

## ABOUT RATIONAL METHODS FOR FINDING OPTIMAL ROUTES IN FUZZY TRAVELING SALESMAN PROBLEMS

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### ABSTRACT

**Context.** This paper presents the results of a study on the use of triangular fuzzy numbers for determining time-optimal routes in the traveling salesman problem under fuzzy representations of travel duration in a transportation network. To formalize the uncertainty and imprecision of input data – associated with the subjectivity in estimating the time intervals required to travel between individual cities-triangular fuzzy numbers are employed. Various approaches to solving fuzzy traveling salesman problems are examined.

**Objective.** The goal of the work is to develop algorithms for solving the fuzzy traveling salesman problem based on the implementation of the Bellman-Zadeh parametric optimization methods, the use of a two-criteria approach with a given weight function and the refinement of the scheme for calculating the center of gravity of the membership function graph for a given curve density.

**Method.** The article considers methods for solving the fuzzy traveling salesman problem, which is formulated as the problem of finding a route to visit a given number of cities without repetitions with a minimum travel time. The parameters of the problem for formalizing the uncertainty and inaccuracy of input data associated with the influence of subjectivity in assessing the duration of time intervals required to travel between individual cities are presented as fuzzy triangular numbers. Different approaches to solving fuzzy traveling salesman problems are considered. The application of the Bellman-Zadeh method, methods taking into account refinements of defuzzified data, and methods based on a multicriteria approach is formalized. Computational experiments are carried out.

**Results.** Rational algorithms for solving the fuzzy traveling salesman problem based on the Bellman-Zadeh parametric optimization model, multicriteria approach and methods for refining the results of defuzzification of fuzzy data have been developed. In the conducted numerical experiments on solving the traveling salesman problem, fuzzy input data are based on the method for calculating the center of gravity (CoG), the center of gravity of homogeneous and non-homogeneous curves, which are determined by the membership function and the specified reliability values of subjective data. A comparison of the results obtained based on solving the crisp traveling salesman problem and the results based on defuzzified duration values for the fuzzy traveling salesman problem is carried out, according to the results of which the dependence of the solution on the defuzzification method is confirmed.

**Conclusions.** The article considers the method of formalizing the algorithm for solving the fuzzy traveling salesman problem with the minimum duration of movement along the route based on the Belman-Zadeh method, methods taking into account the refinements of defuzzified data and methods based on the multicriteria approach. Fuzzy triangular numbers are used to formalize the uncertainty of the input data when assessing the duration of movement between individual towns of the transport network. It was made a conclusion about the feasibility of using fuzzy numbers when solving fuzzy traveling salesman problems in real conditions of logistics transportation.

**KEYWORDS:** fuzzy traveling salesman problem, fuzzy numbers, subjective perception of duration, uncertainty, solution methods, multicriteria approach, defuzzification.

### ABBREVIATIONS

TSP is a traveling salesman problem;  
CoG is a center of gravity.

### NOMENCLATURE

$n$  is a number of cities;  
 $t_{ij}$  are the travel time between all pairs of vertices;  
 $T$  is a matrix of  $t_{ij}$ ;  
 $t_{ij}$  are the travel time between all pairs of vertices;  
 $X$  is a binary matrix of transitions between vertices  
 $x_{ij}$ ;

$x_{ij}$  are the elements of matrix  $X$ , which equal to 0 or 1;  
 $i$  is an index;  
 $j$  is an index;  
 $\tilde{A}$  is a fuzzy set;  
 $\mu_{\tilde{A}}(x)$  is a membership function;  
 $\text{supp } \tilde{A}$  is a support of fuzzy set  $\tilde{A}$ ;  
 $\Delta t_{ij}$  are width of intervals of fuzzy travel time;  
 $\lambda$  is a parameter of optimization Bellman-Zadeh model;  
 $X^{1*}$  is an optimal values for first TSP;

$X^{2*}$  is an optimal values for second TSP;  
 $L_1$  is a lower value of first TSP optimal solution;  
 $L_2$  is a lower value of second TSP optimal solution;  
 $U_1$  is an upper value of first TSP optimal solution;  
 $U_2$  is an upper value of second TSP optimal solution;  
 $\alpha_1$  is a weight coefficient;  
 $\alpha_2$  is a weight coefficient;  
 $\alpha(s)$  is a weight function;  
 $x_C$  is a coordinate  $x$  of CoG of planar curve;  
 $y_C$  is a coordinate  $y$  of CoG of planar curve;  
 $x_C^p$  is a coordinate  $x$  of CoG of inhomogeneous curve;  
 $y_C^p$  is a coordinate  $y$  of CoG of inhomogeneous curve;  
 $\rho(l)$  is a density function;  
 $L$  is a length of membership function grafik;  
 $Z$  is a required reliability level.

## INTRODUCTION

The challenges associated with organizing and operating various spheres of the economy and management possess distinct characteristics that result in difficulties when solving diverse optimization problems. Some of these issues can be addressed by managerial staff, while others require analytical and optimization methods for production and organizational operations, including planning, coordination, decision-making, and control over the movement and storage of goods, services, and information. Mathematical methods are widely used to solve such problems, enabling the search for effective solutions while taking into account the limitations of the applied problem and the specifics of the available data. Solving certain optimization problems requires the use of non-standard methods based on well-known optimization algorithms.

However, the formal use of such methods is often impossible due to the imprecision (uncertainty) of the parameters and limitations of the real-world process models under consideration. These characteristics require the development and implementation of appropriate methods for formalizing and updating data that take uncertainty into account. One of the most well-known and effective methods for representing imprecise data is based on the use of Zadeh fuzzy sets [1], the essence of which allows for the mathematical expression of the subjectivity and uncertainty of parameter values in many applied problems. In recent years, numerous studies and papers have emerged based on this methodology and its application to various optimization tasks, decision support methods, and beyond.

One of the key applied optimization problems that requires a solution under conditions of uncertainty is the logistics-based Traveling Salesman Problem (TSP) [2]. A typical problem of salesman is to determine the route of a

transport network consisting of  $n$  interconnected points (cities). The desired route, along which a traveling salesman must visit all cities in the network, must pass through each city only once and must be optimal in time or length [3].

From a mathematical perspective, the TSP is a combinatorial optimization problem, for which various mathematical programming techniques may be employed.

**The object of study** is a process of finding the optimal route for the fuzzy traveling salesman problem with minimum travel time in the transportation medium.

**The subject of study** is development of efficient algorithms for solving the fuzzy traveling salesman problem based on the Bellman-Zadeh parametric optimization model, a multi-criteria approach and methods for refining the results of defuzzification of fuzzy data.

**The purpose of the work** is to develop algorithms for solving the fuzzy traveling salesman problem based on the implementation of the Bellman-Zadeh parametric optimization method, the use of a two-criteria approach with a given weight function and the refinement of the scheme for calculating the center of gravity of the membership function graph for a given curve density.

## 1 PROBLEM STATEMENT

The task of finding an optimal cyclic tour in the TSP with given pairwise distances or travel durations  $t_{ij}$  between all cities in the network what represented by a matrix  $T = \{t_{ij}\}, i, j = \overline{1, n}$  reduced to determining solution  $X = \{x_{ij}\}, x_{ij} \in \{0, 1\}, i, j = \overline{1, n}$ , of the combinatorial optimization problem [4]

$$E = \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij} \rightarrow \min \quad (1)$$

subject to the constraints

$$\sum_{j=1}^n x_{ij} = 1, i = \overline{1, n}, \sum_{i=1}^n x_{ij} = 1, j = \overline{1, n}, \quad (2)$$

which represents the classical formulation of the problem, where, in addition to route length or travel duration between locations, other criteria may also be considered, such as transportation cost, travel efficiency taking into account cargo volume or weight, and so on. A characteristic feature of all such formulations is the presence of a single optimality criterion in the route selection process.

It should be noted that, in addition to known and objective parameters of the traveling salesman's movement (such as distance between individual locations in the network, the maximum speed of the vehicle, etc.), one may also consider sets of factors that have an uncertain nature of influence. For example, the travel time along a particular route may depend on the time of day,

weather conditions, or even the “congestion level” of a given section of the transport network. Moreover, determining the optimal path requires accounting for their combined influence on the selection of the best route. As an illustration, a specific section of the road may experience different levels of traffic load at different times of the day, implying that the time required to pass through this section varies depending on when it is traversed. On the other hand, weather conditions (fog, rain) are also influential factors affecting travel duration across the considered segment. These situations necessitate solving the problem by incorporating subjective assessments of various motion parameters, leading to the formulation and solution of fuzzy traveling salesman problems (FuzzyTSP) [4].

## 2 REVIEW OF LITERATURE

Numerous works are devoted to the application of fuzzy set theory and fuzzy numbers to solving various applied optimization problems [2–9]. Among the the most recent publications, devoted to solving the traveling salesman problem, one can highlight the work [2], which proposes a method for solving the FuzzyTSP using various membership functions. In [3], a decision-making concept in a fuzzy environment is presented. In [5], a solution to fuzzy linear programming problems with multiple objective functions is proposed. On the other hand, for the classical TSP as a combinatorial optimization problem, many methods based on greedy and heuristic approaches have been proposed, allowing one to find locally optimal solutions (see, for example, [6,7]). Therefore, the integration of these methodologies with the formalization of uncertainty in optimization problems based on fuzzy numbers is of both theoretical and practical interest [4].

This paper examines approaches to improving the objectivity of input parameters in the FuzzyTSP for determining the shortest travel duration under uncertainty. It is proposed to apply and compare the results obtained using the Bellman-Zadeh method [6, 10], methods incorporating refined defuzzified data [11], and approaches based on multi-criteria decision-making [12, 13].

## 3 MATERIALS AND METHODS

Traditional set theory can be viewed as a particular case of fuzzy set theory. In classical mathematics, a set is defined as a collection of elements (objects) that share some common property [4].

According to Zadeh’s theory, fuzzy sets are defined as subsets of the universal set  $X$  as follows:

Definition 1. [4] A fuzzy set  $\tilde{A}$  in a universal set  $X$  is a collection of ordered pairs  $\tilde{A} = \{(\mu_{\tilde{A}}(x), x)\}$ , where  $\mu_{\tilde{A}} : X \rightarrow [0,1]$  is a mapping of the set into the unit interval  $[0,1]$  and is called a membership function of fuzzy set  $\tilde{A}$ .

The value of the membership function  $\mu_{\tilde{A}}(x)$  for element  $x \in X$  indicates the degree to which  $x$  belongs to the fuzzy set  $\tilde{A}$  (see Fig. 1). The interpretation of the

membership degree  $\mu_{\tilde{A}}(x)$  is a subjective measure of how well the element  $x \in X$  corresponds to the concept defined by the fuzzy set  $\tilde{A}$ .

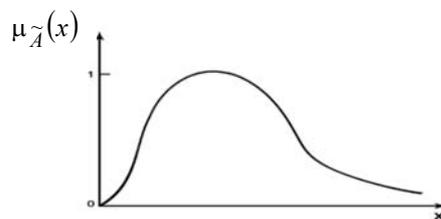


Figure 1 – Example of a membership function

It is noted that for any fuzzy sets  $\tilde{A}$  and  $\tilde{B}$   $\mu_{\tilde{A} \cup \tilde{B}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$ ,  $\mu_{\tilde{A} \cap \tilde{B}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$ .

Definition 2. [14] A fuzzy set  $\tilde{A}$  is called convex if next inequality satisfied

$$\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min(\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y))$$

for all  $x, y \in X, \lambda \in (0,1)$ .

A fuzzy set  $\tilde{A}$  is called normal if there exists  $x \in X$  such that  $\mu_{\tilde{A}}(x) = 1$ .

Crisp sets  $\tilde{A}_\alpha$  are called  $\alpha$ -level sets of fuzzy set  $\tilde{A}$ , where  $\alpha \in [0,1]$  is the height of fuzzy set  $\tilde{A}$  and  $\text{supp } \tilde{A} = \{x \in X : \mu_{\tilde{A}}(x) > 0\}$  is the support of fuzzy set  $\tilde{A}$ .

A fuzzy set  $A$  is considered unimodal if there exists only one element  $x \in X$  such that  $\mu_{\tilde{A}}(x) = h$ .

Let us consider the universal set  $X$  to be the set of real numbers, i.e.,  $X = \mathbb{R}$ .

Definition 3. [4] A fuzzy set  $A$  defined over the set of real numbers  $\mathbb{R}$  is called a fuzzy number if it satisfies the following properties:

- I. the set  $\tilde{A}$  is convex in the sense of definition 2;
- II. the set  $\tilde{A}$  is normal;
- III. the membership function  $\mu_{\tilde{A}}(x)$  is upper semi-continuous;
- IV. the support of the fuzzy number  $\text{supp } p(\tilde{A})$  is a subset of the universal set  $R$ .

Definition 3. [4] A triangular fuzzy number  $\tilde{A}$  is an ordered triplet of real numbers  $(a_1, a_2, a_3)$ ,  $a_1 \leq a_2 \leq a_3$ , with an associated membership function  $\mu_{\tilde{A}}(x)$  defined as:

$$\mu_{\tilde{A}}(x) = \frac{x - a_1}{a_2 - a_1}, x \in [a_1, a_2];$$

$$\mu_{\tilde{A}}(x) = \frac{a_3 - x}{a_3 - a_2}, x \in [a_2, a_3]; \quad (3)$$

$$\mu_{\tilde{A}}(x) = 0, x \notin [a_1, a_3].$$

Clearly, a triangular fuzzy number  $(a_1, a_2, a_3)$  is a special case of a unimodal fuzzy set with a height equal to one [15] (see Fig. 2).

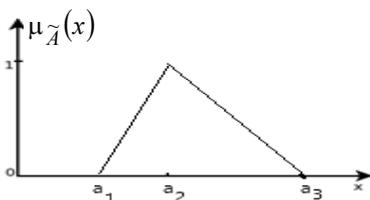


Figure 2 – Example of a triangular fuzzy number

It is well known [4], that triangular fuzzy number  $(a_1, a_2, a_2)$  is called a left triangular fuzzy number if its membership function is defined as

$$\begin{aligned} \mu_{\tilde{A}}(x) &= 0, x \leq a_1; \quad \mu_{\tilde{A}}(x) = \frac{x - a_1}{a_2 - a_1}, x \in (a_1, a_2); \\ \mu_{\tilde{A}}(x) &= 0, x \geq a_2, \end{aligned} \quad (4)$$

and a triangular fuzzy number  $(a_1, a_1, a_2)$  is called a right triangular fuzzy number with the membership function:

$$\begin{aligned} \mu_{\tilde{A}}(x) &= 0, x \leq a_1; \quad \mu_{\tilde{A}}(x) = \frac{a_2 - x}{a_2 - a_1}, x \in (a_1, a_2); \\ \mu_{\tilde{A}}(x) &= 0, x \geq a_3. \end{aligned} \quad (5)$$

**Definition 5.** [4] A trapezoidal fuzzy number  $\tilde{A}$  is an ordered quadruple of real numbers  $(a_1, a_2, a_3, a_4)$ ,  $a_1 \leq a_2 \leq a_3 \leq a_4$ , with a membership function  $\mu_{\tilde{A}}(x)$  defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1}, & \text{if } a_1 \leq x \leq a_2; \\ 1, & \text{if } a_2 \leq x \leq a_3; \\ \frac{a_4 - x}{a_4 - a_3}, & \text{if } a_3 \leq x \leq a_4. \end{cases} \quad (6)$$

**Definition 6.** A rectangular-trapezoidal fuzzy number  $\tilde{A}$  (see Fig. 3) is a special case of a trapezoidal fuzzy number  $(a_1, a_1, a_2, a_3)$ ,  $a_1 \leq a_2 \leq a_3$ , where the membership function  $\mu_{\tilde{A}}(x)$  defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } x < a_1 \text{ or } x > a_3; \\ 1, & \text{if } a_1 \leq x \leq a_2; \\ \frac{a_3 - x}{a_3 - a_2}, & \text{if } a_2 \leq x \leq a_3. \end{cases} \quad (6')$$

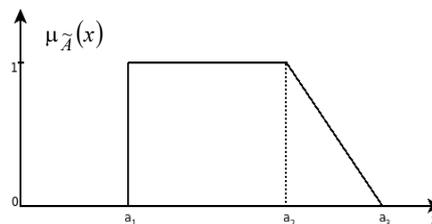


Figure 3 – Trapezoidal fuzzy number

The use of triangular and trapezoidal fuzzy numbers for practical implementation under conditions of uncertainty raises numerous questions regarding their constructiveness and necessitates the development of specialized methods for representing numerical fuzziness. It should be noted that the use of left-sided, right-sided triangular, and rectangular-trapezoidal fuzzy numbers – most commonly employed for formalization – can be considered only within the framework of triangular fuzzy numbers. The inclusion of normalized values from the interval  $[a_1, a_2]$  of a rectangular-trapezoidal fuzzy number does not significantly influence the formation of assessments related to the subjectivity of the formalized uncertainty parameters. In this context, the interval-based representation primarily reflects the predetermined variability of the parameters. From the standpoint of fuzzy evaluation, the interval  $[a_2, a_3]$ , whose corresponding values indicate the degree of subjectivity associated with each estimate, is of critical importance. Therefore, in the future we will limit ourselves to considering only triangular fuzzy numbers for formalizing the uncertainty in travel time in the traveling salesman problem.

#### Fuzzy Traveling Salesman Problem

Let us consider the fundamentals of optimization theory for solving the FuzzyTSP. In article [6] the authors investigated a classical model for solving optimization problems for decision making in a fuzzy environment, which laid the foundation for the development of most results in the theory of fuzzy decision-making. When analyzing the process of solving optimization problems under uncertainty, it becomes evident that both the objective function and the constraints can be fuzzy, each defined by an appropriate membership function. Since the primary goal of optimization problems is to find solutions where the objective function achieves its optimal value under the given constraints, the solution in a fuzzy context is defined analogously. Finding an optimal solution involves identifying an element in the feasible region that simultaneously delivers the best value of the objective function and satisfies all given constraints. Thus, a “solution” under fuzzy conditions can be interpreted as the intersection of the domains defined by the fuzzy constraints and the fuzzy objective function. An example of such a solution in a fuzzy optimization problem is illustrated in Figure 4.

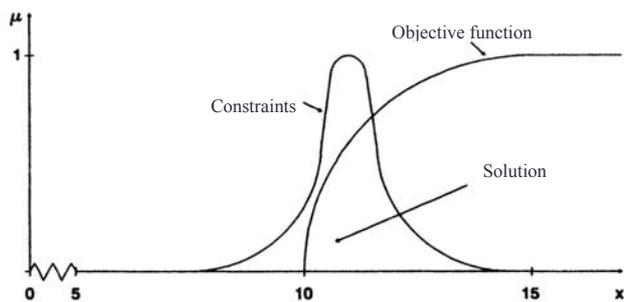


Figure 4 – Example of identifying a solution in a fuzzy optimization problem

It is clear that the “fuzzy linear programming” model cannot be defined unambiguously and that different approaches to implementation are possible depending on the specifics of the real situation that needs to be formalized.

We begin by formulating the basic model of a “fuzzy linear programming problem”. For this purpose, let us consider the standard formulation of the classical linear programming problem: find a solution  $x \in R^n$  that maximizes the objective function

$$c^T x \rightarrow \min, \quad (7)$$

in the feasible solution space defined by the system of constraints

$$Ax \geq b, \quad x \geq 0, \quad (8)$$

where  $x = (x_1, \dots, x_n)^T$  – the vector of decision variables,  $c = (c_1, \dots, c_n)^T$  – the vector of coefficients of the objective function,  $A = (a_{ij}), i = \overline{1, n}, j = \overline{1, m}$ , – the constraint coefficient matrix,  $b = (b_1, \dots, b_m)^T$  – the right-hand side vector.

Let us assume that there is a decision maker who determines the required reliability level  $Z$  that must be achieved in the objective function in model (7)–(8). Also assume that each constraint in the problem is defined using data from a fuzzy set. In this case, we can formulate the general form of the fuzzy linear programming problem (see, [4]): find a vector such that

$$c^T x \lesseqgtr Z, \quad (9)$$

$$Ax \lesseqgtr b, \quad x \geq 0. \quad (10)$$

Here, the symbol “ $\lesseqgtr$ ” is used to denote the fuzzy variant of the non-strict relation “ $\leq$ ” and can be linguistically interpreted as “essentially less than or equal to.” Similarly, the symbol “ $\gtrless$ ”, respectively, denotes the fuzzified version of the relation “ $\geq$ ”, which can be linguistically interpreted as “essentially greater than or equal to”. Formally, the fuzzy relations “ $\lesseqgtr$ ” and “ $\gtrless$ ” can

be treated as particular cases of fuzzy relations “non-strictly less” and “non-strictly greater”, respectively, over the set  $X = R^1$  [16].

In this case, the objective function is defined as the minimization of the linear expression (7), where the value of  $Z$  is considered an upper bound of the optimal solution. Let us introduce the notation.

Let us introduce the notation.  $\begin{pmatrix} -c \\ A \end{pmatrix} = D, \begin{pmatrix} -Z \\ b \end{pmatrix} = p$ , and rewrite the constraints (9)–(10) as follows:

$$Dx \lesseqgtr p, \quad x \geq 0. \quad (11)$$

If the value  $Z$  and the elements of the vector  $b$  are given in the form of fuzzy numbers, then the right-hand side of the system of inequalities (11) will define a vector whose elements can correspond to specific values of the supports of the given fuzzy quantities. Therefore, each of the  $(m + 1)$  inequalities in (11) for a given value of the vector  $p$  has a set of solutions that are characterized by the value of the corresponding membership function, denoted by  $\mu_i(x), i = \overline{1, m+1}$ . Without loss of generality, it can be assumed that all membership functions are monotonically increasing. Then, for any  $\alpha \in [0, 1]$  the values  $\mu_i(x) = \alpha, i = \overline{1, m+1}$ , can be interpreted as degrees of confidence with which the vector  $x$  satisfies

the crisp inequalities  $\sum_{j=1}^n d_{ij} x_j \geq \tilde{p}_i(\alpha), i = \overline{1, m+1}$  (where

$d_{ij}$  – matrix elements  $D$ , and  $\tilde{p}_i(\alpha)$  – are the values obtained based on the  $\alpha$ -level sets of the fuzzy elements of vector  $p$ ). If we assume that the decision-maker (DM) is interested not in a fuzzy set of solutions, but in a crisp “optimal” solution of the original problem, then an improvement of the solution (11) can be proposed by solving the following nonlinear programming problem:

$$\mu_{\tilde{p}}(x) = \min_{i=1, m+1} \mu_i(x). \quad (12)$$

If we assume that the decision maker (DM) is interested not in a fuzzy set of solutions, but in a crisp “optimal” solution to the original problem, the solution (12) can be further refined by solving a nonlinear programming problem

$$\max_{x \geq 0} \min_{i=1, m+1} \mu_i(x) = \max_{x \geq 0} \mu_{\tilde{p}}(x). \quad (13)$$

Let us define the type of membership functions  $\mu_i(x), i = \overline{1, m+1}$ . The values of  $\mu_i(x), i = \overline{1, m+1}$ , should be equal to 0 if the constraints or objective function are significantly violated, and equal to 1 if they are completely satisfied. In addition, the functions are

assumed to increase monotonically from 0 to 1 on the interval  $[0,1]$ , i.e.:

$$\mu_i(x) = \begin{cases} 0, & \sum_{j=1}^n d_{ij}x_j < p_i, \\ \in [0,1], & p_i \leq \sum_{j=1}^n d_{ij}x_j \leq p_i + q_i, \quad i = \overline{1, m+1}, \\ 0, & \sum_{j=1}^n d_{ij}x_j > p_i + q_i, \end{cases} \quad (14)$$

where  $q_i, i = \overline{1, m+1}$ , are subjectively defined tolerance thresholds for admissible deviations in the constraints and the objective function.

Under our assumption regarding fuzzy numbers, the membership functions are linearly decreasing over the corresponding “tolerance intervals”  $[p_i, p_i + q_i]$ ,  $i = \overline{1, m+1}$ :

$$\mu_i(x) = \begin{cases} 0, & \sum_{j=1}^n d_{ij}x_j < p_i, \\ 1 - \left( \sum_{j=1}^n d_{ij}x_j - p_i \right) / q_i, & p_i \leq \sum_{j=1}^n d_{ij}x_j \leq p_i + q_i, \\ 0, & \sum_{j=1}^n d_{ij}x_j > p_i + q_i, \end{cases} \quad i = \overline{1, m+1}. \quad (15)$$

Substituting (15) into (13), and after straightforward algebraic manipulation, we obtain the following decision-making criterion for selecting the optimal solution:

$$\max_{x \geq 0} \min_{i = \overline{1, m+1}} \left( 1 - \left( \sum_{j=1}^n d_{ij}x_j - p_i \right) / q_i \right). \quad (16)$$

We introduce a new variable  $\lambda \in [0,1]$ , which corresponds to the minimal membership level of the fuzzy set of “solutions”  $\tilde{P}$  defined in (12) for the fuzzy model

$$(11): 1 - \left( \sum_{j=1}^n d_{ij}x_j - p_i \right) / q_i \geq \lambda.$$

This yields the Bellman-Zadeh [10] optimization model

$$\max_{x \geq 0} \lambda. \quad (17)$$

under condition

$$\lambda q_i + \sum_{j=1}^n d_{ij}x_j \leq p_i + q_i, \quad i = \overline{1, m+1}, \quad x \geq 0. \quad (18)$$

If the optimal solution of problem (17)–(18) is denoted by a vector  $(\lambda, x^O)$ , then  $x^O$  will be the solution of the maximization problem (13) for the fuzzy optimization model (7)–(8), under the assumption that the membership functions are defined as in (14).

Thus, we come to the conclusion [4], that this optimal solution to the original model (7)–(8) can be found by solving a standard (crisp) linear programming problem with one additional variable and one additional constraint.

Let us apply this approach to solving the fuzzy traveling salesman problem. To solve the FuzzyTSP, it is necessary to take into account the nature of the uncertainty of the problem parameters and develop appropriate methods for finding a route.

In real-world transportation logistics scenarios, it is often impossible to determine precise travel times between cities in a network. When travel times cannot be precisely determined and represented by fuzzy triangular numbers, route finding on a transportation network is formulated as a fuzzy traveling salesman problem.

Let’s formulate a mathematical formulation of the FuzzyTSP. The goal is to find a traveling salesman route, represented by a cyclic permutation of the indexes of cities in the transportation network, that minimizes the total travel time. In other words, we aim to minimize the objective function

$$\sum_{i=1}^n \sum_{j=1}^n \tilde{t}_{ij} x_{ij}, \quad (19)$$

where the travel times between nodes are given by a matrix  $T = \{t_{ij}\}$ ,  $i, j = \overline{1, n}$ , whose elements are triangular fuzzy numbers  $\tilde{t}_{ij} = \tilde{t}_{ji}$ ,  $\tilde{t}_{ij} = (t_{ij}, t_{ij}, t_{ij} + \Delta t_{ij})$ ,  $i, j = \overline{1, n}$ , and the possible connection routes between cities are specified by the matrix  $X = \{x_{ij}\}$ ,  $x_{ij} \in \{0,1\}$ ,  $i, j = \overline{1, n}$  subject to the constraints (2).

To determine the route in the FTSP using the Bellman-Zadeh approach, we solve two crisp TSPs of the form (1), (2), with objective functions

$$Z_l = \min_X \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij}, \quad (20)$$

$$Z_u = \min_X \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + \Delta t_{ij}) x_{ij}, \quad (21)$$

taking into account the constraints (2), respectively. The solutions yield the optimal values of the objective functions  $Z_l$  and  $Z_u$ , which correspond to the optimal total durations of the route under lower and upper bounds of fuzzy travel times in the network.

Let us denote the optimal solutions of problems (20), (2) and (21), (2) as

$$X^{1*} = \arg \min_X \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij},$$

$$X^{2*} = \arg \min_X \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + \Delta t_{ij}) x_{ij},$$

respectively. Then we compute

$$L_1 = \min(Z_l, \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij}^{2*}), \quad U_1 = \max(Z_l, \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij}^{2*}),$$

$$L_2 = \min(Z_u, \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + \Delta t_{ij}) x_{ij}^{1*}),$$

$$U_2 = \max(Z_u, \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + \Delta t_{ij}) x_{ij}^{1*}),$$

which represent, respectively, the lower and upper bounds of the optimal values of the objective functions

$$\sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij} \quad \text{and} \quad \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + \Delta t_{ij}) x_{ij}.$$

Then, the solution to the FuzzyTSP (19), (2) is obtained by solving the parametric Bellman-Zadeh optimization problem [10] of the following form:

$$\begin{aligned} & \max_x \lambda \\ & \lambda(U_1 - L_1) + \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij} \leq U_1, \\ & \lambda(U_2 - L_2) + \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + \Delta t_{ij}) x_{ij} \leq U_2, \\ & \sum_{j=1}^n x_{ij} = 1, \quad i = \overline{1, n}, \quad \sum_{i=1}^n x_{ij} = 1, \quad j = \overline{1, n}, \end{aligned} \quad (22)$$

$x_{ij} = 0$  or  $1$  for all  $i, j = 1, 2, \dots, n$ .

Clearly, solving the FuzzyTSP, taking into account its combinatorial nature, via the corresponding optimization formulation requires significant computational and time resources. Therefore, current research in the domain of FuzzyTSP focuses on improving existing methods and/or developing new approaches.

The main results are associated with the use of techniques that transform the triangular fuzzy parameters into a specific representation format [16]. This transformation allows performing arithmetic operations on fuzzy numbers. The implementation of such an approach is among the most commonly used strategies for constructing route sequences in FuzzyTSP, particularly when averaged values of fuzzy input parameters are calculated based on their centers of gravity [17].

Another direction of research in the development and implementation of FTSP solution methods involves the application of a multicriteria approach. Let us consider the fuzzy traveling salesman problem (19), (2) as a bicriteria optimization problem, where the total travel

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time along the route is to be minimized under fuzzy constraints given by lower and upper bounds. In other words, the classical single-criterion formulation of the TSP is replaced with a formulation containing two criteria:

$$F_1 = \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij} \rightarrow \min, \quad (23)$$

$$F_2 = \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + \Delta t_{ij}) x_{ij} \rightarrow \min, \quad (24)$$

where the values  $t_{ij}, i, j = \overline{1, n}$ , are the elements of the matrix  $T = \{t_{ij}\}, i, j = \overline{1, n}$ , which represent the nominal travel times between all pairs of nodes in the transportation network.

To obtain a compromise solution, we apply the convolution method. Let us introduce weight coefficients  $\alpha_1, \alpha_2 > 0: \alpha_1 + \alpha_2 = 1$ , corresponding to the decision-maker's confidence in the lower and upper bounds of the travel durations. This allows the formation of weighted indicators from the fuzzy input values  $\tilde{t}_{ij}, i, j = \overline{1, n}$ . The solution of problem (8), (9) with two sets of fuzzy parameters  $t_{ij}$  and  $t_{ij} + \Delta t_{ij}, i, j = \overline{1, n}$ , respectively, is sought as the optimal solution to a classical TSP of the form (1), (2) with the following weighted criterion:

$$\begin{aligned} F &= \alpha_1 F_1 + \alpha_2 F_2 = \\ & \alpha_1 \sum_{i=1}^n \sum_{j=1}^n t_{ij} x_{ij} + \alpha_2 \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + \Delta t_{ij}) x_{ij} = \\ & = \sum_{i=1}^n \sum_{j=1}^n \{\alpha_1 t_{ij} + \alpha_2 (t_{ij} + \Delta t_{ij})\} x_{ij} \rightarrow \min, \end{aligned} \quad (25)$$

subject to the constraint (2). It is important to note that the resulting compromise solution depends on the chosen weight coefficients  $w_1$  and  $w_2$ . This approach utilizes only the support intervals of the fuzzy travel durations and does not account for the values of the corresponding membership functions.

Nevertheless, this approach provides a constructive way of solving the fuzzy traveling salesman problem (FTSP) as a bicriteria optimization task for finding a time-optimal travel route based on the criteria defined in (23) and (24), subject to constraints (2).

However, the application of only two criteria-related to the confidence levels of the decision maker in the travel duration bounds – does not fully exploit the convolution method for determining a compromise route in the FTSP defined by (19), (2) as a solution to the associated bicriteria problem.

To generalize the above approach, we introduce confidence indicators through a weight function  $\alpha(s) \geq 0$ , defined on the interval  $s \in [0, 1]$ , which satisfies the condition:



$$\int_0^1 \alpha(s) ds = 1. \quad (26)$$

Let us formulate the optimality criterion in the fuzzy traveling salesman problem (19) using the weight function  $\alpha(s) \geq 0, s \in [0,1]$ , as follows:

$$\int_0^1 \alpha(s) \left\{ \sum_{i=1}^n \sum_{j=1}^n (t_{ij} + s\Delta t_{ij}) x_{ij} \right\} ds = \sum_{i=1}^n \sum_{j=1}^n \left\{ \int_0^1 \alpha(s) (t_{ij} + s\Delta t_{ij}) ds \right\} x_{ij} \rightarrow \min. \quad (27)$$

Assuming that  $\alpha(s) = 0$  for all  $s \notin [0,1]$ , the integral in (27) evaluates the weighted average of the linear travel time functions  $t_{ij} + s\Delta t_{ij}, i, j = \overline{1, n}$ , with respect to the confidence levels specified by the weight function  $\omega(x)$ . In this formulation, the entire range of values within  $\alpha(s)$ , is used to derive defuzzified estimates for fuzzy travel times intervals  $t_{ij} + s\Delta t_{ij}, s \in [0,1], i, j = \overline{1, n}$ , allowing the fuzzy TSP to be reduced to the classical problem (1), (2).

It should be noted that defining an additional confidence function  $\omega(x)$  may impose preferences for certain values  $t_{ij} + s\Delta t_{ij}, s \in [0,1], i, j = \overline{1, n}$ , derived from the support intervals of fuzzy travel durations, without directly accounting for their fuzziness. To incorporate the fuzzy nature of the travel time intervals, corresponding membership functions  $\mu_{\tilde{t}_{ij}}(t_{ij} + s\Delta t_{ij}), s \in [0,1], i, j = \overline{1, n}$  should be used.

Let us denote  $g_{ij}(s) = \mu_{\tilde{t}_{ij}}(t_{ij} + s\Delta t_{ij}) \cdot (t_{ij} + s\Delta t_{ij}), s \in [0,1], i, j = \overline{1, n}$ . Then for any arbitrary weight function

$\alpha(s) \geq 0, s \in [0,1]$ , values  $\int_0^1 \alpha(s) g_{ij}(s) ds$ , will be

considered as the weighted average values for each fuzzy triangular number  $\tilde{t}_{ij}, i, j = \overline{1, n}$ . Then, the defuzzified values of the travel durations on the network are computed by integrating both the membership functions and the preference weights defined by  $\omega(x)$ , and the optimality criterion (12)  $\alpha(s), s \in [0,1]$ , and the optimality criterion (12) for the fuzzy traveling salesman problem becomes:

$$\sum_{i=1}^n \sum_{j=1}^n \left\{ \int_0^1 \alpha(s) g_{ij}(s) ds \right\} x_{ij} \rightarrow \min. \quad (28)$$

Finally, the fuzzy traveling salesman problem with objective function (19) is reduced to a single-criterion optimization problem with the objective function in the

form of (28), by applying a multicriteria approach and a specific linear convolution with an interval-defined weight function. This enables the refinement of route calculations within the fuzzy TSP framework.

Finally, one of the most effective techniques employed in solving fuzzy traveling salesman problems is the defuzzification of fuzzy numbers based on the computation of the center of gravity (CoG) of the fuzzy set [18]. In this case, the “averaged” value of a triangular fuzzy number  $\tilde{A} = (a_1, a_2, a_3)$  is considered to be the value computed as the center of gravity of a planar figure bounded by the abscissa axis and the graph of the membership function of the fuzzy set. For a discrete fuzzy set, the formula is given by:

$$CoG = \frac{\sum_{i=1}^n \mu(x_i) \cdot x_i}{\sum_{i=1}^n \mu(x_i)}, \quad (29)$$

where  $x_i$  are values from the universal set, and,  $\mu(x_i)$  is the membership degree of each value,  $i = \overline{1, n}$ , in the case of a continuous representation of the fuzzy number, the formula takes the form:

$$CoG = \frac{\int_{a_1}^{a_3} x \cdot \mu(x) dx}{\int_{a_1}^{a_3} \mu(x) dx}. \quad (30)$$

The quantity  $CoG$  obtained by this method represents the abscissa of the center of gravity of a homogeneous planar figure. It is important to note that a fuzzy set (or fuzzy number) is defined by the graph of its membership function over the interval of its support. In the case of a triangular fuzzy number, the membership function is piecewise linear, which allows the  $CoG$  value to be replaced by computing the center of gravity of a homogeneous curve  $(x_c, y_c)$ , using the formulas

$$x_c = \frac{\int_{a_1}^{a_3} x \sqrt{1 + [\mu'(x)]^2} dx}{\int_{a_1}^{a_3} \sqrt{1 + [\mu'(x)]^2} dx}, \quad (30)$$

$$y_c = \frac{\int_{a_1}^{a_3} \mu(x) \sqrt{1 + [\mu'(x)]^2} dx}{\int_{a_1}^{a_3} \sqrt{1 + [\mu'(x)]^2} dx}.$$

The center of gravity of a plane curve is defined as a point on the plane at which the static moment about any coordinate axis is equal to the static moment of the curve itself about the same axis. In other words, the center of gravity of the curve corresponds to the “average” value of the triangular fuzzy number.

When applying the above averaging methods based on the CoG concept, it is generally assumed that the density of the membership function curve and the associated plate

is homogeneous. However, one can also assume that the curve has a variable density function  $\rho(l)$ ,  $0 \leq l \leq L$ , where the value of  $\rho(l)$  depends on the position of each point on the graph of the fuzzy number's membership function (here  $L$  is the arc length of the curve). Without loss of generality, we can assume that the density values lie within the interval  $[0,1]$ . In this case, the position of the center of gravity of a continuous inhomogeneous curve is given by

$$x_c^p = \frac{\int_{a_1}^{a_4} \rho(l)x\sqrt{1+[\mu'(x)]^2} dx}{\int_{a_1}^{a_4} \sqrt{1+[\mu'(x)]^2} dx}, \quad (32)$$

$$y_c^p = \frac{\int_{a_1}^{a_4} \rho(l)\mu(x)\sqrt{1+[\mu'(x)]^2} dx}{\int_{a_1}^{a_4} \sqrt{1+[\mu'(x)]^2} dx}.$$

By interpreting the density function  $\rho(x)$  as the credibility (or reliability) function of the membership values, the triangular fuzzy number is thus represented as a type-2 fuzzy number [19]. Accordingly, by expressing the density dependence on the location  $l$  on the curve through an arbitrary function  $\rho(l): R^1 \rightarrow [0,1]$ ,  $0 \leq l \leq L$ , one obtains new "averaged" characteristics of the center of gravity and the corresponding degree of membership (as per equation (31)).

#### 4 EXPERIMENTS

We conduct numerical experiments using various approaches to solve the fuzzy traveling salesman problem in which the travel duration along the network is defined by fuzzy right-triangular numbers. The computations are carried out using a model of a logistic transport network with precisely defined values of travel time for all possible segments of movement [20].

The optimal solution to the classical traveling salesman problem on the specified network corresponds to the route

$$1 \rightarrow 2 \rightarrow 6 \rightarrow 10 \rightarrow 11 \rightarrow 8 \rightarrow 5 \rightarrow 9 \rightarrow 7 \rightarrow 4 \rightarrow 3 \rightarrow 1, \quad (33)$$

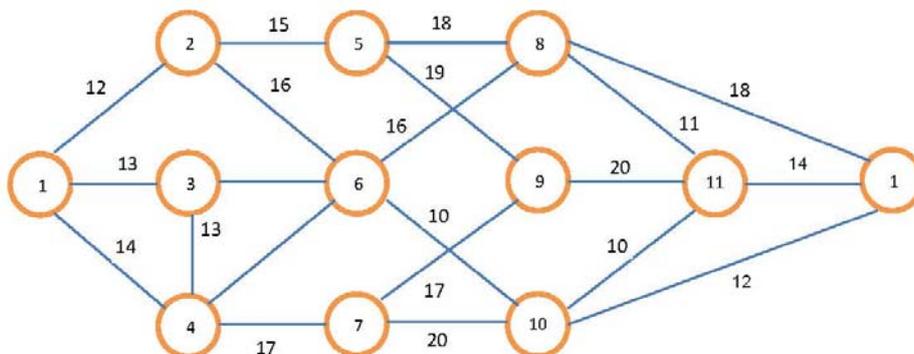


Figure 5 – An example of a transport network in the traveling salesman problem [4]

for which the total duration amounts to 156 units.

To simulate the fuzzy version of the traveling salesman problem, the fuzzy travel time along each path between cities is modeled using the rule:

$$\tilde{t}_{ij} = (t_{ij}, t_{ij}, t_{ij} \cdot 1.8 + 1.5 + (i + j) \cdot 0.75), \quad (34)$$

where  $i, j$  are the indices of the cities and  $t_{ij}$  is the crisp (precisely defined) travel times.

#### 5 RESULTS

Applying different defuzzification methods to the fuzzy travel durations results in solutions that fully coincide with the travel sequence given in (33). Table 1 presents the lower and upper bounds of travel durations between cities  $i$  and  $j$  within the transport network, the weighted travel durations computed using  $\alpha_1 = \alpha_2 = 0.5$ , the  $CoG$  values of the corresponding fuzzy durations, as well as the calculated centroids of homogeneous and inhomogeneous graphs of linear membership functions (using  $\rho(l) = e^{-l}$ ,  $0 \leq l \leq L$ ) and the corresponding optimal values of the total travel time for the salesman (see the row labeled "Duration"), which are obtained by the aforementioned methods. The last column reports the results obtained using the Bellman-Zadeh method with parameter  $\lambda = 0.15$ .

It is evident that the invariance of the route across different solution methods for the fuzzy traveling salesman problem is due to the uniform increase in all fuzzy duration parameters, which is rarely observed under real-world conditions. In this case, the primary objective of the presented results is to visually demonstrate the previously discussed solution techniques for the fuzzy TSP. It is worth noting that the most informative and qualitatively accurate outcomes were achieved in numerical experiments involving fuzzy travel durations between cities, in which the computation of travel time accounts for the density function of the membership graph (see Table 1, column  $x_c^p$ ).

Table 1 – Results of Solving the FuzzyTSP with Fuzzy Travel Duration Values in the Network [4]

$i$	$j$	$t_{ij}$	$t_{ij} + \Delta t_{ij}$	$0.5 t_{ij} + 0.5 (t_{ij} + \Delta t_{ij})$	CoG	$x_c$	$x_c^p$	$x^O$
1	2	12	25.35	24.675	17.2337	17.103	17.01	14.0025
1	3	13	27.90	26.95	18.8037	18.4807	18.17	15.235
1	4	14	30.45	29.225	20.3742	20.0508	20.016	16.4675
1	8	18	40.65	38.325	26.6586	26.3284	26.241	21.3975
1	10	13	31.35	28.675	19.1806	19.160	19.105	15.7525
1	11	14	35.70	31.85	22.0797	22.0195	22.011	17.255
2	5	15	33.75	31.875	22.1870	22.1852	22.156	17.8125
2	6	16	36.30	34.15	23.7584	23.3543	23.256	19.045
3	4	13	30.15	28.075	19.5295	19.512	19.314	15.5725
3	6	15	35.25	32.625	22.6731	22.1125	22.105	18.0375
4	6	16	37.80	34.9	24.2450	24.108	24.092	19.27
4	7	17	40.35	37.175	25.8170	25.6245	25.602	20.5025
5	8	18	43.65	39.825	27.6335	27.3799	27.367	21.8475
5	9	19	46.20	42.1	29.2058	29.0481	29.031	23.08
6	8	16	40.80	36.4	25.2245	25.1274	25.121	19.72
6	10	10	31.50	25.75	17.7688	17.1875	17.085	13.225
7	9	17	44.10	39.05	27.0429	27.0356	27.026	21.065
7	10	20	50.25	45.125	31.2685	31.2386	31.214	24.5375
8	11	11	35.55	28.775	19.8387	19.1381	19.123	14.6825
9	11	20	52.50	46.25	32.0067	31.9191	31.694	24.875
10	11	10	35.25	27.625	19.0165	18.246	18.228	13.7875
Duration		156	396.3	354.15	245.6485	242.1097	241.212	192.045

## 6 DISCUSSION

Several remarks should be noted. The procedure for finding a solution to the fuzzy traveling salesman problem based on the Bellman-Zadeh method is based on multiple solutions of a crisp problem for different parameter values. To solve the problem, a genetic algorithm was used [21], which allowed us to obtain a solution relatively quickly. However, the transport network in the problem under consideration is small, and when it increases, finding a solution based on the Bellman-Zadeh method will be significantly limited. Thus, solving practical problems of finding the optimal traveling salesman route in a fuzzy setting is characterized by low performance and significant requirements for computing resources. To speed up obtaining the result, it is proposed to use methods based on defuzzification of the duration of movements between network nodes by calculating the gravity centroids of the graphs of the corresponding membership functions.

## CONCLUSIONS

This paper presents the results of a study on the use of triangular fuzzy numbers for determining time-optimal routes in the traveling salesman problem under fuzzy representations of travel duration in a transportation network. To formalize the uncertainty and imprecision of input data – associated with the subjectivity in estimating the time intervals required to travel between individual cities-

triangular fuzzy numbers are employed. Various approaches to solving fuzzy traveling salesman problems are examined. The application of the Bellman-Zadeh method, methods incorporating refined defuzzified data, and methods based on multicriteria decision-making are formalized. The interpretation of averaged values for right-sided triangular fuzzy numbers is analyzed. An enhancement of defuzzified values is proposed based on the computation of the center of gravity of the membership function curve and the construction of type-2 fuzzy sets, which allows for improved objectivity of the input parameters and yields better results. In the conducted numerical experiments on solving the traveling salesman problem with fuzzy travel durations, the influence of various defuzzification techniques is demonstrated. These include the use of the center of gravity (CoG), the centroid of homogeneous and inhomogeneous curves defined by membership functions, and the assigned confidence values of subjective data. A comparison is made between the results obtained from solving the crisp version of the traveling salesman problem and those derived from defuzzified values in the fuzzy case. The outcomes confirm the dependence of the solution on the defuzzification method applied. The study concludes that using triangular fuzzy numbers is appropriate and effective for solving fuzzy traveling salesman problems in real-world logistic transportation scenarios.

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## DECLARATIONS

**Conflict of interest:** The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship, or otherwise, that could affect the research and its results presented in this paper.

**Authors' contributions:** Eugene Ivohin: formulation of the general statement of the problem and research methodology; Valery Gavrylenko: discussion of research directions and methods for solving the problem; Konstantin Yushtin: development of a methodology for processing fuzzy numbers and defuzzification methods, Kateryna Ivohina: experimental study of solution search methods.

**Data availability:** The manuscript has no associated data.

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**Use of artificial intelligence tools:** The authors confirm that they did not use artificial intelligence technologies in creating the submitted work.

## REFERENCES

1. Zadeh L. A. Fuzzy sets, *Information and Control*, 1965, No. 8, pp. 338–353.
2. Ghiani G., Laporte G., Musmanno R. Introduction to Logistics Systems Planning and Control. John Wiley & Sons, Ltd, 2013, 377 p. DOI:10.1002/9781118492185
3. Davendra D. Traveling salesman problem: theory and applications. Books on Demand, 2010, 338 p.
4. Gavrylenko V. V., Ivohin E. V., Ivohina K. E., Yushtin K. E. Optimization models of transport and network flows in the problems of supporting decision-making in information management systems, In “*Innovative trends in the development of information control systems and technologies*”. Under the general editorship of Doctor of Economics, Professor Ustenko S. V. Kyiv, KNEU named after Vadym Hetman, 2024, pp. 233–255. <https://ir.kneu.edu.ua/handle/2010/46976>
5. Kumar A., Gupta A. Methods for solving fuzzy assignment problems and fuzzy travelling salesman problems with different membership functions, *Fuzzy Information and Engineering*, 2011, No. 3(1), pp. 3–21.
6. Bellman R. E., Zadeh L. A. Decision making in fuzzy environment, *Management science*, 1970, No. 17, pp. 144–164.
7. Zimmermann H.-J. Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems*, 1978, No.1, pp. 45–55.
8. Christofides N. Vehicle routing in the traveling salesman problem. Lawler, Lenstra, RinooyKan and Shmoys, John Wiley eds., 1985, pp. 431–448.
9. Fuling Tien. Applying interactive fuzzy multi-objective Linear programming to transportation planning decisions, *Journal of information and optimization sciences*, 2006, No. 27(1), pp. 107–126.
10. Kosheleva O., Kreinovich V. Why Bellman-Zadeh approach to fuzzy optimization, *Applied Mathematical Sciences*, 2018, Vol. 12, No. 11, pp. 517–522.
11. Yushtin K., Ivohin E. About defuzzification methods influence on fuzzy traveling salesman problem's solving, *Artificial Intelligence*, 2024, No. 1 (98), pp. 64–72. DOI: 10.15407/jai2024.01.064
12. Ivohin E., Gavrylenko V., Ivohina K. One approach to solving the fuzzy traveling salesman problem based on a multicriteria approach, *Artificial Intelligence*, 2025, No. 2 (103), pp. 84–94. DOI: 10.15407/jai2025 .02.084
13. Ivohin E., Yushtin K. A method for solving a single fuzzy multicriteria traveling salesman problem, *Artificial Intelligence*, 2024, No. 4 (101), pp. 142–150. DOI: 10.15407/jai2024.04.142
14. Bablu Jana, Tapan Kumar Roy Multi-objective fuzzy linear programming and its application in transportation model, *Tamsui Oxford Journal of Mathematical Sciences*, 2005, No. 21(2), pp. 243–268.
15. Kaufman A., Gupta M. M. Introduction to fuzzy arithmetic: theory and applications. Van Nostrand Reinhold Co. Inc., Workingham, Berkshire, 2003, 351 p.
16. Zimmermann H.-J. Fuzzy set theory and its application. Kluwer, Boston, 1992, 525 p. DOI: 10.1007/ 978-94-010-0646-0
17. Voskoglou M. G. Fuzzy sets, fuzzy logic and their applications, *Mathematics*, 2020, 452 p. DOI: 10.3390/ books978-3-03928-521-1
18. Van Broekhoven E., De Baets B. Fast and accurate center of gravity defuzzification of fuzzy system outputs defined on trapezoidal fuzzy partitions, *Fuzzy Sets and Systems*, 2006, Vol. 157, No. 7, pp. 904–918.
19. Mendel J., Robert J. Type-2 fuzzy sets made simple, *IEEE Transactions on Fuzzy Systems*, 2002, No. 10 (2), pp. 117–127. DOI: 10.1109/91.995115.
20. Ivohin E., Gavrylenko V., Ivohina K. On the recursive algorithm for solving the traveling salesman problem on the basis of the data flow optimization method, *Radio Electronics, Computer Science, Control*, 2023, No. 3, pp. 141–147. DOI:10.15588/1607-3274-2023-3-14.
21. Ivohin E. V., Yushtin K. E. Solving the fuzzy traveling salesman problem using genetic algorithm with clustering by ward's method, *Conference Intelligent Transport Systems: Ecology, Safety, Quality, Comfort: ITS ESQC-2024: proceedings*. Springer, 2024, Vol. 1, pp. 199–210. DOI: 10.1007/978-3-031-87376-8

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## ПРО РАЦІОНАЛЬНІ МЕТОДИ ПОШУКУ ОПТИМАЛЬНИХ МАРШРУТІВ У НЕЧІТКИХ ЗАДАЧАХ КОМІВОЯЖЕРА

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### АНОТАЦІЯ

**Актуальність.** Важливою сучасною проблемою є швидке відновлення та оптимізація управління логістикою. В залежності від поставленої задачі існує багато різних математичних методів та підходів до вирішення різних логістичних задач, розв'язування яких набуває широкого практичного впровадження. Його конкретний зміст залежить від характеру проблеми та повноти наявних даних. Іноді для розв'язання відомих задач, однією з яких є задача комівояжера, вдається знайти нетипові методи на основі поєднання декількох обчислювальних схем та методів.

**Ціль.** Мета роботи – розробити алгоритми розв'язання нечіткої задачі комівояжера на основі реалізації методів параметричної оптимізації Беллмана-Заде, використання двокритеріального підходу із заданою ваговою функцією та уточнення схеми розрахунку центру ваги графіка функції належності для заданої щільності кривої.

**Метод.** У статті розглядаються методи розв'язування нечіткої задачі комівояжера, що формулюється як задача знаходження маршруту відвідування заданої кількості міст без повторень з мінімальною тривалістю руху. Параметри задачі для формалізації невизначеності та неточності вхідних даних, пов'язаних з впливом суб'єктивності в оцінках тривалості, необхідних для переміщення між окремими містами проміжків часу, подаються у вигляді нечітких трикутних чисел. Розглянуто різні підходи, що дозволяють розв'язувати нечіткі задачі комівояжера. Формалізовано застосування методу Беллмана-Заде, методів з урахуванням уточнень дефазифікованих даних та методів на основі багатокритеріального підходу. Проведено обчислювальні експерименти.

**Результати.** Розроблено раціональні алгоритми розв'язання нечіткої задачі комівояжера на основі параметричної оптимізаційної моделі Беллмана-Заде, багатокритеріального підходу та методів уточнення результатів дефазифікації нечітких даних. У проведених чисельних експериментах з розв'язання задачі комівояжера з нечітко заданою тривалістю переміщень продемонстровано вплив різних варіантів дефазифікації нечітких вхідних даних на основі методу розрахунку центра тяжіння (CoG), центру ваги однорідної та неоднорідної кривих, які визначаються функцією належності та заданими величинами надійності суб'єктивних даних. Проведено порівняння результатів, отриманих на основі вирішення чіткої задачі комівояжера, та результатів на основі дефазифікованих значень тривалості для нечіткої задачі комівояжера, за ітогами якого підтверджено залежність розв'язку від способу дефазифікації.

**Висновки.** У статті розглянуто метод формалізації алгоритму розв'язання нечіткої задачі комівояжера з мінімальною тривалістю руху за маршрутом на основі методу Беллмана-Заде, методів з урахуванням уточнень дефазифікованих даних та методів на основі багатокритеріального підходу. Для формалізації невизначеності вхідних даних при оцінці тривалості переміщення між окремими містами транспортної мережі використовуються нечіткі трикутні числа. Зроблено висновок про доцільність використання нечітких чисел при розв'язанні нечітких задач комівояжера в реальних умовах логістичних перевезень.

**КЛЮЧОВІ СЛОВА:** нечітка задача комівояжера, нечіткі числа, суб'єктивне сприйняття тривалості, невизначеність, методи розв'язування, багатокритеріальний підхід, дефазифікація.

### ЛІТЕРАТУРА

1. Zadeh L. A. Fuzzy sets / L. A. Zadeh // *Information and Control*. – 1965. – No. 8. – P. 338–353.
2. Ghiani G. Introduction to Logistics Systems Planning and Control / G. Ghiani, G. Laporte, R. Musmanno. – John Wiley & Sons, Ltd, 2013. – 377 p. DOI:10.1002/9781118492185
3. Davendra D. Traveling salesman problem: theory and applications / D. Davendra. – Books on Demand, 2010. – 338 p.
4. Optimization models of transport and network flows in the problems of supporting decision-making in information management systems / [V. V. Gavrylenko, E. V. Ivohin, K. E. Ivohina, K. E. Yushtin] // In “Innovative trends in the development of information control systems and technologies”. – Under the general editorship of Doctor of Economics, Professor Ustenko S.V. – Kyiv, KNEU named after Vadym Hetman, 2024. – P. 233–255. <https://ir.kneu.edu.ua/handle/2010/46976>
5. Kumar A. Methods for solving fuzzy assignment problems and fuzzy travelling salesman problems with different membership functions / A. Kumar, A. Gupta // *Fuzzy Information and Engineering*. – 2011. – No. 3 (1). – P. 3–21.

6. Bellman R. E. Decision making in fuzzy environment / R. E. Bellman, L. A. Zadeh // *Management science*. – 1970. – No. 17. – P. 144–164.
7. Zimmermann H.-J. Fuzzy programming and linear programming with several objective functions / H.-J. Zimmermann // *Fuzzy Sets and Systems*. – 1978. – No. 1. – P. 45–55.
8. Christofides N. Vehicle routing in the traveling salesman problem / N. Christofides. – Lawler, Lenstra, RinnooyKan and Shmoys, John Wiley eds., 1985. – P. 431–448.
9. Fuling Tien Applying interactive fuzzy multi-objective Linear programming to transportation planning decisions / Tien Fuling // *Journal of information and optimization sciences*. – 2006. – No. 27(1). – P. 107–126.
10. Kosheleva O. Why Bellman-Zadeh approach to fuzzy optimization / O. Kosheleva, V. Kreinovich // *Applied Mathematical Sciences*. – 2018. – Vol. 12, No. 11. – P. 517–522.
11. Yushtin K. About defuzzification methods influence on fuzzy traveling salesman problem's solving / K. Yushtin, E. Ivohin // *Artificial Intelligence*. – 2024. – No. 1 (98). – P. 64–72. DOI: 10.15407/jai2024.01.064
12. Ivohin E. One approach to solving the fuzzy traveling salesman problem based on a multicriteria approach / E. Ivohin, V. Gavrylenko, K. Ivohina // *Artificial Intelligence*. – 2025. – No. 2 (103). – P. 84–94. DOI: 10.15407/jai2025.02.084
13. Ivohin E. A method for solving a single fuzzy multicriteria traveling salesman problem / E. Ivohin, K. Yushtin // *Artificial Intelligence*. – 2024. – No. 4 (101). – P. 142–150. DOI: 10.15407/jai2024.04.142
14. Bablu Jana. Multi-objective fuzzy linear programming and its application in transportation model / Bablu Jana, Tapan Kumar Roy // *Tamsui Oxford Journal of Mathematical Sciences*. – 2005. – No. 21(2). – P. 243–268.
15. Kaufman A. Introduction to fuzzy arithmetic: theory and applications / A. Kaufman, M. M. Gupta. – Van Nostrand Reinhold Co. Inc., Workingham, Berkshire, 2003. – 351 p.
16. Zimmermann H.-J. Fuzzy set theory and its application / H.-J. Zimmermann. – Kluwer, Boston, 1992. – 525 p. DOI: 10.1007/978-94-010-0646-0
17. Voskoglou M. G. Fuzzy sets, fuzzy logic and their applications / M. G. Voskoglou // *Mathematics*. – 2020. – 452 p. DOI: 10.3390/books978-3-03928-521-1
18. Van Broekhoven E. Fast and accurate center of gravity defuzzification of fuzzy system outputs defined on trapezoidal fuzzy partitions / E. Van Broekhoven, B. de Baets // *Fuzzy Sets and Systems*. – 2006. – Vol. 157. – No. 7. – P. 904–918.
19. Mendel J. Type-2 fuzzy sets made simple/ J. Mendel, J. Robert // *IEEE Transactions on Fuzzy Systems*. – 2002. – No. 10 (2). – P. 117–127. DOI: 10.1109/91.995115.
20. Ivohin E. V. On the recursive algorithm for solving the traveling salesman problem on the basis of the data flow optimization method / E. Ivohin, V. Gavrylenko, K. Ivohina // *Radio Electronics, Computer Science, Control*. – 2023. – No. 3. – P. 141–147. DOI:10.15588/1607-3274-2023-3-14
21. Ivohin E. V. Solving the fuzzy traveling salesman problem using genetic algorithm with clustering by ward's method / E. V. Ivohin, K. E. Yushtin // *Conference Intelligent Transport Systems: Ecology, Safety, Quality, Comfort: ITS ESQC-2024: proceedings*, Springer, 2024. – Vol. 1. – P. 199–210. DOI: 10.1007/978-3-031-87376-8