73)$$

$$\ddot{\tilde{\boldsymbol{n}}}_B^* = -\dot{\boldsymbol{\omega}}_B^{BE} \times \boldsymbol{n}_B^* + \boldsymbol{p}, \quad (74)$$

$$\ddot{\tilde{\boldsymbol{n}}}_B^* = \tilde{\boldsymbol{\Lambda}}_{EB} \circ \ddot{\boldsymbol{n}}_B^* \circ \boldsymbol{\Lambda}_{EB}. \quad (75)$$

In order for the real direction of the propellers' thrust to coincide with the calculated one, it is necessary to find the control $\boldsymbol{u}_n$ that ensures asymptotic stability of the equilibrium position

$$\boldsymbol{n}_B^* = \boldsymbol{n}_B. \quad (76)$$

To do this, we will use the equation of motion of the vector $\boldsymbol{n}_B^*$ in the form (69), and as a result

$$\ddot{\boldsymbol{n}}_B^* = \boldsymbol{\tau}. \quad (77)$$

Consider the control error

$$\boldsymbol{e} = \boldsymbol{n}_B^* - \boldsymbol{n}_B. \quad (78)$$

Given that $\boldsymbol{n}_B$ is a constant vector, the following equation is valid for the error

$$\ddot{\boldsymbol{e}}_B = \boldsymbol{\tau}. \quad (79)$$

It is obvious that the law of control

$$\boldsymbol{\tau} = -\boldsymbol{K}_1\boldsymbol{e} - \boldsymbol{K}_2\dot{\boldsymbol{n}}_B^* \quad (80)$$

provides asymptotic stability to the equilibrium position $\boldsymbol{e} = 0$, $\dot{\boldsymbol{e}} = 0$. In this case, the control $\boldsymbol{u}_n$ will be determined by the relation (64), and the real control moment $\boldsymbol{M}_u$ by the relation (70). To calculate the moment $\boldsymbol{M}_u$, it is necessary to know the vectors $\dot{\boldsymbol{n}}_E^*$ and $\ddot{\boldsymbol{n}}_E^*$. There are two ways to find these variables: analytical and numerical. Analytical is very cumbersome, so in this case, given that $\boldsymbol{n}_E^*$ is a smooth analytical function of time, it is easier to do it numerically.

To construct an algorithm for stabilizing the yaw angle, let us consider the quaternion $\boldsymbol{\Lambda}_{EB}$. According to [21], the following equation is valid for it

$$\ddot{\boldsymbol{\Lambda}}_{EB} = \boldsymbol{U}_\Lambda - \left\|\dot{\boldsymbol{\Lambda}}_{EB}\right\|^2 \boldsymbol{\Lambda}_{EB}, \quad (81)$$

where $\boldsymbol{U}_\Lambda$ is an arbitrary quaternion, specifying which can form the desired angular motion of the BFF relative EFF. In this case, the moment $\boldsymbol{M}_u$ is determined by the expression

$$\boldsymbol{M}_u = -2\boldsymbol{J}(\boldsymbol{\lambda}_{EB} \times \boldsymbol{u}_\Lambda) + \boldsymbol{\omega}_B^{BE} \times \boldsymbol{J}\boldsymbol{\omega}_B^{BE}, \quad (82)$$

where $\boldsymbol{u}_\Lambda$ is the vector part of a quaternion $\boldsymbol{U}_\Lambda$. The quaternion $\boldsymbol{\Lambda}_{EB}$ is a normalized quaternion and has only three independent coordinates, the fourth coordinate is determined from the condition $\|\boldsymbol{\Lambda}_{EB}\| = 1$. Let us choose its vector part $\boldsymbol{\lambda}_{EB}$ as independent coordinates and consider the equation that describes its change in time

$$\ddot{\boldsymbol{\lambda}}_{EB} = \boldsymbol{u}_\Lambda - \left(\left(\dot{\lambda}_0^{EB}\right)^2 + \dot{\boldsymbol{\lambda}}_{EB}^T\dot{\boldsymbol{\lambda}}_{EB}\right)\boldsymbol{\lambda}_{EB}. \quad (83)$$

In [21] it is shown that the control law

$$\boldsymbol{u}_\Lambda = -k_1\boldsymbol{\lambda}_{EB} - k_2\dot{\boldsymbol{\lambda}}_{EB}, \quad k_1 > 0,\ k_2 > 0 \quad (84)$$

ensures asymptotic stability of the equilibrium position

$$\boldsymbol{\lambda}_{EB}^T = (0\ \ 0\ \ 0), \quad \dot{\boldsymbol{\lambda}}_{EB}^T = (0\ \ 0\ \ 0), \quad (85)$$

and as a consequence the asymptotic stability of the equilibrium position

$$\boldsymbol{\Lambda}_{EB} = 1, \quad \dot{\boldsymbol{\Lambda}}_{EB} = 0, \quad (86)$$

that is, the stabilization of BFF relative to EFF.

4 EXPERIMENTS

A numerical simulation of the proposed algorithms was carried out to analyze the qualitative features of the algorithm. The parameters of the quadcopter model are presented in Table 1. The flight from the starting point to a given one with coordinates $\boldsymbol{r}_E(t_1) = (1000;\ 300;\ -4000)^T$ and hovering over it were simulated. The initial values of the quadcopter angular orientation (for which the graphs below are given) were set as follows: $\omega_B^{BE} = 0$, $\varphi = 10^\circ$, $\psi = -10^\circ$ and $\vartheta = 10^\circ$. After hovering, the quadcopter should turn around at an angle $\psi = 45^\circ$. The flight time from the starting point to the given one was chosen to be 300 seconds.

Table 1 – Quadcopter model parameters

Parameter	Description	Value	Units
g	Gravity	9.81	m/s^2
m	Mass	0.468	kg
ℓ	Distance	0.225	m
J_r	Rotor Inertia	$3.4\cdot10^{-5}$	$kg\cdot m^2$
J_x	Roll Inertia	$4.9\cdot10^{-3}$	$kg\cdot m^2$
J_y	Pitch Inertia	$4.9\cdot10^{-3}$	$kg\cdot m^2$
J_z	Yaw Inertia	$8.8\cdot10^{-3}$	$kg\cdot m^2$
k_p	Proportionality Constant	$2.9\cdot10^{-5}$	
k_m	Proportionality Constant	$1.1\cdot10^{-6}$	

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5 RESULTS

Figure 3 shows graphs of changes in the coordinates of the centre of mass of the quadcopter over time, Fig. 4 shows graphs of changes in the centre of mass velocities over time, Fig. 5 shows the orientation angles of the quadcopter, and Fig. 6 shows the angular velocities.

6 DISCUSSION

In this study, the quadcopter was examined as a control object, its flight mechanics analyzed, and a novel methods for synthesizing algorithms for spatial motion control of the quadcopter were proposed. Based on the obtained quadcopter motion model, algorithms for controlling the quadcopter's spatial motion were developed, namely, an algorithm for determining the required direction of the propeller thrust to translate the quadcopter from its current position to the desired one and algorithms for its angular motion control.

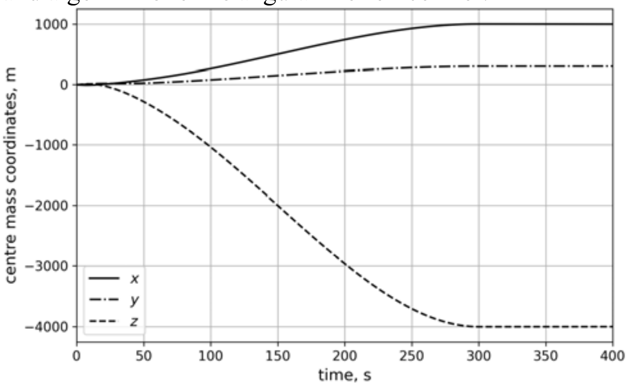

Figure 3 – Change of position coordinates over time

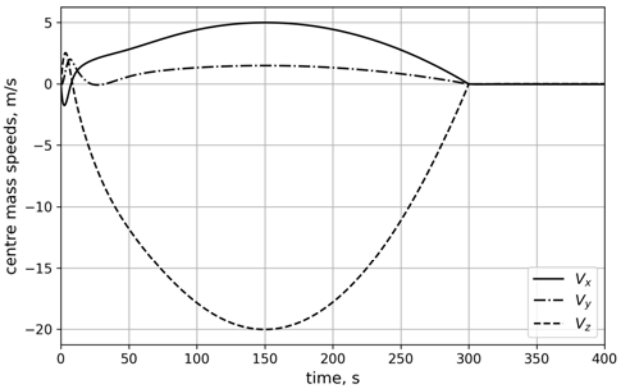

Figure 4 – Change the position coordinate velocities over time

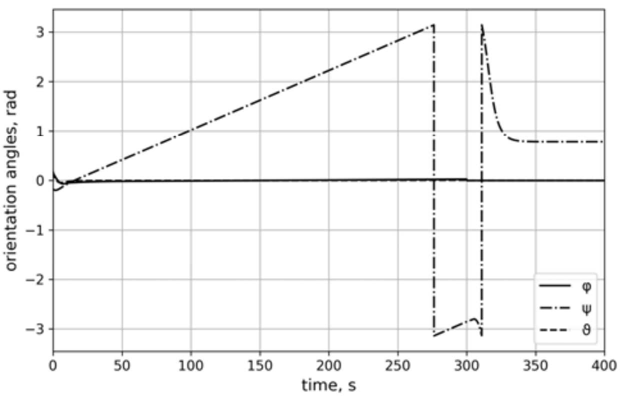

Figure 5 – Change the quadcopter orientation angles over time

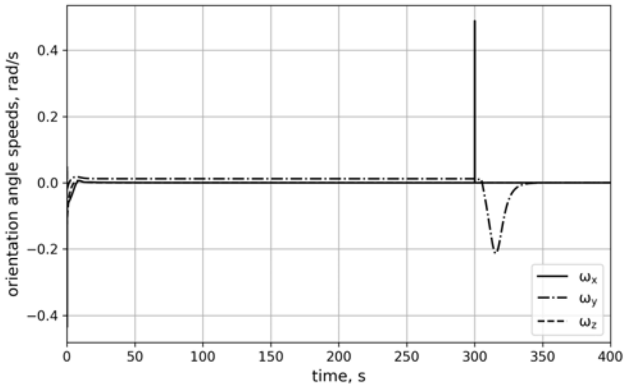

Figure 6 – Change the quadcopter angular speeds over time

The algorithm development process consists of two main parts. The first part is creating an algorithm to move a quadcopter from its current position to a specified one. For this, the approach from [23] was used, its essence is to construct an analytical solution to the boundary value problem for a linear stationary system without control constraints. This approach gives acceptable results when the system is not affected by external disturbances and there are no measurement errors. The calculated direction $\boldsymbol{n}_B^*$ of the propellers' thrust force due to disturbing forces may differ from the actual direction $\boldsymbol{n}_B$. To eliminate this phenomenon, a stabilizing component in the state feedback form was added to the analytical algorithm for solving a two-point boundary value problem.

In the second part, a novel method for constructing a quadcopter angular motion control algorithm was proposed, based on the motion of vector quaternion equation developed by authors in the works [21, 22]. The use of a dynamic quaternion equation of motion of the vector has greatly simplified control synthesis, reducing it to a set of second-order integrating links. In many cases, the control synthesis problem has an analytical solution for such systems. Control algorithms derived from this model are implemented much more simply than those synthesized from the traditional model, which includes the dynamical Euler equation and the kinematic equation for the quaternion.

The proposed algorithms were numerically simulated, and the results demonstrated that the quadcopter flew from the starting point to the target point within 300 seconds. It then turned at a 45-degree angle and hovered over the target location. The results confirmed the proposed algorithms' efficiency in controlling the quadcopter's spatial movement.

CONCLUSIONS

A mathematical model of a quadcopter motion as a control object was developed. Based on this model the algorithms for quadcopter spatial motion control were constructed. Control algorithms include determining the required direction of the propellers' thrust force to transfer the quadcopter from the current position to the given one and control angular motion, which ensures the

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coincidence of the real direction of the propellers' thrust force with the calculated one and yaw angle stabilization.

The scientific novelty of the obtained results is that the quadcopter angular motion control algorithms developed based on the dynamic equation for the quaternion [21]. To eliminate the difference between the real direction of the thrust force and the calculated one a stabilizing component in the form of feedback on the state was added to the known analytical algorithm for solving a two-point boundary value problem. This significantly improved the accuracy of guiding the quadcopter to the given position.

The practical significance of the obtained results is that the developed algorithms allow the implementation of all main modes of quadcopter autonomous flight: stabilization mode in which the quadcopter automatically keeps zero roll and pitch angles and stabilizes the yaw angle; mode of maintaining a given position in which the quadcopter automatically hovers over a given point on the earth's surface; mode of automatic flight along points; modes of automatic takeoff and landing.

Prospects for further research will focus on studying the qualitative aspects of quadcopter control processes affected by external disturbances and onboard sensor errors and developing algorithms for autonomous navigation without using GPS information.

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ТЕРМІНАЛЬНЕ КЕРУВАННЯ ПРОСТОРОВИМ РУХОМ КВАДРОКОПТЕРА

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АНОТАЦІЯ

Актуальність. Побудова алгоритмів керування квадрокоптером є областю підвищеного інтересу, оскільки керування квадрокоптером принципово складна задача, незважаючи на його механічну простоту. Ключовою проблемою систем управління квадрокоптерами є ефективне поєднання трьох поступальних та трьох обертальних ступенів свободи руху для виконання унікальних цільових маневрів. Крім того, ці задачі актуальні у зв'язку з високою затребуваністю квадрокоптерів у різних видах діяльності людини, таких як кадастрова аерофотозйомка для моніторингу важкодоступних територій, доставка вантажів на невеликі відстані, військова справа тощо.

Мета роботи – розробка та обгрунтування нових методів побудови алгоритмів високоточного керування просторовим рухом квадрокоптера, що забезпечують його автономну роботу у всіх основних режимах польоту: режим стабілізації, режим утримання положення, режим автоматичного польоту з точки в точку, режим автоматичного зльоту та посадки.

Метод. Поставлена мета зумовила використання наступних методів дослідження. Для розробки алгоритмів розрахунку програмних траєкторій переведення квадрокоптера з поточного стану в заданий застосовано принцип максимуму Понтрягіна. Для синтезу та аналізу алгоритмів керування кутовим положенням квадрокоптера використано функції Ляпунова та методи модального керування. Для перевірки та підтвердження отриманих теоретичних результатів використано методи чисельного моделювання.

Результати. Запропоновано методику побудови алгоритмів керування просторовим рухом квадрокоптера, що складається з двох частин. Перша частина містить удосконалений метод побудови алгоритма переведення квадрокоптера з поточного положення в задане. У другій частині запропоновано оригінальний метод побудови алгоритмів керування орієнтацією квадрокоптера на основі динамічного рівняння для кватерніону.

Висновки. Запропонована математична модель руху квадрокоптера та методи побудови алгоритмів керування верифіковані чисельним моделюванням та можуть бути застосовані для розробки систем керування квадрокоптерами.

КЛЮЧОВІ СЛОВА: квадрокоптер, кватерніон, принцип максимуму Понтрягіна, гамільтоніан, функції Ляпунова.

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