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METHOD FOR STUDYING THE TIME-SHIFTED MATHEMATICAL MODEL OF A TWO-FRAGMENT SIGNAL WITH NONLINEAR FREQUENCY MODULATION

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ABSTRACT

Context. The further development of the theory and techniques for forming and processing complex radar signals encompasses both the study of existing mathematical models of probing radio signals and the creation of new ones. One of the directions of such research focuses on reducing the maximum side lobe level in the autocorrelation functions of signals with intra-pulse modulation of frequency or phase. In this context, the instantaneous frequency may vary according to either a linear or nonlinear law. Nonlinear frequency modulation laws can reduce the maximum level of side lobes without introducing amplitude modulation in the output signal of the radio transmitting device and, consequently, without causing power loss in the sensing signals. The widespread implementation of nonlinear-frequency-modulated signals in radar technology is constrained by the insufficient development of their mathematical models. Therefore, the development of methods for analyzing existing mathematical models of signals with nonlinear frequency modulation remains an urgent scientific task.

Objective. The purpose of this work is to develop a method for conducting research to evaluate the advantages and disadvantages of a mathematical model of a nonlinear-frequency-modulated signal consisting of two fragments with linear frequency modulation.

Method. This study proposes a method for analyzing mathematical models of signals based on the transition from a shifted time scale to the current time scale. The methodology consists of the following main stages: a formalized description of mathematical models, transition to an alternative time scale, identification of components and determination of their physical essence, and a comparative analysis. The proposed method was validated through simulation modeling.

Results. Using the proposed method, it has been determined that the mathematical operation of time scale shifting is equivalent to the introduction of additional components in the mathematical model. These components simultaneously and automatically compensate for the frequency jump at the junction of fragments, as well as introduce an additional linear phase increment in the second linearly frequency-modulated fragment. This approach provides a clear illustration of the frequency jump compensation mechanism in the studied mathematical model. The applied method enabled the identification of a drawback in the examined mathematical model, namely, the absence of a compensatory component for the instantaneous phase jump during the transition from the first LFM fragment to the second.

Conclusions. A method has been developed to determine the essence and corresponding influence of the components of a mathematical model in a time-shifted, nonlinear, frequency-modulated signal, which consists of two fragments with linear frequency modulation. The model under study is not entirely accurate, as it lacks a component to compensate for the phase jump at the transition from the first signal fragment to the second. The introduction of such a component ensures a further reduction in the maximum level of the side lobes of the signal autocorrelation function.

KEYWORDS: nonlinear-frequency-modulated signals, mathematical model, instantaneous phase jump, autocorrelation function, maximum level of side lobes.

ABBREVIATIONS

ACF is an autocorrelation function;
AFS is an amplitude-frequency spectrum;
ESS is an effective scattering surface;
FM is a frequency modulation;
FMR isa frequency modulation rate;
IPM is an intra-pulse modulation;
LFM is a linear frequency modulation;
MF is a matched filtering;
ML is a main lobe;
NLFM is a non-linear frequency modulation;
MM is a mathematical model;

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MPSLL is a maximum peak side lobe level;
PSLL is a peak side lobe level;
REQ is a radio electronic equipment;
SAR is a synthetic aperture radar;
SL is a side lobe;
WF is a window function.

NOMENCLATURE

f_0 is an initial signal frequency, Hz;

$f(t)$ is an instantaneous frequency of the NLFM signal, Hz;

$f_2(t)$ is an instantaneous frequency of the second fragment of the NLFM signal, Hz;

Δf_1 is a frequency deviation of the first fragments of the NLFM signal, Hz;

Δf_2 is a frequency deviation of the second fragments of the NLFM signal, Hz;

C_2 is a constant of integration;

t is a current time, s;

T_1 is a duration of the first fragments of the NLFM signal, s;

T_2 is a duration of the second fragments of the NLFM signal, s;

$\varphi(t)$ is a instantaneous phase of the NLFM signal, rad;

$\delta\varphi_{12}$ is a compensation component for the instantaneous phase jump at the junction of the first and second LFM fragments, rad;

$\varphi_2(t)$ is an instantaneous phase of the second LFM fragment of the signal, rad;

β_1 is a FMR of the first fragments of the NLFM signal, Hz/s;

β_2 is a FMR of the second fragments of the NLFM signal, Hz/s.

INTRODUCTION

The continuous development of electronic technologies, along with the near-total abandonment of electrovacuum devices in favor of solid-state components, ensures multifunctionality and reduces the weight and dimensions of REQ. Another important direction of expanding the capabilities of REQ is the use of signals with IPM of phase and frequency, commonly referred to as complex signals. Historically, the first systems to incorporate linear frequency-modulated and code-phase manipulated signals emerged [1–10] and are still widely used today. These signals and their MF systems continue to evolve, undergo modifications, and improve [11–22, 24–44].

The implementation of signals with IPM into radio engineering and telecommunications has provided developers of radar systems, radionavigation, and communication technologies with additional opportunities to significantly enhance system performance. This includes extending the operational range of REQ, provided that the peak power of their radio transmitting devices is limited, improving electromagnetic compatibility, enhancing noise immunity and operational security, and increasing the bandwidth of information transmission channels [1–10].

However, despite the numerous advantages of using signals with IPM, a significant drawback occurs – the high MPSLL of the ACF compared to simple signals, which imposes limitations on the achievable dynamic range of the radio receiving device. In a multi-target aerial environment, the reflected signal from a target with a lower ESS may be masked by the side lobes of the signal from a target or a passive obstacle with a higher ESS. Research aimed at reducing MPSLL focuses on both improving MF radio receiving devices and implementing new types of signals with IPM [1–4, 11–44].

Thus, in studies [11–16], methods and devices have been proposed to improve the processing of received echo signals.

In [17–44], several types of NLFM signals have been proposed for use in radio transmitting devices to reduce MPSLL.

A large number of studies have been dedicated to the problem of reducing MPSLL in SAR systems for various purposes [15, 18–22].

Research on the issue of reducing MPSLL by implementing NLFM signals whose fragments have IPM different from LFM has been conducted in studies [23–38].

A number of works are aimed at improving the existing MM of NLFM signals consisting of two or three LFM fragments [39–44].

The authors [45–47] conducted a study analyzing NLFM signal types and estimating their time-frequency parameters in the context of solving problems related to electronic intelligence and REQ suppression.

It should be noted that in the reviewed works, different time scales are used for the formal description of the mathematical model (MM) of pulsed NLFM signals [1–4, 11–44]. In particular, a symmetric time scale relative to zero results in a radio pulse consisting of two opposite fragments. Additionally, a continuous time scale allows for the sequential determination of amplitude values of signal samples in real-time, referred to as the current-time MM. Another approach is to use a mathematical technique in which the mathematical description of both the first and each subsequent fragment of the NLFM signal starts from the zero-time value, thus implementing the time-shifted MM.

In study [40], it is noted that the time-shifted MM of two- and three-fragment NLFM signals has a useful feature, namely, it provides automatic compensation for the frequency jump at the junction of LFM fragments, which occurs at the moment of a change in the FMR value.

A detailed analysis of the compensation mechanism and the study of the peculiarities of MM operation with a time shift, due to the complexity of the mathematical description, have not been conducted in known studies, so it is advisable to conduct such an analysis.

To conduct the study, this paper proposes a method based on the mathematical transformation of a time-shifted MM into an equivalent current-time MM, i.e., the representation of models on a unified time scale, followed

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by a detailed analysis of their fine structure and properties.

Using the example of the MM of a two-fragment NLFM signal, the validity of this approach is demonstrated, and the equivalence of the current-time and time-shifted MM is substantiated. Based on the results of the conducted analysis, previously unaccounted distortion components of the NLFM signal are identified, and a method for their compensation is proposed.

The object of study is the process of synthesizing NLFM signals using a time-shifted MM based on two LFM fragments with different FMR values.

The subject of study is the MM of a time-shifted NLFM signal consisting of two LFM fragments.

The purpose of the work is to identify and analyze the components of the time-shifted MM of a two-fragment NLFM signal by converting it into a current-time model.

1 PROBLEM STATEMENT

For further analysis, we will write the expressions for the instantaneous frequency and phase of the two-fragment NLFM signal in the current time.

$$f(t)=\begin{cases} f_0 \pm \beta_1 t,\ 0 \le t \le T_1; \\ f_0 \pm \beta_1 T_1 \pm \beta_2 t,\ T_1 < t \le T_1 + T_2; \end{cases} \quad (1)$$

$$\varphi(t)=2\pi\begin{cases} f_0 t \pm \beta_1 \dfrac{t^2}{2},\ 0 \le t \le T_1; \\ (f_0 \pm \Delta f_1)t \pm \beta_2 \dfrac{t^2}{2},\ T_1 < t \le T_1 + T_2, \end{cases} \quad (2)$$

where $\beta_1 = \dfrac{\Delta f_1}{T_1}$; $\beta_2 = \dfrac{\Delta f_2}{T_2}$.

The '+' or '–' sign in (1), (2), and the subsequent expressions for the uncertainty ' ' is chosen depending on whether the frequency of the LFM fragments increases or decreases with time.

The descriptions (1) and (2) are obtained by removing the third fragment of the MM of the NLFM signal introduced in [2].

For describing time-shifted NLFM signals, we will use the MM introduced by the authors in [40, 42–44]. The expression for the instantaneous frequency is as follows:

$$f(t)=\begin{cases} f_0 \pm \beta_1 t, 0 \le t \le T_1; \\ f_0 \pm \beta_1 T_1 \pm \beta_2 (t - T_1), T_1 < t \le T_2, \end{cases} \quad (3)$$

for the description of the instantaneous phase:

$$\varphi(t)=2\pi\begin{cases} f_0 t \pm \dfrac{\beta_1}{2} t^2, 0 \le t \le T_1; \\ (f_0 \pm \Delta f_1)(t - T_1) \pm \beta_2 \left(\dfrac{t^2}{2} - T_1 t\right), \\ T_1 < t \le T_2. \end{cases} \quad (4)$$

The defining difference between (3), (4) and (1), (2) is the use of a different time scale. The initial count of the frequency and phase of the second LFM fragment in (3), (4) is shifted to the zero mark. During the studies conducted earlier [40], it was determined that the MMs (3), (4) provide automatic compensation for the instantaneous frequency jump at the junction of the fragments, but the mechanism of such compensation has not been studied in the known works.

2 REVIEW OF THE LITERATURE

The fundamental principles of the construction and functioning of REQ for various purposes, including the use of signals with IPM, are discussed in papers [1–10]. It is noted that in devices using MF based on the compression of complex signals, various methods for reducing the MPSLL of the ACF are employed.

The time WF method is most widely used in the radio receiving device, which rounds the radio pulse envelope, resulting in a reduction of the MPSLL at the MF output [12, 16, 24].

The authors [1, 2] proposed emitting radio signals with a rounded AFS, which is equivalent to its WF in the time domain.

It is possible to further improve the results of the window function (WF) and achieve lower values of the MPSLL. To this end, signals with polynomial FM [1, 14, 15] and NLFM signals [2, 17–22] are proposed. In papers [1, 2], the achievable MPSLL for the proposed signals was determined by calculation to be –42.8 dB. However, in study [2], it is noted that for low-base signals – those for which the product of signal duration and spectrum width is less than 100 – a MPSLL value of –30.0 dB is considered a significant achievement. The authors [37], by improving the MM of the form [1] and using a genetic algorithm to optimize the time-frequency parameters, achieved an MPSLL for the low-base signal at –34.9 dB.

The distinctive feature of the NLFM signal proposed in [2] is that it consists of three LFM fragments, with the FMR changing as it transitions to a new fragment. Further improvement of the MM of the form [2] was conducted by the authors [34–44]. The main difference in the MMs they propose is the use of a time-shifted scale and fragments with both linear and nonlinear frequency modulation laws.

The authors [34–37, 40–42] have developed MM for two- and three-fragment NLFM signals, in which compensation for frequency-phase distortions at the junctions of the fragments is implemented at the moments when the FMR changes. The fragments of such signals follow both linear and nonlinear frequency modulation laws. These NLFM signals, which simultaneously include fragments with both linear and nonlinear FM, have been proposed to be classified as combined signals [34–36].

It is shown in [43, 44] that the feature of the time-shifted MM is the automatic compensation for the frequency jump at the junction of fragments, which allows the synthesis of signals while adhering to the specified time-frequency parameters without the introduction of

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any additional compensation components. However, the compensation mechanism itself has not been studied.

3 MATERIALS AND METHODS

For further research, we will use the proposed method, which involves replacing the time scale for describing the second fragment (3), i.e., transitioning from shifted time to current time. By expanding the brackets and combining like terms, we obtain:

$$f_2(t) = f_0 \pm \beta_2 t \mp (\beta_2 - \beta_1)T_1, T_1 < t \le T_2. \quad (5)$$

The analysis of (5) indicates that, compared to (1), the expression for the second fragment has undergone a transformation of the constant component of the frequency change from $\beta_1 T_1$ to the component $(\beta_2 - \beta_1)T_1$, which is accounted with the opposite sign relative to the main one and is interpreted as compensatory.

Let us compare (5) with the current-time MM of the two-fragment NLFM signal with compensation for instantaneous phase and frequency jumps at the junction of fragments [41]:

$$f(t) = \begin{cases} f_0 \pm \beta_1 t, \; 0 \le t \le T_1; \\ f_0 \pm \beta_2 t \mp (\beta_2 - \beta_1)T_1, \; T_1 < t \le T_1 + T_2; \end{cases} \quad (6)$$

$$\varphi(t) = 2\pi \begin{cases} f_0 t + \dfrac{\beta_1 t^2}{2}, \; 0 \le t \le T_1; \\ \left[f_0 + (\beta_2 - \beta_1)T_1\right] t + \dfrac{\beta_2 t^2}{2} + \dfrac{(\beta_2 - \beta_1)T_1^2}{2}, \\ T_1 \le t \le T_1 + T_2. \end{cases} \quad (7)$$

The change in instantaneous frequency is defined by (6), from the second expression of which we can observe that the compensation for the instantaneous frequency jump at the junction of the signal fragments occurs due to the component $(\beta_2 - \beta_1)T_1$.

From this, an important conclusion can be drawn: the time-shifting operation in the second expression of the system of equations (3) is equivalent to the introduction of a compensatory component for the frequency jump, which is what needed to be proven.

We now turn to the current time scale in the description of the instantaneous phase of the second LFM fragment of the MM (4). Through simple transformations, we obtain:

$$\varphi_2(t) = 2\pi \begin{cases} \left[f_0 \mp (\beta_2 + \beta_1)T_1\right] t \pm \dfrac{\beta_2 t^2}{2} \mp \Delta f_1 T_1, \\ T_1 < t \le T_1 + T_2. \end{cases} \quad (8)$$

The analysis of (8) shows that the component $(\beta_2 + \beta_1)T_1$ in square brackets compensates for the linear incursion of the instantaneous phase caused by the compensatory component of the frequency jump, while the component $\Delta f_1 T_1$ outside the square brackets compensates for the total phase incursion of the first LFM fragment. That is, as intended by the authors of MM (3), (4), the use of a shifted time scale should ensure a zero initial count for both the frequency and phase of the signal for the second LFM signal fragment. However, in this case, a jump in the instantaneous phase occurs due to the instantaneous frequency jump $T_1(\beta_2 + \beta_1)$ at the junction of the fragments, requiring additional compensation. The development of such an MM was carried out in study [40], resulting in the following:

$$\varphi(t) = 2\pi \begin{cases} f_0 t \pm \dfrac{\beta_1 t^2}{2}, 0 \le t \le T_1; \\ (f_0 \pm \Delta f_1)(t - T_1) \pm \beta_2 \left(\dfrac{t^2}{2} - T_1 t\right) \mp \delta\varphi_{12}, \\ T_1 < t \le T_1 + T_2, \end{cases} \quad (9)$$

where $\delta\varphi_{12} = \dfrac{1}{2} T_1^2 (\beta_2 + \beta_1)$. (10)

It should be noted that the time scale shift operation in (10) led to a sign change from '–' to '+' in the parentheses compared to (7).

To derive (9), the graph-analytical method was applied in [40]; let us attempt to obtain this MM purely analytically. As a result of integrating the second equation (3), we obtain:

$$\varphi_2(t) = \begin{cases} 2\pi \int_t f_2(t - T_1)dt = (f_0 \pm \Delta f_1)(t - T_1) \pm \beta_2 \left(\dfrac{t^2}{2} - T_1 t\right) + C_2, \\ T_1 < t \le T_2. \end{cases} \quad (11)$$

The integration constant C_2 is determined considering the initial conditions:

$$C_2 = \varphi_2(t)\Big|_{t=T_1} = \mp \beta_2 \frac{T_1^2}{2}, \quad (12)$$

has the physical meaning of a compensatory component concerning the phase jump at the junction of the fragments caused by the instantaneous frequency jump.

Comparison of (10) and (12) shows that they do not correspond, and therefore we conclude that expression (11) is not entirely correct. In order to compensate for the phase jump at the junction of fragments caused by a frequency jump for a time-shifted MM, it is necessary to integrate the frequency jump component $(\beta_2 + \beta_1)T_1$, which is used to calculate the linear phase incursion in (8). This is quite logical since the time-shift operation ensures a zero initial phase value for the second LFM fragment, while the final phase of the first LFM fragment is not zero.

Thus, it has been established that the application of the time scale shift to a zero point for the second LFM fragment in MM (3) is equivalent to the introduction of a

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compensatory component for the instantaneous frequency jump at the junction of the fragments. The obtained result can be similarly extended to NLFM signals with a greater number of fragments.

As a result of the conducted study, it has been established that MM (4), used by the authors in [43, 44], does not provide complete compensation for the phase distortions of the resulting NLFM signal that occur when transitioning to a new fragment due to the instantaneous change in the value of the FMR. Only the additional linear phase incursion caused by the frequency jump at the moment of this transition is compensated. Therefore, obtaining the resulting signal with a reduced MPSLL is not a general rule and is only achieved for specific sets of frequency-time parameters of the LFM fragments, which are typically determined through selection.

We will verify the obtained theoretical results through simulation modeling. It should be noted that MM (7) and (9) are equivalent and provide identical results, so it is sufficient to use one of them for the simulation.

4 EXPERIMENTS

Mathematical modeling was carried out using the Matlab software package. The studies were conducted through a comparative analysis of the results obtained using MM (4) and (9) for NLFM signals with identical frequency-time parameters, namely: $\Delta f_1 = \Delta f_2 = 200$ kHz, $T_1 = 40$ µs, $T_2 = 100$ µs. A classical LFM signal with a duration of 140 µs and a frequency deviation of 400 kHz was also modeled.

The ACF parameters obtained as a result of modeling the specified signals are summarized in Table 1.

Table 1 – Values of ACF Parameters for the Signals

The name of the ACF parameter	LFM	NLFM (4)	NLFM (9)
Width of the ML of the ACF, µs	2.21	2.37	2.43
MPSLL, dB	–13.5	–14.59	–17.14
Rate of SL decay, dB/decade	19.35	19.8	21.25

The result of using the time-shifted MM without compensation for instantaneous phase jumps (4) is shown in Fig. 1. Figure 1a demonstrates the change in instantaneous frequency without a jump at the junction of the LFM fragments (at the current time of 40 µs). To simplify the analysis of the oscillogram of the resulting signal (Fig. 1b), we take $f_0 = 0$. In the oscillogram, at the moment of transition from the first fragment of the NLFM signal to the second at 40 µs, an instantaneous phase jump is observed, with a magnitude exceeding 180^0. The presence of a significant phase jump causes a dip in the signal spectrum between the LFM components at a frequency of 200 kHz (Fig. 1c), which also results in distortion of its peak and the presence of ripples on the side slopes and "wings" of the spectrum.

The analysis of the ACF of the signal (Fig. 1d) indicates the merging of the ML with the nearby SLs, which led to the expansion of the ML and, consequently, a deterioration in range resolution. Due to instantaneous phase distortion at the junction of the fragments, the resulting MPSLL is higher than expected, and the SL ripples exhibit level fluctuations and an irregular pattern of changes. The MPSLL of the ACF is –14.59 dB, and the ML width at the 0.707 level is 2.37 µs. Compared to the classical LFM signal, the signal of type (4) demonstrated an 8% reduction in the MPSLL of the ACF, a 7% increase in the ML width at the 0.707 level of the maximum, and a slight increase (approximately 2%) in the rate of decline in the SL of the ACF.

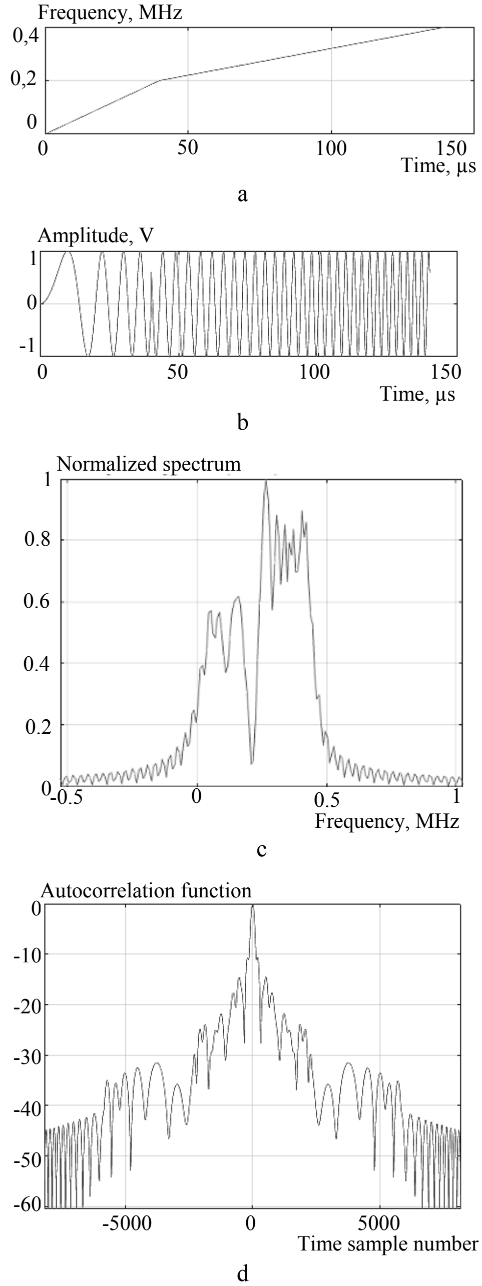

Figure 1 – Oscillogram (a), instantaneous frequency change graph (b), spectrum (c), ACF (d) of the NLFM signal according to the model (4)

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The simulation results corresponding to (9) are presented in Fig. 2. Figure 2a shows that the frequency of the signals (4) and (9) changes within the same range; therefore, their frequency variation graphs are in full compliance. The frequency jump at the junction of the fragments is compensated. The oscillogram in Fig. 2b, in contrast to Fig. 1b, is smooth, i.e., the compensatory phase component was calculated correctly, ensuring this result. Accordingly, the spectrum of the resulting NLFM signal acquired the expected shape. Due to the higher FMR of the first LFM fragment, its power spectral density is lower, as seen in the spectrograms of Figs. 1c and 2c. However, Fig. 2c shows no dips, peak distortion, or ripples on the slopes and 'wings' of the spectrum.

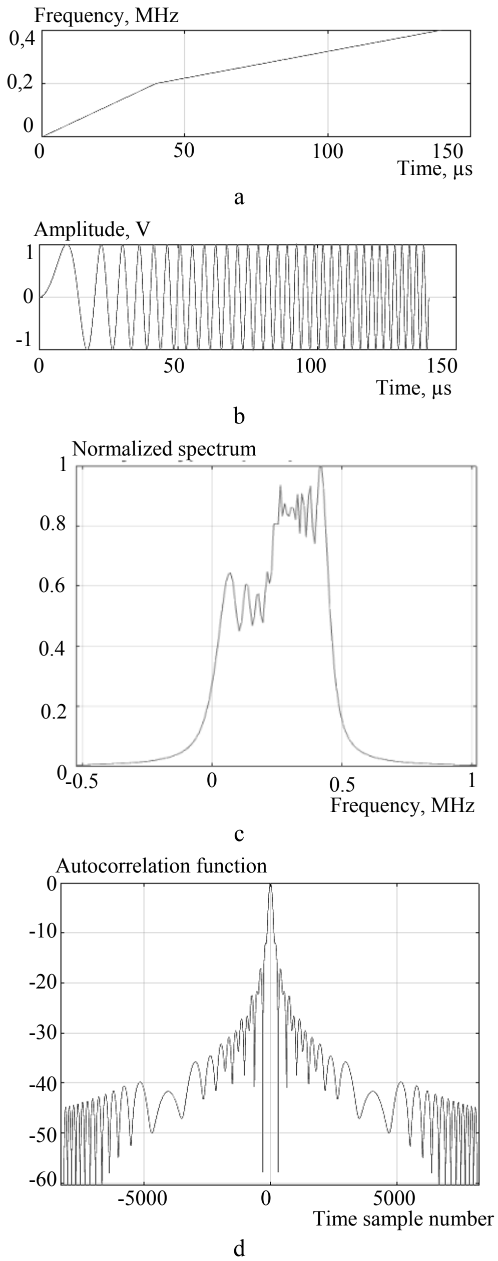

Figure 2 – Oscillogram (a), instantaneous frequency change graph (b), spectrum (c), ACF (d) of the NLFM signal according to the model (9)

The absence of a phase jump at the moment of transition to the second fragment of the signal ensured an improvement in the shape and a decrease in the MPSLL of the ACF, which is –17.14 dB. However, the ML width at the 0.707 level from the maximum increased to 2.43 µs. Compared to the LFM signal, the resulting NLFM signal of type (9) provided a 27% reduction in the MPSLL of the ACF, the ML width at the 0.707 level of the maximum increased by 10%, and the rate of decline in the SL of the ACF increased by 10%.

5 RESULTS

As a result of mathematical modeling, it was established that for the given identical frequency and time parameters of the two-fragment NLFM and classical LFM signals, the use of the studied MM for the NLFM signal (4) compared to the LFM signal resulted in a decrease in the MPSLL by 1.09 dB, a slight increase in the decay rate of the PSLL by 0.45 dB/decade, and a slight expansion of the ML, which corresponds to an increase in the resolving power for a range of 24 m. After transitioning to a continuous time scale, in accordance with the proposed method, a phase jump at the junction of fragments was identified, which had not been taken into account previously. The further introduction of a compensatory component for the phase jump improved the well-known MM (9) and reduced the MPSLL compared to the LFM signal by 3.64 dB. In relation to MM (4), the reduction was more than three times greater. The corresponding PSLL decay rate increased by 1.9 dB/decade, i.e., it increased fourfold compared to MM (4), while the resolving power for a range, compared to the LFM signal, increased by 33 m, which is approximately 1.4 times higher than MM (4).

6 DISCUSSION

The study of MM (3), (4) conducted using the proposed method enabled a comparison with MM (1), (2) and analogues [2, 43, 44].

The analysis of MM (3), (4), carried out using the current time scale, allowed for the formalization of the analytical expressions of the compensatory components:

– the frequency jump at the junction of fragments in the second expression of (3);

– the linear increment of the instantaneous phase caused by the frequency jump at the junction of fragments in the second expression of (4).

The applied approach provides a clear illustration of the frequency jump compensation mechanism in MM (3), (4).

The applied method enabled the identification of a drawback in the examined MM, namely, the absence of a compensatory component for the instantaneous phase jump at the moment of transition from the first LFM fragment to the second (second expression in (4)).

This can be explained by the fact that when the time scale is shifted to zero, the initial phase of the second LFM fragment becomes zero, while the final phase of the first LFM fragment can take any value within the interval from 0 to 2π, which explains the mechanism of the phase

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jump occurrence. This is confirmed by the fact that direct integration of the second expression in model (3) does not allow for obtaining the complete set of compensatory components needed to calculate the instantaneous phase values of the second LFM fragment of the signal.

In addition to directly introducing a compensatory component for the phase jump, it is possible to avoid such a jump by using the method of providing an integer value for the number of periods of the LFM oscillation in the first fragment of the NLFM signal, as proposed in [42].

CONCLUSIONS

The study is based on the use of the proposed method for transitioning to a continuous current time scale, followed by a comparative analysis of the MM of NLFM signals, which consist of two LFM fragments. The study was made possible through the use of results obtained in [40, 41].

This approach is advisable for comparing different MMs of NLFM signals with equivalent time-frequency parameters. The feasibility of the method is confirmed through mathematical calculation and verified by simulation modeling.

The scientific novelty. A method for studying the time-shifted MMs of NLFM signals is proposed, which involves transitioning to the current time scale followed by a detailed analysis of its structure and properties.

For the first time, the mechanism of automatic compensation for the instantaneous frequency jump at the junction of fragments is explicitly highlighted. It is shown that performing the time scale shift operation for the second LFM fragment in (3) and (4) is equivalent to the emergence of compensatory components for the frequency jump and the linear increment of the instantaneous phase. This ensures the absence of a gap in the spectrum of the resulting signal and the specified frequency deviation.

The use of the proposed method allowed for determining the advantages and disadvantages of the time-shifted MM (3), (4) relative to the MM (1), (2). It is shown that the studied time-shifted MM (4) does not function correctly in the process of determining the instantaneous phase increments of the second LFM fragment, and the reason for this is the failure to account for the instantaneous phase jump at the junction of the fragments.

The practical significance of the obtained results lies in providing the scientific community with a new mechanism for studying time-shifted MM. This mechanism involves applying a mathematical technique to transform such a model into a current-time MM, which facilitates the simplification and detailed analysis of the model's components.

Prospects for further research involve applying the proposed approach to the study of time-shifted MM for NLFM signals with a greater number of LFM fragments and with FM laws different from linear in one or both fragments.

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МЕТОД ДОСЛІДЖЕННЯ МАТЕМАТИЧНОЇ МОДЕЛІ ЗСУНУТОГО ЧАСУ ДВОФРАГМЕНТНОГО СИГНАЛУ З НЕЛІНІЙНОЮ ЧАСТОТНОЮ МОДУЛЯЦІЄЮ

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АНОТАЦІЯ

Актуальність. Подальший розвиток теорії та техніки формування і оброки складних радіолокаційних сигналів передбачає дослідження існуючих та створення нових математичних моделей зондувальних радіосигналів. Один із напрямків таких досліджень спрямовується на зниження максимального рівня бічних пелюсток автокореляційних функцій сигналів з внутрішньо імпульсною модуляцією частоти або фази. При цьому миттєва частота може змінюватися за лінійним або ж нелінійним законом. Нелінійні закони частотної модуляції можуть забезпечити зниження максимального рівня бічних пелюсток без амплітудної модуляції вихідного сигналу радіопередавального пристрою, а значить без втрат потужності зондувальних сигналів. Широке запровадження нелінійно-частотно модульованих сигналів в радіолокаційну техніку стримується недостатньою проробкою їх математичних моделей. Тому розроблення методів для дослідження існуючих математичних моделей сигналів з нелінійною частотною модуляцією є актуальною науковою задачею.

Метою роботи є розробка методу для виконання досліджень стосовно визначення переваг та недоліків математичної моделі нелінійно-частотно модульованого сигналу у складі двох фрагментів з лінійною модуляцією частоти.

Метод. У цьому дослідженні запропоновано метод аналізу математичних моделей сигналів, який базується на переході від шкали зсунутого часу до шкали поточного часу. Методологія включає такі основні етапи: формалізований опис математичних моделей, перехід до іншої шкали часу, виділення складових та визначення їх фізичної сутності, проведення порівняльного аналізу. Перевірку працездатності методу виконано шляхом імітаційного моделювання.

Результати. З використанням запропонованого методу визначено, що математична операція зсуву шкали часу є еквівалентною появі в математичній моделі додаткових складових, що здійснюють одночасну автоматичну компенсацію стрибка частоти на стику фрагментів, а також додаткового лінійного приросту фази у другому лінійно-частотно модульованому фрагменті. Застосований підхід забезпечує наочну ілюстрацію механізму компенсації стрибка частоти у математичної моделі, що досліджувалася. Використаний метод дозволив виявити недолік розглянутої математичної моделі, який полягає у відсутності компенсаційної складової стрибка миттєвої фази у момент переходу від першого ЛЧМ фрагменту до другого.

Висновки. Розроблено метод для визначення сутності та відповідного впливу складових математичної моделі у зсунутому часі нелінійно-частотно модульованого сигналу, до складу якого входять два фрагменти з лінійною модуляцією частоти. Досліджувана модель є не зовсім коректною, оскільки не має у собі складової для компенсації стрибка фази у момент переходу від першого фрагменту сигналу до другого. Введення такої складової забезпечує подальше зниження максимального рівня бічних пелюсток автокореляційної функції сигналу.

КЛЮЧОВІ СЛОВА: сигнали з нелінійною частотною модуляцією; математична модель; стрибок миттєвої фази; автокореляційна функція; максимальний рівень бічних пелюсток.

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