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ABOUT SPECIAL CASES OF LAGRANGIAN INTERSTRIPATION OF APPROXIMATIONS OF FUNCTIONS OF TWO VARIABLES

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ABSTRACT

Context. The problem of approximating the values of continuous functions of two variables based on known information about them on stripes, the boundaries of which are parallel to the coordinate axes, is considered. The object of the study is the process of approximating the values of functions based on incomplete information about them, which is given on the system of stripes.

Objective. The goal of the work is the review of information operators of Lagrangian interstripation and features of the construction of information approximation operators for some cases of the mutual arrangement of stripes in some region, which allow to significantly simplify the calculation of approximate values of the function in unknown subregions of the region.

Method. Methods for approximating the values of continuous functions of two variables with incomplete information about them on some limited area are proposed. Information about the function is known only on a system of stripes limited by straight lines parallel to the coordinate axes. A method for approximating the values of continuous functions of two variables, information about which is known on two stripes, as a result of union of which only some rectangular subregion remains unknown in the region, is proposed. A method for approximating the values of continuous functions of two variables, information about which is known on three stripes, as a result of union of which only some rectangular subregion remains unknown in the region, is proposed. A method for approximating the values of continuous functions of two variables, information about which is known on four stripes, as a result of union of which only some rectangular subregion remains unknown in the region, is proposed. A method for approximating the values of continuous functions of two variables, the information about which is known on two stripes, as a result of union of which four rectangular subregions remain unknown in the region, is proposed. For all the considered cases, approximation operators are given that allow calculating the approximate form of the function in the unknown subregions in the analytical form.

Results. The information operators of Lagrangian interstripation are implemented programmatically and investigated in problems of approximating the values of functions of two variables from known information about them on the systems of stripes.

Conclusions. The experiments confirmed the accuracy of approximation of the values of continuous functions of two variables of the proposed information interstripation operators for different systems of stripes. Approximation operators are given for special cases of the location of stripes in the region, the difference of which from the information interstripation operators of the general form lies in the significant simplification of the approximation operators without losing the accuracy of the approximation with a smaller number of arithmetic operations, which can be a decisive factor in some cases. Prospects for further research lie in the application of the proposed information operators in the problems of digital image processing, seismic mineral exploration data and remote sensing data etc.

KEYWORDS: numerical methods, mathematical modeling, information operators, interlination, interstripation.

NOMENCLATURE

$f(x, y)$ is a function of two variables;

D is a rectangular region;

$S_{x,k}$ is a known data stripe with restrictions on the variable x, k – serial number of the strip;

$S_{y,p}$ is a known data stripe with restrictions on the variable y, p – serial number of the strip;

$\bar{S}_{x,k,k+1}$ is an unknown data stripe with restrictions on the variable x, which is located between k and $k + 1$ strips;

$\bar{S}_{y,p,p+1}$ is an unknown data stripe with restrictions on the variable y, which is located between p and $p + 1$ strips;

$\{x_k\}_{k=1}^{n}$ is a sequence of indexed variables x from 1 to n;

$f(x, y)|_S$ is a trace of the function $f(x, y)$ on the stripe S;

$f(x)|_\Gamma$ is a trace of the function $f(x, y)$ on the line Γ, which is a function of one variable x;

$f(y)|_\Gamma$ is a trace of the function $f(x, y)$ on the line Γ, which is a function of one variable y;

$Of(x, y)$ is an interstripation operator;

$Lf(x, y)$ is an approximation operator.

INTRODUCTION

Methods of approximation of functions with incomplete information are an important part of modern applied mathematics and are widely used in various fields of science and technology. Their main goal is to restore or simulate a function for which only a limited number of values are known, or these values contain noise or errors. Such situations arise in real conditions, when a complete description of an object or process is not available due to technical, financial or physical constraints.

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One of the key areas of application of these methods is signal and image processing. For example, in the compression or restoration of digital images, approximation methods are used to obtain the most accurate reconstruction from partial data. In machine learning, function approximation is the basis for building models that predict or classify data based on incomplete or limited samples.

In natural and technical sciences, approximation methods help solve inverse problems – for example, in medical tomography or geophysical research, where it is necessary to reconstruct the internal structure of an object from indirect measurements. In economics, these methods are used to forecast market indicators, estimate demand or production functions based on incomplete statistical data.

Due to their versatility and ability to work with imprecise or incomplete data, function approximation methods have become a powerful tool in conditions of uncertainty, opening up new possibilities for analysis, modeling, and decision-making in a wide variety of applied problems.

The object of study is the process of approximating functions based on incomplete information about them, given on a system of stripes.

The subject of study is mathematical methods and methods of information operators of O. M. Lytvyn, which allow building approximate models of functions based on partial data.

The purpose of the work is to analyze the main methods of approximating functions in cases of incomplete information, to investigate their effectiveness and areas of application, and to justify the choice of approaches for practical use in conditions of limited data.

1 PROBLEM STATEMENT

Let $f(x, y)$ be a function of two variables defined on some rectangular region $D=[a,b]\times[c,d]$, called the interstripation region. Suppose that the values of this function are known exactly on some system of stripes:

$$S_k = \{(x, y) : \alpha_k \le \omega_k(x, y) \le \beta_k\},\ k \in \mathbb{N}.$$

Let's assume that the boundaries of the stripes are straight lines parallel to the coordinate axes. In this case, the stripe system can be written in a simplified form if the stripe boundaries are parallel to each other and parallel to the axis O*y*:

$$S_{x,k} = \{(x, y) : a_k \le x \le b_k\},\ k = \overline{1,n}$$

or, if the boundaries of the stripes are parallel to each other and parallel to the O*x* axis:

$$S_{y,p} = \{(x, y) : c_p \le y \le d_p\},\ p = \overline{1,m}.$$

Let's consider the following sequences of values, which contain the coordinates of the boundaries of the interstripation region and the boundaries of the stripes of the form:

$$\{x_k\}_{k=1}^{2n} : a_1 < b_1 \le a_2 < b_2 \le \ldots \le a_{n-1} < b_{n-1} \le a_n < b_n,$$

$$\{y_p\}_{p=1}^{2m} : c_1 < d_1 \le c_2 < d_2 \le \ldots \le c_{m-1} < d_{m-1} \le c_m < d_m.$$

Therefore, at points in the region D that do not belong to any of the above stripes $S_{x,k}$ or $S_{y,p}$ the information about the function is unknown. Let us denote such regions by:

$$\bar{S}_{x,k,k+1} = \{(x, y) : b_k < x < a_{k+1}\},\ k = \overline{1,n-1},$$

$$\bar{S}_{y,p,p+1} = \{(x, y) : d_p < y < c_{p+1}\},\ p = \overline{1,m-1}.$$

The problem is to approximate the values of the function in those parts of the region D where information about it is unknown.

2 REVIEW OF THE LITERATURE

The problem of function approximation with incomplete, sparse, or noisy data is relevant in many fields of science and technology, therefore its research is interdisciplinary in nature. These may be, for example, values of a function at individual points, approximate observations, or partial information about derivatives. In such cases, various approximation methods are used, which can be classified according to analytical, statistical, or numerical approaches [1–2].

The first systematic approaches to function approximation were developed within the framework of classical analysis and numerical mathematics. A significant contribution to the development of this field was made by such scientists as Weierstrass, Chebyshev, Hilbert, Gauss and others. The Weierstrass and Stone-Weierstrass theorems became the basis for the formal justification of the possibility of approximating continuous functions by polynomials. Chebyshev's work on optimal approximations was also of great importance, especially for the construction of uniform approximations [3–4].

One of the classical methods is interpolation, in which a function is constructed that passes exactly through given points. In the simplest case, polynomial interpolation is used: the function is approximated by a polynomial that passes through all given values. Among the popular forms are Lagrange's interpolation formula and Newton's formula. However, as the number of points increases, these methods become unstable: the Runge effect appears, when the polynomial fluctuates strongly between nodes [1], [5].

To avoid these problems, splines are used – piecewise polynomial functions that combine local approximations with smoothness conditions. The most common is the cubic spline, which ensures the continuity of the function and its derivatives up to the second order. Splines work well for large sets of points, allowing you to preserve both accuracy and smoothness [5–6].

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In the case when the data are noisy, it is more expedient to use approximation methods, in particular the least squares method. In this case, it is not required to pass through all points exactly, but only to minimize the mean square error. This provides a smooth approximation that is less sensitive to random errors in the data. Regression methods – both linear and nonlinear – are also used to build approximating models that generalize the behavior of the function [7–8].

In many applied problems, function recovery is accompanied by instability problems, when a small change in the input data can cause large deviations in the result. For such cases, regularization is used – the introduction of additional restrictions or penalty terms into the functional being minimized. The most famous method is Tikhonov regularization, which provides a compromise between accuracy and stability of the solution [9–11].

When not only a sample of function values but also some a priori knowledge about its properties is available, statistical and probabilistic methods, in particular Gaussian process models, are effective. In this approach, the function is considered as a random process for which not only the expected value but also an estimate of the error can be calculated. This is especially important in conditions of uncertainty or data limitations [12].

In recent years, machine learning methods have been actively developed, demonstrating high efficiency in the approximation of complex functions. In particular, artificial neural networks are able to approximate arbitrary continuous functions by learning from a large number of examples. They are extremely flexible, but require significant computational resources and careful tuning [13].

Among modern research, it is also worth noting works devoted to compressed sensing, which are based on the assertion that a function can be accurately reconstructed even from a much smaller number of measurements than required by traditional methods, provided that it has a certain "sparseness" or structure. This direction has been actively developing since the early 2000s [10].

Also, in cases where only general properties of a function are known (for example, monotonicity, convexity, or boundedness), optimization methods are used to find a function that best matches the available information and satisfies the given constraints. In such problems, the approximation of a function is formulated as a problem of minimizing the functional considering structural requirements [14].

A review of the literature shows that the choice of approximation method depends on the nature of the available information about the function: the amount of data, the presence of noise, requirements for accuracy, smoothness or stability of the solution. Modern approaches increasingly combine classical mathematical and modern machine learning tools to achieve maximum accuracy under conditions of limited resources and data.

A relatively new direction in information operators are information operators proposed by O. M. Lytvyn [15–16]. The proposed information operators allow using information about the object of study not only on a system of points, but also on lines, stripes and flats.

Such information operators are used for a wide class of applied problems in various fields of science and technology.

3 MATERIALS AND METHODS

Let's introduce the concept of the trace of a function on a strip and on a line.

The trace of a function $f(x,y)$ on a strip S is called a function of two variables $f(x,y)\big|_S$, which at each point of this stripe takes the same values as the function $f(x,y)$, and outside the stripe is unknown.

The trace of a function $f(x,y)$ on a line Γ is a function of one variable $f(x)\big|_\Gamma$ or $f(y)\big|_\Gamma$, which at each point of this line takes the same values as the function $f(x,y)$, and outside the line is unknown.

The interstripation of a continuous function of two variables $f(x,y)$ on a system of stripes S_k, $k\in\mathbb{N}$ is called its recovery using its traces on the strips $f(x,y)\big|_{S_k}$, $k\in\mathbb{N}$.

The general form of the interstripation operator, which on each stripe returns the value of the function in the form of a trace of the function on the stripe, and between stripes approximates the function according to the known information about it on the stripe system, has the form:

$$\mathrm{O}f(x,y)=\begin{cases} f(x,y)\big|_{S_{x,k}}, & (x,y)\in S_{x,k},\\ f(x,y)\big|_{S_{y,p}}, & (x,y)\in S_{y,p},\\ Lf(x,y), & (x,y)\in D\setminus(S_{x,k}\cup S_{y,p}),\end{cases}$$
$$k=\overline{1,n},\ \ p=\overline{1,m}.$$

The most general case is when the information about the function is known on a system of stripes whose boundaries are mutually perpendicular to each other. That is, the traces of the function are known on the stripes $S_{x,k}$ whose boundaries are parallel to the axis Oy and on the strips $S_{y,p}$ whose boundaries are parallel to the axis Ox. The union of the information from all stripes forms a system of intersecting stripes with rectangular regions where information about the function is unknown:

$$\bar{S}_{k,k+1,p,p+1}=\left\{(x,y):(x,y)\in\left(b_k,a_{k+1}\right)\times\left(d_p,c_{p+1}\right)\right\},$$
$$k=\overline{1,n-1},\ \ p=\overline{1,m-1}.$$

To approximate the values of the function using information from such a system of stripes, it is proposed to use an interstripation operator in the Lagrange form:

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$$Lf(x,y)=\sum_{k=1}^{2n} f(x_k,y)\prod_{i=1,i\neq k}^{2n}\frac{x-x_i}{x_k-x_i}+$$
$$+\sum_{p=1}^{2m} f(x,y_p)\prod_{j=1,j\neq p}^{2m}\frac{y-y_j}{y_p-y_j}-$$
$$-\sum_{k=1}^{2n}\sum_{p=1}^{2m} f(x_k,y_p)\prod_{i=1,i\neq k}^{2n}\frac{x-x_i}{x_k-x_i}\prod_{j=1,j\neq p}^{2m}\frac{y-y_j}{y_p-y_j}.$$

Properties of the interstripation operator for approximating function values on a system of stripes whose boundaries are mutually perpendicular and parallel to the coordinate axes:

$$Of(x,y)\big|_{S_{x,k}}=f(x,y)\big|_{S_{x,k}},\ k=\overline{1,n},$$
$$Of(x,y)\big|_{S_{y,p}}=f(x,y)\big|_{S_{y,p}},\ p=\overline{1,m}.$$

In case if $b_k=a_{k+1}$, $k=\overline{1,n-1}$ and $d_p=c_{p+1}$, $p=\overline{1,m-1}$, then the interstripation operator will exactly approximate the function using the given traces on each stripe $S_{x,k}$, $S_{y,p}$.

This approximation can be used to arbitrarily divide a region into stripes whose boundaries are parallel to the coordinate axes. However, for some special cases, the form of the information interstripation operator can be significantly simplified.

Let's assume that the information about the function $f(x,y)$ is given by its traces on two stripes $S_{x,1}$ and $S_{y,1}$. Moreover, it is known that for each of the stripes one of the boundaries whose belong to the interstripation region *D*, and the other on the boundary of the interstripation region ∂D.

As a result of the union of the information about the function from both stripes, we obtain some rectangular region in *D*, where the information about the function is unknown. The operator $Lf(x,y)$ in the interstripation operator in this case can be simplified to the following form:

$$Lf(x,y)=f(y)\big|_{\Gamma_1}+f(x)\big|_{\Gamma_2}-f(\tilde{x},\tilde{y}),$$
$$\Gamma_1: x=\tilde{x},\ \Gamma_2: y=\tilde{y},$$
$$\tilde{x}=\begin{cases}a_1,\ a_1\notin\partial D,\\ b_1,\ b_1\notin\partial D,\end{cases}$$
$$\tilde{y}=\begin{cases}c_1,\ c_1\notin\partial D,\\ d_1,\ d_1\notin\partial D.\end{cases}$$

Let's assume that the information about the function $f(x,y)$ is given by its traces on three stripes $S_{x,1}$, $S_{x,2}$ and $S_{y,1}$. Moreover, as in the previous case, it is known that for each of the strips one of the boundaries whose belong to the interstripation region *D*, and the other on the boundary of the interstripation region ∂D.

As a result of the union of the information about the function from the three stripes, we obtain some rectangular region in *D*, where the information about the function is unknown. The operator $Lf(x,y)$ in the interstripation operator in this case can be simplified to the following form:

$$Lf(x,y)=\frac{x-\tilde{x}_2}{\tilde{x}_1-\tilde{x}_2}\left(f(y)\big|_{\Gamma_1}-f(\tilde{x}_1,\tilde{y})\right)+$$
$$+\frac{x-\tilde{x}_1}{\tilde{x}_2-\tilde{x}_1}\left(f(y)\big|_{\Gamma_2}-f(\tilde{x}_2,\tilde{y})\right)+f(x)\big|_{\Gamma_3},$$
$$\Gamma_1: x=\tilde{x}_1,\ \Gamma_2: x=\tilde{x}_2,\ \Gamma_3: y=\tilde{y},$$
$$\tilde{x}_1=\begin{cases}a_1,\ a_1\notin\partial D,\\ b_1,\ b_1\notin\partial D,\end{cases}$$
$$\tilde{x}_2=\begin{cases}a_2,\ a_2\notin\partial D,\\ b_2,\ b_2\notin\partial D,\end{cases}$$
$$\tilde{y}=\begin{cases}c_1,\ c_1\notin\partial D,\\ d_1,\ d_1\notin\partial D.\end{cases}$$

Note that a similar operator $Lf(x,y)$ can be obtained for the stripes $S_{x,1}$, $S_{y,1}$ and $S_{y,2}$, for which similar conditions on the location of the stripe boundaries in the interstripation region *D* will be satisfied.

Let's assume that the information about the function $f(x,y)$ is given by its traces on the four stripes $S_{x,1}$, $S_{x,2}$, $S_{y,1}$ and $S_{y,2}$. Moreover, as in the previous case, it is known that for each of the stripes one of the boundaries whose belong to the interstripation region *D*, and the other on the boundary of the interstripation region ∂D.

As a result of the union of the information about the function from the four stripes, we obtain some rectangular region in *D*, where the information about the function is unknown. The operator $Lf(x,y)$ in the interstripation operator in this case can be simplified to the following form:

$$Lf(x,y)=\frac{x-\tilde{x}_2}{\tilde{x}_1-\tilde{x}_2}f(y)\big|_{\Gamma_1}+\frac{x-\tilde{x}_1}{\tilde{x}_2-\tilde{x}_1}f(y)\big|_{\Gamma_2}+$$
$$+\frac{y-\tilde{y}_2}{\tilde{y}_1-\tilde{y}_2}f(x)\big|_{\Gamma_3}+\frac{y-\tilde{y}_1}{\tilde{y}_2-\tilde{y}_1}f(x)\big|_{\Gamma_4}-$$
$$-\frac{x-\tilde{x}_2}{\tilde{x}_1-\tilde{x}_2}\left(\frac{y-\tilde{y}_2}{\tilde{y}_1-\tilde{y}_2}f(\tilde{x}_1,\tilde{y}_1)+\frac{y-\tilde{y}_1}{\tilde{y}_2-\tilde{y}_1}f(\tilde{x}_1,\tilde{y}_2)\right)-$$
$$-\frac{x-\tilde{x}_1}{\tilde{x}_2-\tilde{x}_1}\left(\frac{y-\tilde{y}_2}{\tilde{y}_1-\tilde{y}_2}f(\tilde{x}_2,\tilde{y}_1)+\frac{y-\tilde{y}_1}{\tilde{y}_2-\tilde{y}_1}f(\tilde{x}_2,\tilde{y}_2)\right),$$
$$\Gamma_1: x=\tilde{x}_1,\ \Gamma_2: x=\tilde{x}_2,\ \Gamma_3: y=\tilde{y}_1,\ \Gamma_4: y=\tilde{y}_2,$$
$$\tilde{x}_1=\begin{cases}a_1,\ a_1\notin\partial D,\\ b_1,\ b_1\notin\partial D,\end{cases}$$

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$$\tilde{x}_2 = \begin{cases} a_2, \ a_2 \notin \partial D, \\ b_2, \ b_2 \notin \partial D, \end{cases}$$

$$\tilde{y}_1 = \begin{cases} c_1, \ c_1 \notin \partial D, \\ d_1, \ d_1 \notin \partial D, \end{cases}$$

$$\tilde{y}_2 = \begin{cases} c_2, \ c_2 \notin \partial D, \\ d_2, \ d_2 \notin \partial D. \end{cases}$$

Let's assume that the information about the function $f(x,y)$ is given by its traces on two stripes $S_{x,1}$ and $S_{y,1}$. Moreover, it is known that for each of the stripes both boundaries belong to the interstripation region *D*.

As a result of the union of the information about the function from both stripes, we obtain in the general case four rectangular regions in *D*, where the information about the function is unknown. The operator $Lf(x,y)$ in the interstripation operator in this case can be simplified to the following form:

$$Lf(x,y) = \frac{x-b_1}{a_1-b_1} f(y)\big|_{\Gamma_1} + \frac{x-a_1}{b_1-a_1} f(y)\big|_{\Gamma_2} +$$
$$+\frac{y-d_1}{c_1-d_1} f(x)\big|_{\Gamma_3} + \frac{y-c_1}{d_1-c_1} f(x)\big|_{\Gamma_4} -$$
$$-\frac{x-b_1}{a_1-b_1}\left(\frac{y-d_1}{c_1-d_1} f(a_1,c_1) + \frac{y-c_1}{d_1-c_1} f(a_1,d_1)\right) -$$
$$-\frac{x-a_1}{b_1-a_1}\left(\frac{y-d_1}{c_1-d_1} f(b_1,c_1) + \frac{y-c_1}{d_1-c_1} f(b_1,d_1)\right),$$
$$\Gamma_1 : x = a_1, \ \Gamma_2 : x = b_1, \ \Gamma_3 : y = c_1, \ \Gamma_4 : y = d_1 .$$

For all the above interstripation operators, the same properties will be valid as for the general interstripation operator.

4 EXPERIMENTS

For computational experiments, a computer program was developed in the Python programming language using additional libraries (for example, Matplotlib – for visualization, plotting function graphs, SymPy – for finding function approximations in analytical form).

The interstripation region $D = [-3,3] \times [-3,3]$ was chosen for computational experiments.

The following function was chosen as the test function:

$$f(x,y) = Ae^{-\frac{(x-x_0)^2}{2\sigma_x^2}} + Be^{-\frac{(y-y_0)^2}{2\sigma_y^2}} .$$

Test function parameters for computational experiments:

$$A = 0, \ B = 0, \ x_0 = 0, \ y_0 = 0, \ \sigma_x = 1, \ \sigma_y = 1 .$$

Analytical form of the test function after parameter substitution:

$$f(x,y) = e^{-0.5x^2} + e^{-0.5y^2} .$$

For each computational experiment, a system of stripes was given. The function itself $f(x,y)$ was chosen in the traces of the function on each of the stripes in the systems (i.e., the function on the stripes was considered to be known exactly, without errors).

Four computational experiments were conducted for each of the above special cases of the location of stripes on the interstripation region.

For the first computational experiment, the following system of stripes was specified:

$$S_{x,1} = \{(x,y): -3 \le x \le 0\},$$
$$S_{y,1} = \{(x,y): 0 \le y \le 3\}.$$

For the second computational experiment, the following system of stripes was specified:

$$S_{x,1} = \{(x,y): -3 \le x \le -1\},$$
$$S_{x,2} = \{(x,y): 1 \le x \le 3\},$$
$$S_{y,1} = \{(x,y): 1 \le y \le 3\}.$$

For the third computational experiment, the following system of stripes was specified:

$$S_{x,1} = \{(x,y): -3 \le x \le -1\},$$
$$S_{x,2} = \{(x,y): 1 \le x \le 3\},$$
$$S_{y,1} = \{(x,y): -3 \le y \le -1\},$$
$$S_{y,2} = \{(x,y): 1 \le y \le 3\}.$$

For the fourth computational experiment, the following system of stripes was specified:

$$S_{x,1} = \{(x,y): -1 \le x \le 1\},$$
$$S_{y,1} = \{(x,y): -1 \le y \le 1\}.$$

5 RESULTS

For each of the computational experiments conducted, an analytical form of the information approximation operator $Lf(x,y)$ was obtained, and graphs of approximations and absolute errors of approximations were also constructed.

Case one. Traces of the function on the boundaries of stripes which belong to the interstripation region ($\tilde{x} = 0$, $\tilde{y} = 0$):

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$$f(y)\big|_{\Gamma_1} = f(0,y) = 1 + e^{-0.5y^2},$$

$$f(x)\big|_{\Gamma_2} = f(x,0) = e^{-0.5x^2} + 1,$$

$$\Gamma_1 : x = 0,\ \Gamma_2 : y = 0.$$

The value at the intersection point of the boundaries of stripes which belong to the interstripation region:

$$f(\tilde{x},\tilde{y}) = f(0,0) = 2.$$

Fig. 1 graphically illustrates the union of the test data on the stripes.

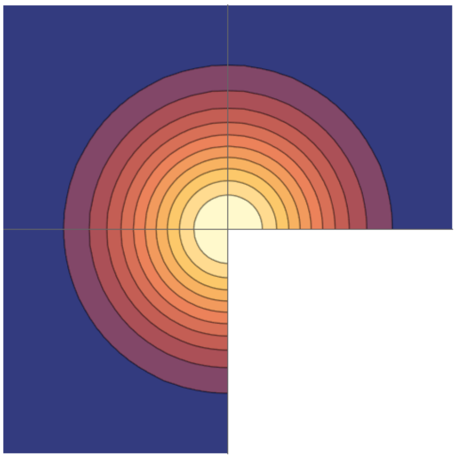

Figure 1 – The known information about the test function based on traces on two stripes for the first computational experiment

After substituting the test data into the corresponding formula for the operator $Lf(x,y)$, we obtain the analytical form of the approximation of the function in the subregion of the region *D*, where the function is unknown:

$$Lf(x,y) = e^{-0.5x^2} + e^{-0.5y^2}.$$

Case two. Traces of the function on the boundaries of stripes which belong to the interstripation region ($\tilde{x}_1 = -1$, $\tilde{x}_2 = 1$, $\tilde{y} = 1$):

$$f(y)\big|_{\Gamma_1} = f(-1,y) = e^{-0.5} + e^{-0.5y^2},$$

$$f(y)\big|_{\Gamma_2} = f(1,y) = e^{-0.5} + e^{-0.5y^2},$$

$$f(x)\big|_{\Gamma_3} = f(x,1) = e^{-0.5x^2} + e^{-0.5},$$

$$\Gamma_1 : x = -1,\ \Gamma_2 : x = 1,\ \Gamma_3 : y = 1.$$

The values at the intersection points of the boundaries of stripes which belong to the interstripation region:

$$f(\tilde{x}_1,\tilde{y}) = f(-1,1) = 2e^{-0.5},$$

$$f(\tilde{x}_2,\tilde{y}) = f(1,1) = 2e^{-0.5}.$$

Fig. 2 graphically illustrates the union of the test data on the stripes.

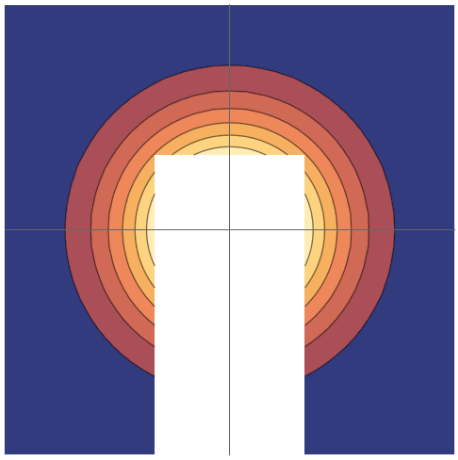

Figure 2 – The known information about the test function based on traces on three stripes for the second computational experiment

After substituting the test data into the corresponding formula for the operator $Lf(x,y)$, we obtain the analytical form of the approximation of the function in the subregion of the region *D*, where the function is unknown:

$$Lf(x,y) = -\frac{x-1}{2}\left(e^{-0.5} + e^{-0.5y^2} - 2e^{-0.5}\right) + \frac{x+1}{2}\left(e^{-0.5} + e^{-0.5y^2} - 2e^{-0.5}\right) + e^{-0.5x^2} + e^{-0.5}.$$

After combining the similar ones, we get the following expression for $Lf(x,y)$:

$$Lf(x,y) = e^{-0.5x^2} + e^{-0.5y^2}.$$

Case three. Traces of the function on the boundaries of stripes which belong to the interstripation region ($\tilde{x}_1 = -1$, $\tilde{x}_2 = 1$, $\tilde{y}_1 = -1$, $\tilde{y}_2 = 1$):

$$f(y)\big|_{\Gamma_1} = f(-1,y) = e^{-0.5} + e^{-0.5y^2},$$

$$f(y)\big|_{\Gamma_2} = f(1,y) = e^{-0.5} + e^{-0.5y^2},$$

$$f(x)\big|_{\Gamma_3} = f(x,-1) = e^{-0.5x^2} + e^{-0.5},$$

$$f(x)\big|_{\Gamma_4} = f(x,1) = e^{-0.5x^2} + e^{-0.5},$$

$$\Gamma_1 : x = -1,\ \Gamma_2 : x = 1,\ \Gamma_3 : y = -1,\ \Gamma_4 : y = 1.$$

The values at the intersection points of the boundaries of stripes which belong to the interstripation region:

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$$f(\tilde{x}_1,\tilde{y}_1)=f(-1,-1)=2e^{-0.5},$$
$$f(\tilde{x}_1,\tilde{y}_2)=f(-1,1)=2e^{-0.5},$$
$$f(\tilde{x}_2,\tilde{y}_1)=f(1,-1)=2e^{-0.5},$$
$$f(\tilde{x}_2,\tilde{y}_2)=f(1,1)=2e^{-0.5}.$$

Fig. 3 graphically illustrates the union of the test data on the stripes.

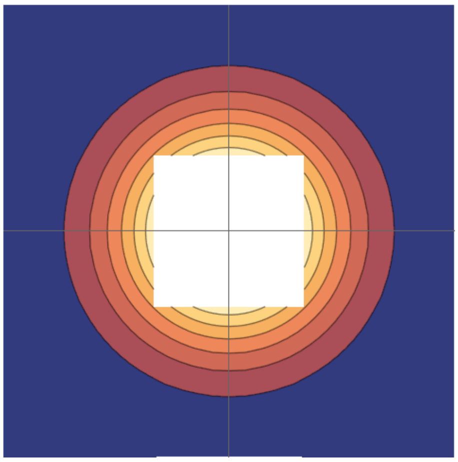

Figure 3 – The known information about the test function based on traces on four stripes for the third computational experiment

After substituting the test data into the corresponding formula for the operator $Lf(x,y)$, we obtain the analytical form of the approximation of the function in the subregion of the region *D*, where the function is unknown:

$$Lf(x,y)=-\frac{x-1}{2}\left(e^{-0.5}+e^{-0.5y^2}\right)+\frac{x+1}{2}\left(e^{-0.5}+e^{-0.5y^2}\right)-$$
$$-\frac{y-1}{2}\left(e^{-0.5x^2}+e^{-0.5}\right)+\frac{y+1}{2}\left(e^{-0.5x^2}+e^{-0.5}\right)+$$
$$+\frac{x-1}{2}\left(-(y-1)e^{-0.5}+(y+1)e^{-0.5}\right)-$$
$$-\frac{x+1}{2}\left(-(y-1)e^{-0.5}+(y+1)e^{-0.5}\right).$$

After combining the similar ones, we get the following expression for $Lf(x,y)$:

$$Lf(x,y)=e^{-0.5x^2}+e^{-0.5y^2}.$$

Case four. Traces of the function on the boundaries of stripes:

$$f(y)\big|_{\Gamma_1}=f(-1,y)=e^{-0.5}+e^{-0.5y^2},$$
$$f(y)\big|_{\Gamma_2}=f(1,y)=e^{-0.5}+e^{-0.5y^2},$$
$$f(x)\big|_{\Gamma_3}=f(x,-1)=e^{-0.5x^2}+e^{-0.5},$$
$$f(x)\big|_{\Gamma_4}=f(x,1)=e^{-0.5x^2}+e^{-0.5},$$
$$\Gamma_1: x=-1,\ \Gamma_2: x=1,\ \Gamma_3: y=-1,\ \Gamma_4: y=1.$$

The values at the intersection points of the boundaries of stripes:

$$f(-1,-1)=2e^{-0.5},$$
$$f(-1,1)=2e^{-0.5},$$
$$f(1,-1)=2e^{-0.5},$$
$$f(1,1)=2e^{-0.5}.$$

Fig. 4 graphically illustrates the union of the test data on the stripes.

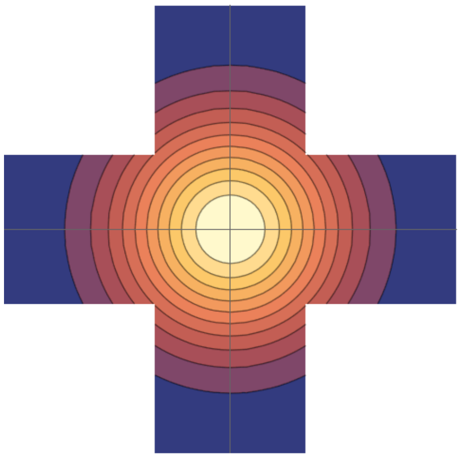

Figure 4 – The known information about the test function based on traces on two stripes for the fourth computational experiment

After substituting the test data into the corresponding formula for the operator $Lf(x,y)$, we obtain the analytical form of the approximation of the function in the subregion of the region *D*, where the function is unknown:

$$Lf(x,y)=-\frac{x-1}{2}\left(e^{-0.5}+e^{-0.5y^2}\right)+\frac{x+1}{2}\left(e^{-0.5}+e^{-0.5y^2}\right)-$$
$$-\frac{y-1}{2}\left(e^{-0.5x^2}+e^{-0.5}\right)+\frac{y+1}{2}\left(e^{-0.5x^2}+e^{-0.5}\right)+$$
$$+\frac{x-1}{2}\left(-(y-1)e^{-0.5}+(y+1)e^{-0.5}\right)-$$
$$-\frac{x+1}{2}\left(-(y-1)e^{-0.5}+(y+1)e^{-0.5}\right).$$

After combining the similar ones, we get the following expression for $Lf(x,y)$:

$$Lf(x,y)=e^{-0.5x^2}+e^{-0.5y^2}.$$

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6 DISCUSSION

As can be seen from the results of the computational experiments, for all special cases of the location of the stripes in the interstripation region, the given operators accurately restore the value of the function from the given traces of the function on the system of stripes.

This result is achieved due to the fact that for the experiments the function traces on the stripes were chosen in such a way that they exactly coincide with the value of the function.

In practice, to describe the traces of a function on stripes in this way is a problematic and sometimes even impossible task. Therefore, the traces of a function that one has to deal with in practice already contain the quality of inaccuracy and error. Or they are given on some system of points and to apply the informational operators of interstripation it is necessary to first apply interpolation operators, which also approximate the values with some error.

However, interstripation information operators can potentially be applied in many fields of science and technology.

CONCLUSIONS

This work is devoted to methods for approximating the values of continuous functions of two variables, which are given on a system of stripes.

The scientific novelty of the obtained results lies in the fact that for the first time the approximation of continuous functions of two variables, given by their traces on systems of stripes of a special form, was considered. Their difference from information operators of interstripation of a general form lies in the significant simplification of the approximation operators without losing the accuracy of the approximation with a smaller number of arithmetic operations, which can be a decisive factor in some cases.

The practical significance of the obtained results lies in the fact that software has been developed that implements the approximation of continuous functions of two variables by information operators of interstripation in the Lagrange form. The results of the experiments prove the accuracy of the approximation of functions on a given system of strips, the boundaries of which are parallel to the coordinate axes.

Prospects for further research are the application of the proposed information operators in the tasks of digital image processing, seismic exploration data, remote sensing data etc.

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ПРО ОСОБЛИВІ ВИПАДКИ ЛАГРАНЖЕВОЇ ІНТЕРСТРІПАЦІЇ НАБЛИЖЕННЯ ФУНКЦІЙ ДВОХ ЗМІННИХ

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АНОТАЦІЯ

Актуальність. Розглянуто задачу наближення значень неперервних функцій двох змінних за відомою інформацією про неї на смугах, границі яких парарлельні осям координат. Об'єктом дослідження є процес наближення значень функцій на основі неповної інформації про них, що задана на системі смуг.

Мета роботи – огляд інформаційних операторів інтерстріпації Лагранжа та особливостей побудови інформаційних операторів наближення для деяких випадків взаємного розташування смуг у просторі, які дозволяють значно спростити обчислення наближених значень функції у невідомих підобластях області дослідження.

Метод. Запропоновано методи наближення значень неперервних функцій двох змінних за неповної інформацією про неї на деякій обмеженій області. Інформація про функцію є відомою лише на системі смуг, обмежених прямими, що парарлельні осям координат. Запропоновано метод наближення значень неперервних функцій двох змінних, інформація про яку є відомою на двох смугах, в результаті об'єднання яких у області залишається невідомою лише деяка прямокутна підобласть. Запропоновано метод наближення значень неперервних функцій двох змінних, інформація про яку є відомою на трьох смугах, в результаті об'єднання яких у області залишається невідомою лише деяка прямокутна підобласть. Запропоновано метод наближення значень неперервних функцій двох змінних, інформація про яку є відомою на чотирьох смугах, в результаті об'єднання яких у області залишається невідомою лише деяка прямокутна підобласть. Запропоновано метод наближення значень неперервних функцій двох змінних, інформація про яку є відомою на двох смугах, в результаті об'єднання яких у області залишається невідомими чотири прямокутні підобласті. Для всіх розглянутих випадків наведено оператори наближення, які дозволяють обчислити наближений вигляд функції у невідоміх підобластях у аналітичному вигляді.

Результати. Інформаційні оператори інтерстріпації Лагранжа реалізовані програмно і досліджені в задачах наближення значень функцій двох змінних за відомою інформацією про неї на системах смуг.

Висновки. Проведені експерименти підтвердили точність наближення значень неперервних функцій двох змінних запропонованих інформаційних операторів інтерстріпації для різних систем смуг. Наведено оператори наближення для особливих випадків розташування смуг у області дослідження, відмінність яких від інформаційних операторів інтерстрпації загального вигляду полягає у значному спрощенні операторів наближення не втрачаючи при цьому точності наближення при меншій кількості арифметичних операцій, що може бути вирішальним фактором в деяких випадках. Перспективи подальших досліджень полягають у застосуванні запропонованих інформаційних операторів у задачах цифрової обробки зображень, даних сейсмічної розвідки та даних дистанційного зондування планети.

КЛЮЧОВІ СЛОВА: чисельні методи, математичне моделювання, інформаційні оператори, інтерлінація, інтерстріпація.

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