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CONTROL IN TECHNICAL SYSTEMS

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OPTIMIZATION OF FUEL CONSUMPTION IN THE PROBLEM OF STABILIZING THE ANGULAR POSITION OF AN AXISYMMETRIC SPACECRAFT

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ABSTRACT

Context. The problem of maintaining the angular orientation of a spacecraft is critical, especially when subjected to impulsive external disturbances that cause sharp deviations in angular velocities. The relevance of solving this problem is determined by the limited fuel supply on board, particularly for the class of spacecraft designed to provide artificial gravity, where precise and efficient control is paramount.

Objective. The main objective of this work is to minimize the consumption of energy resources (fuel) for the stabilization of the angular position of a specific class of spacecraft. This goal is achieved through the sequential execution of two interrelated tasks: 1) damping sharp deviations in the spacecraft’s angular velocities; 2) stabilizing the final angular position.

Method. A two-stage approach is proposed. To solve the first task (damping), optimal control is synthesized using a combination of Pontryagin’s maximum principle and the phase plane method. This allows for the creation of optimal switching curves that divide the phase plane into regions with corresponding optimal control values. To solve the second task (stabilization), a modal approach based on a proposed method of indeterminate coefficients is used, which ensures the specified dynamic characteristics of the transient stabilization processes.

Results. Modeling of the dynamics of the spacecraft’s angular motion was carried out. The simulation results confirm the high effectiveness of using the proposed combined approach for solving the problem of stabilizing the angular position of the spacecraft after significant external disturbances.

Conclusion. The joint application of Pontryagin’s maximum principle with the phase plane method for fuel-efficient damping of angular velocities, followed by the implementation of an optimal stabilization law based on the proposed method of indeterminate coefficients, represents an effective procedure for controlling the orientation and stabilization of a spacecraft with minimal fuel consumption.

KEYWORDS: axisymmetric spacecraft, maximum principle, phase plane, optimal switching and disconnection lines, modal synthesis, method of undetermined coefficients.

NOMENCLATURE

$x(t)$ – n-dimensional state vector;

$u(t)$ – m-dimensional control vector;

$A(t)$ – matrices of size parameters $n \times n$;

$B(t)$ – matrices of size parameters $n \times m$ respectively;

$F(x, u)$ – quality functional;

$H(x, u)$ – Hamilton function;

K – gravitational parameter of the Earth;

$r(\boldsymbol{n})$ – radius-vector of the center of mass of the spacecraft;

$F[N]$ – sum of external forces;

m [kg] – mass of the spacecraft;

I_X, I_Y, I_Z (kg m^2 – the main central moments of inertia of the spacecraft;

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ω_i – projections of the angular velocities and the moments of the spacecraft;

ψ – Euler angles (roll) rotation of the apparatus around its transverse axis *x*;

φ – Euler angles (pitch) rotation of the apparatus around its longitudinal axis *x*;

θ – Euler angles (yaw), rotation of the apparatus around its transverse axis y which runs from one wing to the other;

$M_i[N \cdot m]$ – control moments;

$M_{iз}[N \cdot m]$ – perturbing moments;

T [s] – time interval of the stabilization process;

$T_{\min}$ [s] – stabilization time in a system that is optimal in terms of minimizing the time of the transient process;

$\psi_i(t)$ – variables determined from the solution of the conjugate system;

α – coefficient in the equations of motion;

β – coefficient under control actions;

ε[rad/s] – specified area for the final state of angular velocities;

k – weighting coefficient in the quality functional;

c[rad/s] – constant angular velocity of the spacecraft;

λ – roots (poles) of the characteristic polynomial of the closed system

G – diagonal matrices with constant coefficients of the corresponding dimension , penalizes deviations from the target state (zero angles and velocities);

Q – diagonal matrices with constant coefficients of the corresponding dimension, penalizes the expenditure of the control resource (fuel);

a^i_j and b^i_j – coefficients, corresponding the coordinates of the center of that cicle arc, which is the optimal trajectory of switching;

R^i_j – coefficients that determines the radius of the circle arc;

x^T – spacecraft angular motion variables;

f_j – coefficient at λj in *i*-th considered determinant.

INTRODUCTION

At present, space technology is represented by a wide range of vehicles that differ in their intended use, overall weight characteristics, and the composition of onboard equipment. New tasks solved by spacecraft put forward high demands on onboard systems, in particular, the attitude control and attitude stabilization system. Since the accuracy of stabilization of the angular position of spacecraft is significantly affected by external disturbances, in particular, vibrations of attached elastic elements, such as solar panels, antennas, etc., as well as the damping of the angular oscillations of the spacecraft after separation from the launch vehicle or upper stage. it is necessary to improve the methods and algorithms for controlling the orientation and stabilization of the spacecraft, taking into account the specifics of the tasks it solves [1–3].

Energy consumption is one of the most important characteristics of the spacecraft control systems. Accordingly, the consumption of the working body of the engines to maintain the required angular position of the spacecraft in the control mode of its orientation should be minimal. This problem is quite relevant for modern cosmonautics, and its solution based mainly on the methods of the theory of automatic control, and, in particular, the methods of optimal control. It should be noted that, in contrast to the task of minimizing the time of transients, three-level control with zero control value zone is implemented in fuel consumption optimization tasks [4].

To implement these tasks, the most effective control system, which most often used in practice, is the active control system for jet nozzles. The control moment in this system occurs when the mass of the working fluid ejected from the nozzle of a small jet engine, the axis of which does not pass through the center of mass of the spacecraft. The control torque depends on the flow rate of the working fluid, as well as on the size of the lever acting on the engine's tractive effort.

The specified position of the spacecraft is determined in a certain coordinate system, the direction of the axes of which in space known in advance. This coordinate system called the basic reference system. Axes of this system must be set on board the spacecraft using special devices and devices. The second Newton's law used to create the equations of motion of the centers of mass of the spacecraft [1]:

$$m \cdot \frac{d^2 r}{dt^2} = -\frac{K}{r^2} \cdot \frac{\vec{r}}{r} + \vec{F}. \quad (1)$$

In this paper the equations of rotational motion of the spacecraft are described by the Euler dynamic equations look like [5, 6]:

$$\begin{aligned} I_X \cdot \frac{d\omega_X}{dt} + \omega_Y \cdot \omega_Z \cdot (I_Z - I_Y) &= \sum M_X; \\ I_Y \cdot \frac{d\omega_Y}{dt} + \omega_Y \cdot \omega_Z \cdot (I_Z - I_Y) &= \sum M_Y; \\ I_Z \cdot \frac{d\omega_Z}{dt} + \omega_X \cdot \omega_Z \cdot (I_Y - I_X) &= \sum M_Z. \end{aligned} \quad (2)$$

$$\begin{aligned} \omega_x &= coa\varphi \ \cos\psi \cdot \frac{d\theta}{dt} - \sin\varphi \frac{d\psi}{dt}; \\ \omega_y &= coa\theta \frac{d\psi}{dt} - \sin\varphi \ \cos\psi \frac{d\theta}{dt}; \\ \omega_z &= \frac{d\varphi}{dt} - \sin\psi \frac{d\theta}{dt}. \end{aligned} \quad (3)$$

The object of study is an axisymmetric cylindrical spacecraft, a simplified diagram of which with the location of jet engines is shown in Fig. 1. Angular control moments $\sum M_i (I = x, y, z)$ creates by jet engines.

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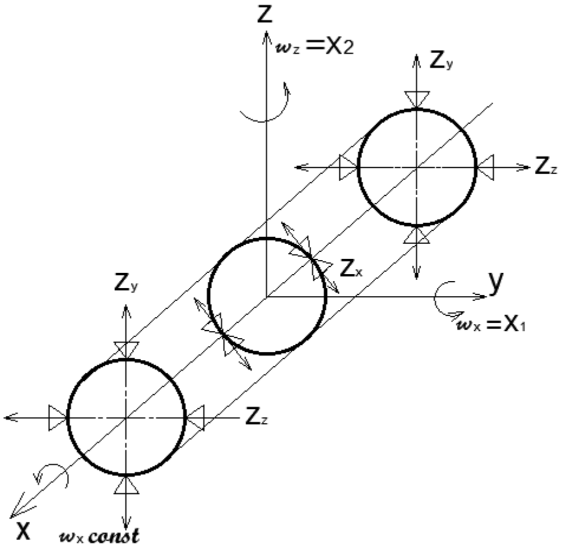

Figure 1 – Location of jet engines on an axisymmetric spacecraft

Introduce the following assumptions:

– spacecraft is axisymmetric with respect to the axis $0x(Iz = IY = I; M_y = M_z = M)$;

– the perturbing forces of the M_{ip} in comparison with the control moments can be ignored $M_{ip} = 0(i = x, y, z)$;

– the angular velocity of the spacecraft around the symmetry axis 0x is constant $(\omega_x(t) = \omega_{x0} = \text{const} = \text{c})$.

Such assumptions may be due to the creation of artificial gravity on the spacecraft. The idea of artificial gravity due to the rotation of an axisymmetric cylindrical spacecraft based on the principle of equivalence of the force of gravity and the force of inertia. In other words, if the inert mass and gravitational mass are equal, it is impossible to distinguish which force acts on the body-the gravitational or inertia force, i.e. the centrifugal force will “push” the astronaut away from the center of rotation, and he will be able to stand on the “floor”.

This article considers the problem of maintaining the angular orientation of the spacecraft during sudden deviations of the angular velocities from impulsive external disturbances. Its solution is proposed to be carried out with the sequential execution of two interrelated tasks that guarantee the minimization of fuel consumption for control:

– damping sudden deviations of the angular velocities of the spacecraft (task 1);

– stabilization of the angular position of the spacecraft (task 2).

The relevance of this problem statement is obvious, since the spacecraft have a limited supply of fuel.

It should be noted that this approach is explained by the fact that in practice, due to the presence of measurement errors and delays in the processing of control signals, it is impossible to reduce strictly to zero the impulse perturbations of the angular velocities. In real conditions, with relay control, this leads to the occurrence of undamped self-oscillations at the end point of control, and, moreover, does not guarantee the preservation of the original angular orientation of the spacecraft.

1 PROBLEM STATEMENT

Task 1. To solve task 1, only the system of equations (2) is considered. The system of equations (2) is sufficient to describe the angular movements of the spacecraft, if the influence of internal moments acting on it can be ignored [3, 5, 6].

Introduce the following notation:

$$x_1 = \omega_y; x_2 = \omega_z; a = \omega_{x0}\frac{(I - I_x)}{I};$$
$$\beta = \frac{M}{I}; u_1 = \frac{M_y}{M}; u_2 = M_z / M. \quad (4)$$

Taking into account (4), the system (2) will take the form:

$$\frac{dx_1(t)}{dt} = ax_1(t) + \beta u_1(t); \quad (5)$$

$$\frac{dx_2(t)}{dt} = ax_2(t) + \beta u_2(t). \quad (6)$$

The boundary conditions of the optimization problem are:

$$x_1(0) = x_{10}; x_2(0) = x_{20}; x_1(T) = x_2(T) = 0\,. \quad (7)$$

Finally, as is customary in many works on optimal control, we will consider normalized controls for the convenience of analyzing the results obtained:

$$|u_1(t)| \le 1; |u_2(t)| \le 1. \quad (8)$$

Task 1 of the optimal fuel consumption damping of sharp deviations of the angular velocities of the spacecraft is formulated as follows: to determine from the permissible range (8) the control actions $u_1(t)$ and $u_2(t)$ and the boundary conditions (7) that ensure the transfer of the system (5), (6) at the stabilization time interval $0 \le t \le T$ from an arbitrary initial state x_{10}, x_{20} to a given final state, defined by some area $|x_1(T)| \le \varepsilon$, $|x_2(T)| \le \varepsilon$, and minimize the quality functional:

$$I_K = \int_0^T \left[k + \sum_{i=1}^{2} |u_i(t)|\right] dt,$$
$$T - not \;\; fixed, 0 \le k \le \infty. \quad (9)$$

In other words, the task 1 is to extinguish sharp deviations of the angular velocities ω_y and ω_z from their zero values.

Task 2. To solve task 2 the system of equations (2) and (3) for the spacecraft (Fig. 1) is considered under assumptions (4) and the following notation:

$$x_1 = \omega_y; x_2 = \omega_z; x_3 = \psi; x_4 = \varphi;$$
$$\alpha = \omega_{x0}\frac{(I - I_x)}{I}; \beta = \frac{M}{I}. \quad (10)$$

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Taking into account (10) and the values of trigonometric functions for small values of their parameters, system (2), (3) takes the form:

$$\begin{aligned} \frac{dx_1}{dt} &= ax_2 + \beta u_1; \\ \frac{dx_2}{dt} &= -ax_1 + \beta u_2; \\ \frac{dx_3}{dt} &= x_1 + cx_4; \\ \frac{dx_4}{dt} &= x_2 - cx_3. \end{aligned} \qquad (11)$$

We accept as boundary conditions:

$$x_i(0) = x_{i0}; x_i(T) = 0 (i = 1,2,3,4). \qquad (12)$$

We also assume that system (11) is completely controllable and completely observable. To assess the quality of transient processes of spacecraft orientation stabilization, we will use a quadratic functional of the form

$$\begin{gathered} x_1(0) = x_{10}; x_2(0) = x_{20}; x_1(T) = x_2(T) = 0, \\ F(x,u) = \int_0^{\infty} (x^T Gx + u^T Qu)dt. \end{gathered} \qquad (13)$$

Task 2 is formulated as follows: to find the optimal values of the control u_1, u_2, which transfer the system (11) from the given initial values of the variables to the final ones according to the boundary conditions (12) and minimize the functional (13).

2 REVIEW OF THE LITERATURE

Quite a lot of works are devoted to the problems of optimal control of space vehicles. In particular, in [7], an optimal control law was obtained that stabilizes the program motion of the spacecraft. The stabilizing properties of the proposed regulators are proved by the Lyapunov method. In [8], an approximate optimal method for stabilizing the relative motion of a spacecraft is proposed, based on the dynamic programming method and the averaging method. In [9] the problem of control efficiency was solved with priorities in managing based on Pontryagin maximum and the mathematical apparatus of quaternions. In [10], a block diagram was developed for monitoring the angle of rotation of the steering mechanism and its angular velocity for the operation of disturbing forces. In [11], an anti-perturbing inverse control scheme for the movement and rotation of a rigid spacecraft with external disturbances and drive limitation was implemented. In [12], the maneuver of satellites with a minimum orientation time is studied using Control Moment Gyros (CMG) gyroscopes. In [13], an optimal energy-saving problem for a rendezvous mission based on linear-quadratic optimization is considered. In [14], a control law with output feedback was proposed for the problem of spacecraft reorientation based on the Lyapunov method. In [15], a significant control of the engine switch was implemented for an accurate study of the spacecraft. The article [16] proposes a PID controller architecture for controlling the attitude of a spacecraft, and the concepts of the nonlinear control theory H_∞ are applied to obtain stability properties.

It should be noted that despite the significant number of articles related to the control of the angular motion of spacecraft, in contrast to the problem of linear quadratic optimization and minimum transient time, there are practically no articles on the problems of optimizing fuel consumption during orientation and stabilization of the angular position of spacecraft. In this sense, we can note the work [17], which considers the problem of the optimal turn of the spacecraft. Turnaround time is kept to a minimum, as is the functionality that matters in terms of fuel consumption.

This article proposes algorithm for stabilizing the initial orientation of a spacecraft in the event of sudden disturbances based on the sequential solution of the above two interrelated tasks.

3 MATERIALS AND METHODS

The choice of functional (9) is scientifically attractive in the sense that it provides the necessary compromise between the fuel consumption and the stabilization time of the angular position of the spacecraft by the given value *k*. In this case, for *k*=0, the problem of pure fuel consumption is solved, and for *k*→∞, the problem of minimizing the transition process time is solved. It should be noted that in the case of pure fuel consumption, when *k*=0 in the functional (9), the condition for existence of optimal control is the condition $T >$, $T_{\min}$, when *k*→∞ in functional (9).

To solve the task 1 use the mathematical apparatus of the maximum principle (in our case, the minimum principle) [4, 9] and the phase space method (in our case, the phase plane) [18, 19, 24]. According to the minimum principle [4], the Hamiltonian of this task has the form:

$$H = k + \beta|u_1| + \beta|u_2| + \psi_1(ax_2 + \beta u_1) + \psi_2(-ax_2 + \beta u_2). \qquad (14)$$

It follows from the analysis of the Hamiltonian (14) that the controls that minimize it satisfy the following conditions:

$$u_i(t) = 0, if \left|\psi_i(t)\right| \le 1; \qquad (15)$$

$$u_i(t) = -sig\psi_i(t), if \left|\psi_i(t)\right| < 1; \qquad (16)$$

$$0 \le u_i(t) \le 1, if \left|\psi_i(t)\right| = -1; \qquad (17)$$

$$-1 \le u_i(t) \le 0, if \left|\psi_i(t)\right| = 1. \qquad (18)$$

Due to the non-degeneracy of this problem, which is quite easy to justify in accordance with [4], conditions (17) and (18) are excluded from consideration. The optimal values of the control actions $u_1(t), u_2(t)$ from (15), (16) are clearly illustrated in Fig. 2, from which it follows that the following control sequences are optimal

$$\begin{gathered} \ldots -1. -1 \to -1,0 \to -1,1 \to 0,1 \to -1,1 \to 0,1 \\ \to 1,1 \to 1,0 \to 1, -1 \to 0, -1 \to -1,1 \ldots \end{gathered} \qquad (19)$$

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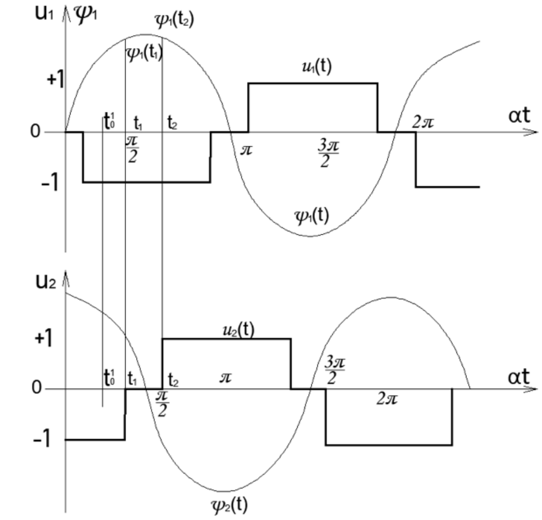

Figure 2 – Optimal sequence of values of control actions

It should be noted that [4] provides a strict justification for the structure of optimal fuel consumption processes with three-level control. Moreover, as in our case, in the presence of complex-conjugate roots, the number of switches depends on the initial conditions. By entering the inverse time *z=T–t*, integrate the system for $u_1 = +/-1, u_2 = +/-1$ and, excluding z in the obtained solutions, obtain circles on the phase plane (αx_1, αx_2), which are described by equations:

$$[ax_1 - (2i-1)\beta u_2]^2 + (ax_2 + \beta u_1)^2 = 2\beta^2; u_1 = -u; \tag{20}$$

$$(ax_1 - \beta u_2)^2 [ax_2 - (2i-1)\beta u_1]^2 = = 2\beta^2; u_1 = -u_2 = +/-1. \tag{21}$$

The area of turning off one of the control actions in sequence (19) can be found as the locus of displaying the points of the switching lines found above for the time-optimal control system (Fig. 3). Omitting intermediate mathematical calculations, the optimal disabling curves of one of the controls in the sequence (19) are determine in the coordinate system $(\tilde{x}_1, \tilde{x}_2)$

$$\begin{aligned} \tilde{x}_1 &= ax_1 \cos\varphi + ax_2 \sin\varphi; \\ \tilde{x}_2 &= ax_2 \cos\varphi + ax_1 \sin\varphi; \\ \varphi &= \frac{\text{arctg}\beta}{k+\beta}, \end{aligned} \tag{22}$$

i.e. they are described as arcs of circles

$$\left(\tilde{x}_2 + \alpha_j^i u_l^i\right)^2 + \left(\tilde{x}_1 + b_j^i u_2^i\right)^2 = \left(R_j^i\right)^2. \tag{23}$$

On Fig. 3 the disabling curves of one of the control actions in the sequence (19) are denoted as $0Q_j^i K_j^i L_j^i \ldots (i = 1,2,3,4)$. Here are the values of optimal controls for each of the regions of the phase plane (ax_1, ax_2). The value of the coefficient k in the functional (9) is selected based on the practical requirements for fuel consumption and stabilization time. In addition, Fig. 3 also shows the optimal phase trajectory, which is a spiral from point $A(ax_{20}, ax_{10})$ to the origin. It consists of arcs of a circle with a center and radius determined by the values of optimal controls from the area of the phase plane, where the point of the current values of the angular velocities of the spacecraft is located.

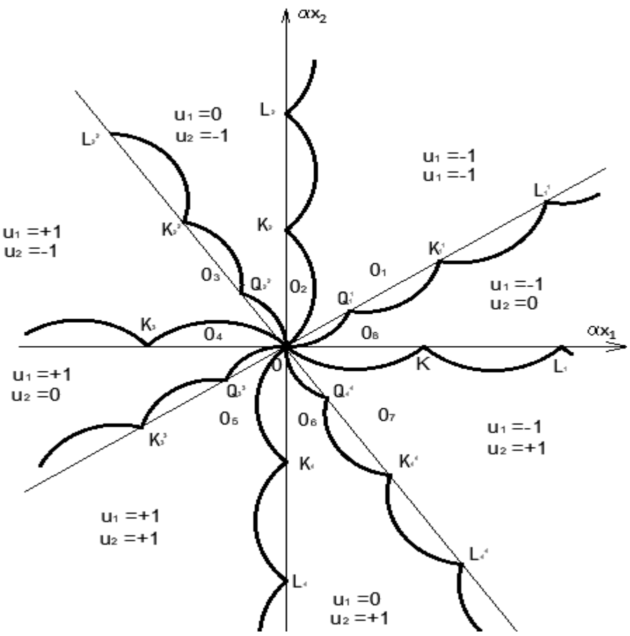

Figure 3 – Phase portrait of the optimal control system

In task 2 the controls u_1, u_2 are determined by a linear combination of deviations *xi* (i=1,…,4), i.e:

$$u_k = \sum p_{ki} x_i (i = 1,...,4; k = 1,2); \tag{24}$$

It is known that the main problem of the analytical design of optimal controllers is the absence of a direct relationship between the coefficients of the functional and the dynamic indicators of transient stabilization processes. Therefore, in this paper, it is proposed to use the modal synthesis of optimal control based on the method of indefinite coefficients [20], which in this article is generalized to the vector case of control using the superposition principle. Write system (11) in the following vector form:

$$\frac{dx}{dt} = Ax + Bu. \tag{25}$$

Without losing the generality of the results obtained below, we use the principle of superposition to determine the required coefficients $p_{ki} (i = 1,...,4; k = 1,2)$.

Step 1. Accept $(u = u_{1,0}), u_1 = u$. It is known that for completely observable systems of the form (25) in the case of a quadratic performance criterion (13), the extre-

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mal control u is a linear state function. We write control of the system (25) in vector form:

$$u = \overline{p}^T \overline{x}; \tag{26}$$

To determine the vector of feedback coefficients $\overline{p}$, we use the modal approach proposed by the authors in [20] based on the method of indeterminate coefficients.

Modal synthesis is based on the fact that the vector of feedback coefficients $\overline{p}$ can be chosen in such a way that the poles of the closed system (25) will be located at given arbitrary points that provide the required dynamic properties of transient processes of stabilizing the angular orientation of the spacecraft [21, 22].

In the general case, it was proved in [20] that the unknown coefficients in the optimal control law (26) enter linearly into the expression for the coefficients of the characteristic polynomial of the closed system (25), i.e.:

$$H(\lambda) = \lambda^n + \left(\sum_{i=1}^{n} c_{n-1,i} p_i + d_{n-1}\right)\lambda^{n-1} + \ldots + \left(\sum_{i=1}^{n} c_{0,i} p_i + d_0\right), \tag{27a}$$

or

$$H(\lambda) = \lambda^n + \left(\overline{c}_{n-1}^T \overline{p} + d_{n-1}\right)\lambda^{n-1} + \ldots + \left(\overline{c}_0^T \overline{p} + d_0\right) \tag{27b}$$

Characteristic determinant of the closed system (25) has next form:

$$\det(\lambda) = \left|A + B\overline{p}^T - I\right|\lambda = \begin{vmatrix} a_{11} + b_1 p_1 - \lambda \ldots a_{1j} p_j \ldots a_{1n} + b_1 n \\ a_{ji} + b_j p_1 \ldots a_{jj} + b_j p_j - \lambda \ldots a_{jn} + b_j p_n \\ a_{n1} + b_n p_1 \ldots a_{nj} + b_n p_j \ldots a_{nn} - \lambda \end{vmatrix}. \tag{28}$$

Define the unknown parameters $c_{ji}, d_j (j = \overline{0, n} - 1; i = \overline{1, n})$ using the undetermined coefficients method [23]. To do this, we put $p_i = 0(i = \overline{1, n})$ in the characteristic determinant (28) at the first step and explanded characteristic determinant (28) by one of the known numerical methods [23]. The coefficients found for different powers of λ determine the unknown coefficients $d_i = (j = \overline{01 - n})$ in the expressions for the characteristic polynomial of the closed system for the corresponding powers of λ. In the next n steps, setting sequentially one of the coefficients $p_i = (i = \overline{1, n})$ equal to one while others remain zero and explanded the characteristic determinant, we obtain expressions for the unknown parameter c_{ji} for the corresponding power $\lambda^j = (j = \overline{0, n-1})$ in the characteristic polynomial of the closed system:

$$c_{ji} = (f_j - d_i), \tag{29}$$

On the other hand, the characteristic polynomial of the closed system (25) with desired roots $\lambda_1, \lambda_2, \ldots \lambda_n$ has the form:

$$F(\lambda) = \prod_{i=1}^{n} (\lambda - \lambda_i) = \sum_{j=0}^{n-1} 1_j \lambda^j + \lambda^n. \tag{30}$$

As a result, taking into account (27b), (29), (30) to determine the feedback coefficients, we obtain a system of linear algebraic equations:

$$col\left(\overline{c}^T, \overline{c}^T, \ldots, \overline{c}^T\right)\overline{p} = \overline{1} - \overline{d}, \tag{31}$$

where $i = \left(1_{n-1}, 1_{n-1}, \ldots, 1_0\right) d = \left(d_{n-1}, d_{n-2}, \ldots, d_0\right)$.

According to this approach, for the chosen poles $\lambda_1, \ldots \lambda_4$ of a closed optimal stabilization system, the control u1 is defined as:

$$u_2, p_{11} x + \ldots + p_{14} x_4. \tag{32}$$

The resulting controls (32) and (33) are components of the optimal vector control (26).

Step 2. We close system (25) to the found control u1 from (32). We accept u=(0,u2, 0,…, 0), u2=u. Similarly to step 1, using the modal approach based on the method of uncertain coefficients, we obtain for the same poles the optimal control u2 in the form:

$$u_2 = p_{11} x_1 + \ldots + p_{24} 4 x_4. \tag{33}$$

4 EXPERIMENTS

Without losing the generality of the results obtained, to simulate the optimal stabilization problems considered above, the following values of the parameters of equations (5), (6), (11) α = β =c= 1 were accepted.

The simulation was aimed at research in order to confirm the effectiveness of the proposed combined approach to the angular stabilization of the spacecraft, as well as the impossibility of maintaining the original angular orientation only by solving task 1. In addition, the processes of angular stabilization were studied in the presence of errors in assessing the state of the spacecraft and delays in the control loops u_1, u_2. The simulation was carried out in the Mathcad environment. Evaluate the process of angular stabilization of the spacecraft when the values of angular velocities enter the region of specified values of task 2.

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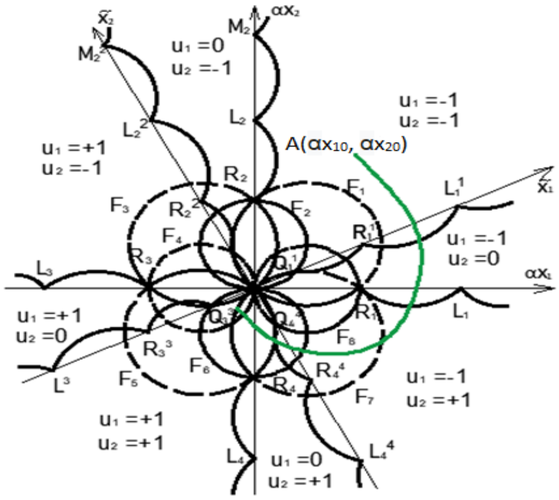
5 RESULTS

For task 1, based on equations (5), (6) at α = β = 1, the dynamics of the transition process was simulated from the given initial conditions $x_1(0) = 3[0/s]; x_2(0) = 4[0/s]; x_3(0) = 0; x_4(0) = 0$ to zero final ones $x_1(T) = x_2(T) = 0[0/s]$ (Fig. 4).



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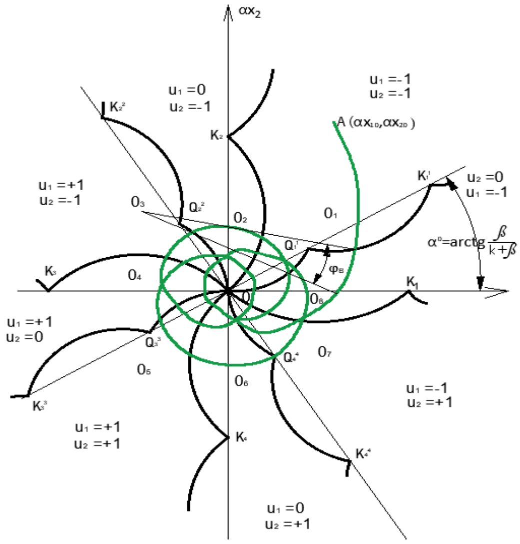


b

Figure 4 – Phase portraits and phase trajectories of the transient process in the absence (Fig. 4a) and presence (Fig. 4b) of delay in the control loop

Also, for the initial ones $x_1(0) = 3[0/s]; x_2(0) = 4[0/s]; x_3(0) = 0; x_4(0) = 0$, and final $x_1(T) = x_2(T) = 0[0/s]$ conditions, based on equation (11), modeling of the dynamics of the transient process in the time domain was carried out (Fig. 5 and Fig. 6), if there is an error in assessing the state spacecraft. Here $z_{n,j} = x_i$, and the real time t_{real} is determined through the model t_{mod} as $t_{real} = 0.02 t_{\text{mod}} = 0.02t$.

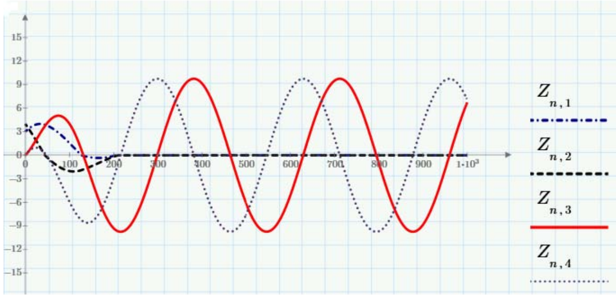


Figure 5 – Continuous self-oscillations x_3, x_4

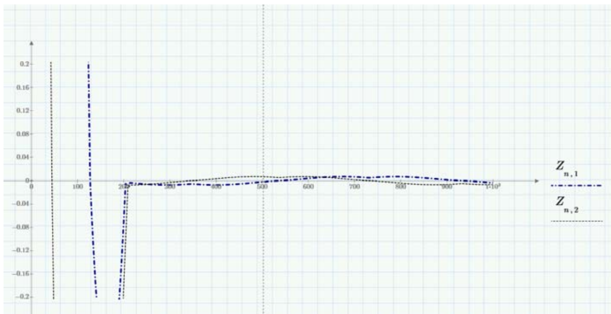
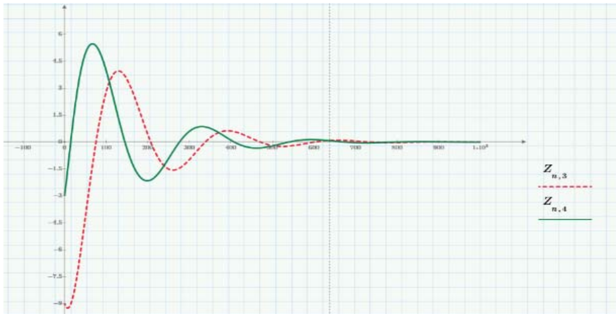
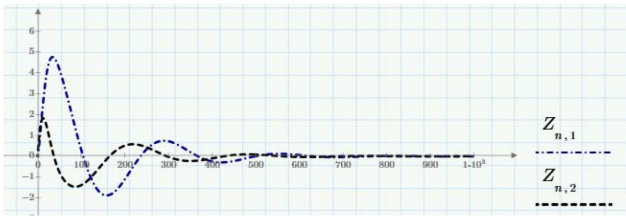


Figure 6 – Continuous self-oscillations x_1, x_2

For task 2, the system of equations (11) was simulated with the values $\alpha - \beta = c = 1; \varepsilon = 0.2[0/s]$ with initial conditions that were final for task 1, i.e. $x_1(T) \le 0.2[0/s]; x_2(T) \le 0.2[0/s]$; and finite zero coordinates $x_1(\infty) = x_2(\infty) = x_3(\infty) = x_4(\infty) = 0$ At the same time, a version of the optimal control actions u_1 and u_2 was synthesized, obtained for the desired spectrum of roots $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} = \{-0.1, -0.5, -1, -2\}$ based on the above modal approach. Graphs of transient processes are shown in Fig. 7a, b.



a



b

Figure 7 – Transient graphs: a) x_3, x_4; b) x_1, x_2

6 DISCUSSION

With all the obvious advantages of the synthesized optimal fuel consumption law for stabilizing the angular position of the spacecraft (task 1), in which, by choosing

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the value of k in (9), it is possible to vary between the time of the transition process and fuel consumption, in real conditions with relay control in the region of zero coordinates undamped self-oscillations may occur in the presence of unaccounted errors in assessing the state of the spacecraft (Fig. 5) and/or delay (Fig. 4b) in the formation of optimal control actions u_1 and u_2. The error in assessing the state of the spacecraft is simulated by approximating the optimal switching curves with straight lines. In Fig. 6 for greater clarity the presence of self-oscillations x_1 and x_2 , the scale of Fig. 5 along the vertical axis. In addition, the use of only the algorithm of task 1 does not guarantee the preservation of the original angular orientation of the spacecraft in the region of zero values x_1 and x_2 at t ≈4c. (Fig. 5). When implementing task 2, it is possible to avoid the occurrence of undamped self-oscillations, ensure the specified dynamic properties of transient processes in the region of zero coordinates and restore the stable initial angular orientation of the spacecraft, i.e. $x_1(0) = 0(i = 1,2,2,4)$. Thus, the combined approach proposed in the article to solving the problem of optimal stabilization of the angular position of the spacecraft allows to optimally damp sudden deviations in angular velocities in terms of fuel consumption and to maintain the initial orientation of the spacecraft.

CONCLUSION

This article discusses the problem of maintaining the angular orientation of a spacecraft during sharp deviations of angular velocities from pulsed external disturbances. Its solution is proposed to be carried out by sequentially performing two interrelated tasks: damping sharp deviations in the angular velocities of the spacecraft (task 1); stabilization of the angular position of the spacecraft in the region of zero coordinates (task 2). As part of the solution to task 1, based on the methods of the Pontryagin maximum principle and the phase space (in this case, the phase plane), optimal switching curves were synthesized that unambiguously divide the phase plane into regions with the corresponding values of optimal controls. To solve task 2, a modal approach based on the method of undetermined coefficients is proposed. This approach makes it possible to provide specified dynamic indicators of transient processes for stabilizing the angular position of the spacecraft.

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DECLARATIONS

Conflict of interest. The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship, or otherwise, that could affect the research and its results presented in this paper.

Authors' contributions. **Alexandr Stenin** developed the conceptual framework and methodology based on Pontryagin's maximum principle and provided overall supervision. **M. Soldatova** performed the formal mathematical analysis and investigated the integration of the two-stage stabilization approach. **V. P. Pasko** was responsible for the writing of the original draft and describing the spacecraft orientation processes. **Drozdovich** developed to performed the visualization of results, and served as the project administrator for the final manuscript preparation.

Data availability. The manuscript has no associated data

Software availability. The manuscript has no associated software

Use of artificial intelligence tools. The authors confirm that they did not use artificial intelligence technologies in creating the submitted work.

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ОПТИМІЗАЦІЯ ВИТРАТ ПАЛИВА В ЗАДАЧІ СТАБІЛІЗАЦІЇ КУТОВОГО ПОЛОЖЕННЯ ОСЕСИМЕТРИЧНОГО КОСМІЧНОГО АПАРАТУ

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АНОТАЦІЯ

Актуальність. Проблема підтримки кутової орієнтації космічного апарата є критичною, особливо в умовах імпульсних зовнішніх збурень, що спричиняють різкі відхилення кутових швидкостей. Актуальність розв'язання цієї задачі визначається обмеженим запасом палива на борту, зокрема для класу космічних апаратів, призначених для забезпечення штучної гравітації, де точне та ефективне керування має першочергове значення.

Мета роботи. Основною метою цієї роботи є мінімізація споживання енергетичних ресурсів (палива) для стабілізації кутового положення певного класу космічних апаратів. Ця мета досягається шляхом послідовного виконання двох взаємопов'язаних завдань: 1) демпфування різких відхилень кутових швидкостей космічного апарата; 2) стабілізації кінцевого кутового положення.

Метод. Запропоновано двоетапний підхід. Для розв'язання першого завдання (демпфування) синтезується оптимальне керування з використанням комбінації принципу максимуму Понтрягіна та методу фазової площини. Це дозволяє побудувати оптимальні криві перемикання, які однозначно поділяють фазову площину на області з відповідними значеннями оптимальних керувань. Для розв'язання другого завдання (стабілізації) використовується модальний підхід на основі запропонованого методу невизначених коефіцієнтів, що забезпечує задані динамічні показники перехідних процесів стабілізації.

Результати. Було проведено моделювання динаміки кутового руху космічного апарата. Результати моделювання підтверджують високу ефективність використання запропонованого комбінованого підходу для розв'язання задачі стабілізації кутового положення космічного апарата після значних зовнішніх збурень.

Висновки. Спільне застосування принципу максимуму Понтрягіна та методу фазової площини для паливно-ефективного демпфування кутових швидкостей, з подальшою реалізацією оптимального закону стабілізації на основі запропонованого методу невизначених коефіцієнтів, є ефективною процедурою керування орієнтацією та стабілізацією космічного апарата з мінімальними витратами палива.

КЛЮЧОВІ СЛОВА: осесиметричний КА, принцип максимуму, фазова площина, оптимальні лінії перемикання та відключення, модальний синтез, метод невизначених коефіцієнтів.

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