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COMPARISON OF SOME ORTHOGONAL FUNCTION SYSTEMS FOR KOLMOGOROV-WIENER FORECASTING OF MFSD PROCESS

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ABSTRACT

Context. We investigate such orthogonal function systems as the Walsh function one and the Bessel function one for the search of the weight function of the Kolmogorov-Wiener (KW) filter for the forecasting of the heavy-tail MFSD (multifractal fractional sum-difference) random process. The results for the MAPE (mean absolute percentage error) of the misalignment of both sides of the Wiener-Hopf integral equation are compared to that obtained by the Chebyshev polynomial expansion in our previous paper.

Objective. The objective is to derive the weight function of the KW continuous filter via the truncated expansions in Walsh and Bessel functions and to compare results with the results obtained via the Chebyshev polynomials in our previous paper.

Method. Galerkin method with the Walsh functions and the Bessel functions orthogonal on the required interval as the method basis is used.

Results. It is shown that the choice of Walsh functions leads to better results for the above-described MAPE than the choice of the Chebyshev polynomials and Bessel functions. It is shown that the choice of the orthogonal polynomials is more effective than the choice of the Bessel functions. It is also obtained that of the approximation of 128 Walsh functions leads to the MAPE less than 0.5%.

Conclusions. The weight function of the KW filter in the continuous case is investigated for the forecasting of a random stationary heavy-tail process in the MFSD model. The Walsh functions and Bessel functions are chosen to be the basis of the Galerkin method described in the paper. The results are compared to that obtained with the choice of the Chebyshev polynomials. It is obtained that the Walsh functions lead to the most reliable results among the above-mentioned orthogonal function systems. The calculated results may be applied for the practical forecasting of traffic in telecommunications and also they may be applied to the treatment of random processes in other fields of knowledge: in agriculture, etc.

KEYWORDS: Kolmogorov-Wiener filter weight function, Walsh functions, telecommunication traffic, MFSD model, Bessel functions, Galerkin method.

ABBREVIATIONS

KW is a Kolmogorov-Wiener;
WH is a Wiener-Hopf;
GFSD is a Gaussian fractional sum-difference;
MFSD is a multifractal fractional sum-difference;
MAPE is a mean absolute percentage error;
FGN is a fractional Gaussian noise.

NOMENCLATURE

T is an interval where the input data are taken;
$\mathrm{p/s}$ packets per second;
z is a length of the interval where the forecasting is provided;
$h(t)$ is a weight function of the KW filter;
α is a rate of the packet transfer;
θ is a quantity which varies with the rate of the packet transfer;
ξ is a quantity which varies with the rate of the packet transfer;
λ is a quantity which varies with the rate of the packet transfer;
$R(t)$ is a correlation function of a MFSD process;
$\rho(t)$ is a correlation function of a GFSD process;
$\Gamma(x)$ is a gamma function;
d is a fractional differencing parameter;
a is a constant;
b is a constant;

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n is an amount of Bessel and Walsh functions in the approximations;

g_s are coefficients on the Walsh functions and the Bessel functions;

$\mathrm{wal}_s(t)$ are Walsh functions which are orthogonal on $t \in [0, T]$;

$J_\nu(x)$ is a Bessel function;

$\mu_{\nu i}$ are Bessel functions zeros;

$\mathrm{Left}(t)$ is a left-hand side of the WH integral equation;

$\mathrm{Right}(t)$ is a right-hand side of the WH integral equation;

G_{ks} are integral brackets;

B_k are free terms;

N is an amount of points for the numerical integration;

$W_{jl}^{(n)}$ are Walsh functions values;

$X_l(t)$ are auxiliary integrals;

V_{ls} are auxiliary integrals;

Q_s are auxiliary integrals.

INTRODUCTION

Telecommunication traffic is considered as a random heavy-tail process; see, for example, [1–3]. Accurate traffic forecasting is urgent for effective network management such as resource allocation, traffic scheduling, detection of anomalies, etc., see [4–6].

There are many approaches to traffic forecasting, see [7–10]. Recently a couple of models of the stationary heavy-tail traffic were proposed, such as the generalized FGN model [11, 12], the MFSD and GFSD models [13], etc.

Recently we investigated such a method of traffic forecasting as the KW filter. This method is rather simple and is applicable to the forecasting of a heavy-tail stationary process if the process is enough smooth [14]. As is known (see, for example, [14]), if the data amount is large, one may apply the continuous KW filter instead of the discrete one, and the weight function of the filter obeys the WH integral equation, which is the Fredholm integral equation of the first kind. Such an equation may be solved on the basis of a truncated expansion in the orthogonal function system (the so-called Galerkin method). The Galerkin method is widely used for telecommunications [15] and for other fields of knowledge (see [16, 17]).

Recently we developed the theory for the KW filter construction for GFSD and generalized FGN process forecasting [18, 19]. The investigation for the MFSD model was also made with the use of the orthogonal Chebyshev polynomials [15]. However, a question occurs may another orthogonal system enhance the misalignment of the sides of the WH integral equation for the derived solutions. Moreover, the forecasting described in [14] was made with the help of the orthogonal Walsh functions. So, in this paper we solve the WH equation via the Walsh functions and Bessel functions and compare the results with that based on the polynomial functional system [15].

The object of study is the continuous KW filter applied for the forecasting of the MFSD process.

The subject of study is the KW filter weight function.

The aim of the work is to derive approximate solutions for the weight function via the Galerkin method with the choise of the Walsh functions and Bessel functions as the basis and to compare the results with that based on the polynomial functional system.

1 PROBLEM STATEMENT

Weight function of the KW filter if the filter is a continuous one is the solution of the WH integral equation (see [14]):

$$\int_0^T d\tau h(\tau) R(t-\tau) = R(t+z). \tag{1}$$

The problem statement is to obtain approximate solutions for the unknown weight function $h(\tau)$ from the equation (1) via the Galerkin method with the choice of the Walsh functions and Bessel functions as the basis and to compare the results with that based on the polynomial system [15].

2 REVIEW OF THE LITERATURE

In our recent papers we investigated the KW filter for the forecasting of stationary processes in different models such as the GFSD model [18], generalized FGN model [19], the MFSD model [15], etc. In [15] the WH integral equation was treated on the basis of the Chebyshev polynomial expansion. The corresponding MAPE of the misalignment of both sides of the WH integral equation is lower than 1% but higher than 2%, see [15], here the results of the thriteen-polynomial approximation are given. The increasing of the number of polynomials does not lead to the increasing of the corresponding accuracy, maybe it is connected with the fact that the corresponding integrals may be calculated only approximately. Moreover, the problem of the multiplying of very high and very low numbers may be present in the above-mentioned polynomial approach. However, for example, the choice of the Walsh functions for the forecasting of a generalized FGN may lead to the MAPE less than 1% [19]. So, a question arises whether the use if a non-polynomial system of orthogonal functions may enhance the corresponding MAPE results for the traffic in the MFSD model. So, in this paper we deal with two non-polynomial orthogonal function sets: Walsh functions and Bessel functions. We compare the MAPE results for the above-mentioned function sets with the results obtained for the Chebyshev polynomials in [15].

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3 MATERIALS AND METHODS

The correlation function of a discrete MFSD process is as follows [13]:

$$R(t)=\frac{e^{\xi(\alpha)\rho(t)}-1}{e^{\xi(\alpha)}-1}, \quad (2)$$

where $\Gamma(x)$ is the Gamma function (see definition in [20]), the other quantities which take place in (2) are written in (3), (4):

$$\rho(t)=\left(1-\theta(\alpha)\right)\frac{2(1-d)t^2-(1-d)^2}{t^2-(1-d)^2}\times$$
$$\times\frac{\Gamma(1-d)}{\Gamma(d)}\frac{\Gamma(t+d)}{\Gamma(t-d+1)}, \quad (3)$$
$$\theta(\alpha)=\frac{2^{-7.21}\alpha^{0.75}}{2^{-7.21}\alpha^{0.75}+1},$$

and

$$\xi(\alpha)=\ln\left(\Gamma\left(1+\frac{2}{\lambda(\alpha)}\right)\right)-2\ln\left(\Gamma\left(1+\frac{1}{\lambda(\alpha)}\right)\right),$$
$$\lambda(\alpha)=\frac{2^{-5.36}\alpha^{0.63}}{2^{-5.36}\alpha^{0.63}+1}, \quad (4)$$

the packet rate $\alpha\in\left[2^{10.22}\,\mathrm{p/s},2^{17.5}\,\mathrm{p/s}\right]$, $d=0.31$, see [13]. In [13] it is stressed that the expressions (2) and (3) are valid for $t\ge 1$, so in [15, 18] the function $\rho(t)$ is redefined for the continuous case:

$$\rho(t)=\begin{cases} a|t|^b+1, |t|\le 1; \\ \left(1-\theta(\alpha)\right)\dfrac{2(1-d)t^2-(1-d)^2}{t^2-(1-d)^2}\times \\ \times\dfrac{\Gamma(1-d)}{\Gamma(d)}\dfrac{\Gamma(|t|+d)}{\Gamma(|t|-d+1)}, |t|\ge 1, \end{cases} \quad (5)$$

the values of the coefficients a, b are chosen according to the following statements:

$$\lim_{t\to 1-0}\rho(t)=\lim_{t\to 1+0}\rho(t),\ \lim_{t\to 1-0}\frac{d\rho(t)}{dt}=\lim_{t\to 1+0}\frac{d\rho(t)}{dt}. \quad (6)$$

In what follows we take the function $R(t)$ as (2) where $\rho(t)$ is taken on the basis of (5) and (6).

First o: all we describe the solution of integral equation (1) via the Galerkin method with the choice of the Walsh functions as the method basis. We seek the weight function as

$$h(\tau)=\sum_{s=1}^{n}g_s \mathrm{wal}_s(\tau), \quad (7)$$

where $\mathrm{wal}_s(\tau)$ are the Walsh functions in the Walsh numeration, their detailed description is given in [21]. The Walsh functions obey the orthogonally property [22]

$$\int_0^T \mathrm{wal}_s(\tau)\mathrm{wal}_k(\tau)d\tau=T\delta_{ks},\ \delta_{ks}=\begin{cases}1, k=s.\\ 0, k\ne s.\end{cases} \quad (8)$$

In such a case, as is known [21], the coefficients g_s are the solutions of the following system of equations:

$$\sum_{s=1}^{n}g_s G_{ks}=B_k,\ k=\overline{1,n},$$
$$G_{ks}=\int_0^T\int_0^T d\tau dt \mathrm{wal}_k(t)\mathrm{wal}_s(\tau)R(t-\tau), \quad (9)$$
$$B_k=\int_0^T dt \mathrm{wal}_k(t)R(t+z).$$

The correlation function $R(t)$ is an even one and the Walsh functions are piecewise constant functions, so similarly to [21] we obtain

$$G_{jk}=\sum_{l,s=1}^{n}W_{jl}^{(n)}W_{ks}^{(n)}V_{ls},$$
$$W_{jl}^{(n)}=\mathrm{wal}_j\left(\frac{(2l-1)T}{2n}\right), \quad (10)$$
$$V_{ls}=\int_{(l-1)T/n}^{lT/n}\int_{(s-1)T/n}^{sT/n}dtd\tau R(t-\tau),$$

see also [19]. The quantities V_{ls} and the integral brackets G_{ks} obey the properties

$$V_{ls}=V_{l+1,s+1},\ V_{ls}=V_{sl},\ G_{ks}=G_{sk}, \quad (11)$$

see [19, 21], so only V_{1l}, $l=\overline{1,n}$ should be calculated direclty, the other V_{ls} may be calculated on the basis of V_{1l} and (11). Moreover, only the integral brackets G_{ks} for $k\ge s$ should be calculated directly, the other brackets may be calculated on the basis of (11). The explicit expression for $R(t)$ is rather cumbersome, and the analyti-

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cal integration for V_{1l} can hardly be provided. So, they are calculated as:

$$V_{1l} = \int_0^{T/n} \int_{(l-1)T/n}^{lT/n} dt d\tau R(t-\tau) \approx \approx \sum_{i=0}^{N}\sum_{j=0}^{N} R\left(\frac{i}{N}\frac{T}{n} - \left(\frac{(l-1)T}{n} + \frac{j}{N}\frac{T}{n}\right)\right)\left(\frac{T}{n}\frac{1}{N}\right)^2, \quad (12)$$

$N \gg 1$, the value $N = 10^3$ is chosen. The Walsh functions are piecewise constant functions, so similarly to [21] we obtain

$$B_k = \sum_{s=1}^{n} W_{ks}^{(n)} Q_s, \; Q_s = \int_{(s-1)T/n}^{sT/n} dt R(t+z). \quad (13)$$

The analytical integration can hardly be provided, so we calculate Q_s as

$$Q_s = \int_{(s-1)T/n}^{sT/n} dt R(t+z) \approx \approx \frac{1}{2}\sum_{j=1}^{N}\left(R\left(\frac{(s-1)T}{n} + \frac{(j-1)}{N}\frac{T}{n} + z\right) + + R\left(\frac{(s-1)T}{n} + \frac{j}{N}\frac{T}{n} + z\right)\right)\frac{T}{n}\frac{1}{N}, \quad (14)$$

$N \gg 1$, the value $N = 10^3$ is chosen.

The integral brackets G_{ks} are calculated via (10)–(12), the quantities B_k are calculated via (13) and (14), and with the help of (9) we calculate g_s:

$$\begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{pmatrix} = \begin{pmatrix} G_{11} & G_{21} & \cdots & G_{n1} \\ G_{12} & G_{22} & \cdots & G_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ G_{1n} & G_{2n} & \cdots & G_{nn} \end{pmatrix}^{-1} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix}. \quad (15)$$

The accuracy of approximate solutions is estimated vai the MAPE:

$$\text{MAPE} = \frac{1}{T}\int_0^T \left|\frac{\text{Left}(t) - \text{Right}(t)}{\text{Right}(t)}\right| dt \cdot 100\%, \quad (16)$$

where

$$\text{Left}(t) = \int_0^T d\tau h(\tau) R(t-\tau), \; \text{Right}(t) = R(t+z), \quad (17)$$

are the sides of the equation (1). The Walsh functions are piecewise constant functions, so similarly to [21] we obtain

$$\text{Left}(t) = \sum_{l=1}^{n} h_l X_l(t), \; h_l = h\left(\frac{(2l-1)T}{2n}\right), \quad X_l(t) = \int_{(l-1)T/n}^{lT/n} d\tau R(t-\tau), \quad (18)$$

the function $h(\tau)$ has the form (7), the calculation of the coefficients g_s is described in detail in what precedes. The analytical integration can hardly be provided, so the following approximate expressions are used:

$$X_l(t) = \int_{(l-1)T/n}^{lT/n} d\tau R(t-\tau) \approx \approx \frac{1}{2}\sum_{j=1}^{N}\left(R\left(t - \left(\frac{(l-1)T}{n} + \frac{(j-1)}{N}\frac{T}{n}\right)\right) + + R\left(t - \left(\frac{(l-1)T}{n} + \frac{j}{N}\frac{T}{n}\right)\right)\right)\frac{T}{n}\frac{1}{N}, \quad (19)$$

$N \gg 1$, the value $N = 10^3$ is chosen. The MAPE (16) is roughly estimated as

$$\text{MAPE} \approx \frac{1}{N}\sum_{j=0}^{N-1}\left|\frac{\text{Left}\left(\frac{jT}{N}\right) - \text{Right}\left(\frac{jT}{N}\right)}{\text{Right}\left(\frac{jT}{N}\right)}\right| \cdot 100\%, \quad (20)$$

$N \gg 1$, the value $N = 10^2$ is chosen, the same choice is made in [15]. The MAPE results for the above-described solutions and corresponding comparison graphs of the left-hand side and of the right-hand side are given in the next section.

Now let us describe the Galerkin method via the Bessel functions. The definition of the Bessel functions $J_\nu(x)$ is given in [23], they obey the orthogonality property [24]

$$\int_0^T x J_\nu\left(\mu_{\nu i}\frac{x}{T}\right) J_\nu\left(\mu_{\nu j}\frac{x}{T}\right) dx = \frac{T^2}{2} J_{\nu+1}^2(\mu_{\nu i})\delta_{ij}, \quad (21)$$

see the definition of δ_{ij} in (8), $\mu_{\nu i}$ are the zeros of the function $J_\nu(x)$. In this paper we investigate only the use of $J_0(x)$ functions. We seek the solution in the form

$$h(\tau) = \sum_{s=1}^{n} g_s J_0\left(\mu_{0s}\frac{\tau}{T}\right), \quad (22)$$

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here and in what follows the coefficients g_s are redefined according to (22). Similarly to the obtaining of (9) it can be derived that the coefficients g_s obey the system of equations

$$\sum_{s=1}^{n} g_s G_{ks} = B_k, \ k = \overline{1,n},$$

$$G_{ks} = \int_0^T \int_0^T d\tau dt J_0\left(\mu_{0k}\frac{t}{T}\right) J_0\left(\mu_{0s}\frac{\tau}{T}\right) R(t-\tau), \quad (23)$$

$$B_k = \int_0^T dt J_0\left(\mu_{0s}\frac{t}{T}\right) R(t+z),$$

quantities G_{ks} and B_k are redefined according to (23). The integral brackets are calculated as

$$G_{ks} = \int_0^T \int_0^T d\tau dt J_0\left(\mu_{0k}\frac{t}{T}\right) J_0\left(\mu_{0s}\frac{\tau}{T}\right) R(t-\tau) \approx$$
$$\approx \left(\frac{T}{N}\right)^2 \cdot \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\left(J_0\left(\mu_{0k}\frac{i+0.5}{N}\right) J_0\left(\mu_{0s}\frac{j+0.5}{N}\right)\times \right. \quad (24)$$
$$\left. \times R\left((i-j)\frac{T}{N}\right)\right),$$

$N \gg 1$, the value $N = 3\cdot 10^3$ is chosen for $\alpha = 2^{11}$ p/s and $\alpha = 2^{13}$ p/s; the value $N = 6\cdot 10^3$ is chosen for $\alpha = 2^{15}$ p/s and $\alpha = 2^{17}$ p/s, the same values for N are chosen in [15] where the Galerkin method was investigated via the orthogonal polynomials.

The free terms B_k are calculated as follows:

$$B_k = \int_0^T dt J_0\left(\mu_{0s}\frac{t}{T}\right) R(t+z) \approx$$
$$\approx \frac{1}{2}\sum_{j=0}^{N-1}\left(J_0\left(\mu_{0s}\frac{j\tau}{N}\right) R\left(\frac{jT}{N}+z\right) + \right. \quad (25)$$
$$\left. + J_0\left(\mu_{0s}\frac{(j+1)\tau}{N}\right) R\left(\frac{(j+1)T}{N}+z\right)\right),$$

$N \gg 1$, the value $N = 10^3$ is chosen. The coefficients g_s are calculated with the help of (15) with account for (24) and (25). Both sides of equation (1) are as described in (17), the approximate expression for the function $\text{Left}(t)$ is as follows

$$\text{Left}(t) = \int_0^T d\tau h(\tau) R(t-\tau) \approx$$
$$\approx \frac{1}{2}\sum_{j=0}^{N-1}\left(h\left(j\frac{T}{N}\right) R\left(t - j\frac{T}{N}\right) + \right. \quad (26)$$
$$\left. + h\left(\frac{(j+1)T}{N}\right) R\left(t - \frac{(j+1)T}{N}\right)\right),$$

$N \gg 1$, the value $N = 10^4$ is chosen (the sale value of N was chosen in [15] for the derivation of $\text{Left}(t)$). The expression for $h(\tau)$ is (22). The corresponding MAPE is roughly estimated as described in (20). The MAPE results for the solutions based on the Bessel functions and corresponding comparison graphs of the both sides are given in the next section.

4 EXPERIMENTS

The numerical values of the parameters T and z are taken from [15]: $T = 100$, $z = 3$.

The values of a, b for various packet rates are taken from [15].

The results for the Walsh functions are given in Table 1. The results for the Bessel functions are given in Table 2 and Table 3.

Table 1 – MAPE for the n Walsh functions approximations

$\alpha = 2^{11}$ p/s		$\alpha = 2^{13}$ p/s	
n	MAPE, %	n	MAPE, %
2	14.3	2	13.9
4	6.43	4	6.24
8	3.00	8	2.91
16	1.50	16	1.46
32	0.83	32	0.81
64	0.56	64	0.55
128	0.39	128	0.39
$\alpha = 2^{15}$ p/s		$\alpha = 2^{17}$ p/s	
n	MAPE, %	n	MAPE, %
2	13.6	2	13.0
4	6.16	4	6.21
8	2.89	8	2.95
16	1.46	16	1.49
32	0.81	32	0.82
64	0.55	64	0.55
128	0.39	128	0.39

Table 2 – MAPE for the approximations of n Bessel functions $J_0(x)$ for rather low packet rates

$\alpha = 2^{11}$ p/s		$\alpha = 2^{13}$ p/s	
n	MAPE, %	n	MAPE, %
1	19.0	1	18.2
2	18.9	2	18.1
3	10.6	3	9.98
4	10.5	4	9.95
5	7.25	5	6.80
6	7.24	6	6.79
7	5.49	7	5.12
8	5.49	8	5.12
9	4.40	9	4.09
10	4.42	10	4.11
11	3.66	11	3.40
12	3.67	12	3.41
13	3.15	13	2.88
14	3.16	14	2.91
15	2.76	15	2.53
16	2.77	16	2.56
17	2.46	17	2.26
18	2.46	18	2.28

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Table 3 – MAPE for the approximations of n Bessel functions $J_0(x)$ for rather high packet rates

$\alpha = 2^{15}$ p/s		$\alpha = 2^{17}$ p/s	
n	MAPE, %	n	MAPE, %
1	18.0	1	17.1
2	17.9	2	17.5
3	9.84	3	9.35
4	9.82	4	9.50
5	6.69	5	6.38
6	6.69	6	6.46
7	5.04	7	4.82
8	5.04	8	4.88
9	4.01	9	3.89
10	4.04	10	3.93
11	3.34	11	3.27
12	3.35	12	3.32
13	2.83	13	2.85
14	2.86	14	2.89
15	2.49	15	2.54
16	2.51	16	2.58
17	2.22	17	2.33
18	2.24	18	2.35

As one can see, the MAPE for the approximations of 32, 64 and 128 Walsh functions is lower than the MAPE of the 13-polynomial approximation [15]. As for the method based on the Bessel functions, in fact the same parameters as in [15] are used, but, as can be seen, the MAPE results for the 13-polynomial approximation [15] are better than the results of the approximation of 17 or 18 Bessel functions. Here, for simplicity, we restrict ourselves to the approximation of 18 Bessel functions, because the enlarging of the number of Bessel functions may significantly enlarge the computation time.

The results of the paper are obtained in the Wolfram Mathematica package.

5 RESULTS

Some results are illustrated on Fig. 1 and Fig. 2.

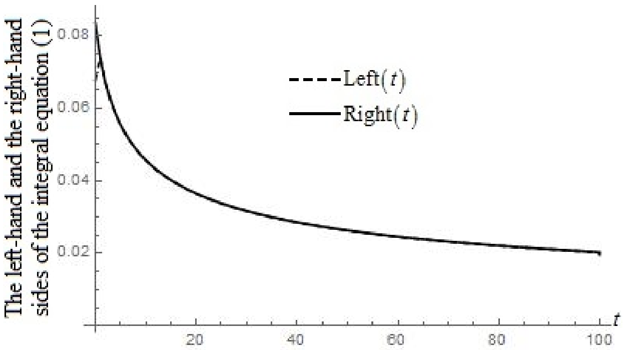

Figure 1 – Comparison of both sides of eq. (1) for $\alpha = 2^{11}$ p/s for the approximation of 128 Walsh functions

The comparison graphs for $\alpha = 2^{13}$ p/s, $\alpha = 2^{15}$ p/s and $\alpha = 2^{17}$ p/s are similar to the graphs for $\alpha = 2^{11}$ p/s shown on Fig. 1 and Fig. 2. As can be seen, the sides of the equation (1) are close for the derived solutions, but the use of the Walsh functions allows one to obtain more accurate results via the Galerkin method.

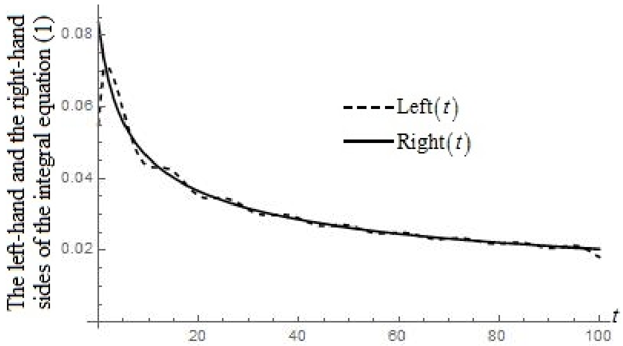

Figure 2 – Comparison of both sides of eq. (1) for $\alpha = 2^{11}$ p/s for the approximation of 17 Bessel functions

6 DISCUSSION

We improved the development [15] of the theory of the continuous KW filter for the forecasting of a stationary MFSD process.

We investigate the Galerkin method of the obtaining of the solution of the WH integral equation for the process under consideration. In [15] corresponding solutions were obtained via the polynomial expansion of the weight function. In this paper we derive the solutions via the Walsh functions and Bessel functions $J_0(x)$.

The approximations of 2^k Walsh functions are investigated for $k = \overline{1,7}$, it is shown that the MAPE of misalignment of both sides of the integral equation (1) is less than 0.5% for the approximation of 128 Walsh functions. The solutions based on the Bessel functions are investigated up to the approximation of 18 Bessel functions, however, the corresponding MAPE is greater than 2%. So, the use of the Walsh functions is more effective that the use of the polynomials [15] (the corresponding MAPE in [15] for 13-polynomial approximation is more than 1%), however, the use of the Bessel functions is less effective that the use of the polynomials (the corresponding MAPE in [15] for 13-polynomial approximation is less than 2%).

So, among the above-mentioned functions, the use of the Walsh functions is the most effective for the problem under consideration. It also should be stressed that the computation time for the calculation of approximation of 128 Walsh functions is less than the corresponding time required for the calculation of the approximation of 18 Bessel functions, especially for $\alpha = 2^{15}$ p/s and $\alpha = 2^{17}$ p/s.

The future plans are following ones. The practical applications of the investigated theoretical approach may be of interest. The investigation of other orthogonal function systems may be also provided, for example, the investigation of the trigonometric orthogonal system or the investigation of the Bessel functions $J_\nu(x)$ with $\nu \neq 0$. The random processes, including the heavy-tail ones, are important both for the traffic forecasting and for other fields of knowledge, for example: agriculture [25–27], estimation of risks in weather derivatives [28], and so on. The

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proposed method may be applicable both for telecommunication traffic forecasting and other fields of knowledge.

The above-mentioned plans may be carried out in other papers.

CONCLUSIONS

The weight function of the KW filter for the forecasting of the MFSD process is obtained via the Galerkin method with the choice of the Walsh and Bessel functions $J_0(x)$ as the method basis. Approximations up to the 128 Walsh functions and up to the 18 Bessel functions are investigated. The results are compared with the corresponding results based on the polynomial expansions [15]. It is shown that among the considered functions the use of the Walsh functions leads to the best coincidence of both sides of the WH integral equation for the obtained approximate solutions.

The scientific novelty is that the for the first time the weight function of the KW filter for the forecasting of the MFSD process is obtained via the Walsh functions and the Bessel functions $J_0(x)$. The choice of the Walsh functions leads to the better coincidence of the sides of the considered integral equation than the choice of the polynomials and Bessel functions.

The practical significance is that the obtained results may be applied to the forecasting of stationary random processes in different fields of knowledge.

Prospects for further research are to investigate the Galerkin method via other orthogonal function systems, for example, the Bessel functions $J_\nu(x)$ with $\nu \neq 0$. The practical applications of the investigated theoretical approach may be of interest.

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DECLARATIONS

Conflict of interest: The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship, or otherwise, that could affect the research and its results presented in this paper.

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ПОРІВНЯННЯ ДЕЯКИХ СИСТЕМ ОРТОГОНАЛЬНИХ ФУНКЦІЙ ДЛЯ ПРОГНОЗУВАННЯ КОЛМОГОРОВА-ВІНЕРА MFSD ПРОЦЕСУ

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АНОТАЦІЯ

Актуальність. Ми досліджуємо такі ортогональні системи функцій, як систему функцій Уолша та систему функцій Бесселя як основу пошуку вагової функції фільтра Колмогорова-Вінера для прогнозування випадкового MFSD процесу з важким хвостом. Результати для середньої абсолютної відсоткової помилки нев’язки лівої та правої частин інтегрального рівняння Вінера-Хопфа порівнюються з результатами, отриманими в нашій попередній статті на основі розвинення за поліномами Чебишова.

Мета роботи. Метою роботи є отримати наближений розв’язок для вагової функції фільтра Колмлгорова-Вінера на основі обірваного розвинення по функціях Уолша та Бесселя та порівняти отримані результати з результатами, отриманими в нашій попередній статті на основі поліномів Чебишова.

Метод. Для обчислення вагової функції використано метод Галеркіна, що базується на функціях Уолша та Бесселя, ортогональних на відповідному проміжку.

Результати. Показано, що використання функцій Уолша призводить до кращих результатів для вище наведеної середньої абсолютної відсоткової помилки нев’язки лівої та правої частин інтегрального рівняння Вінера-Хопфа, ніж використання поліномів Чебищова та функцій Бесселя. Також показано, що використання ортогональних поліномів є ефективнішим за використання функцій Бесселя. Також показано, що наближення 128 функцій Уолша призводить до помилки нев’язки меншої за 0,5%.

Висновки. Досліджено теорію побудови неперервного фільтра Колмогорова-Вінера для прогнозування випадкового процесу з важким хвостом в MFSD моделі. Фунції Уолша та Бесселя розглянуто як основу методу Галеркіна. Результати порівняно з результатами, отриманими на основі поліномів Чебишова. Показано, що використання функцій Уолша дає найкращі результати серед перелічених вище систем ортогональних фунцій. Отримані результати можуть бути застосовними для прогнозування телекомунікаційного трафіку на практиці, а також вони можуть бути корисними для обробки випадкових процесів в інших галузях знань: в сільському господарстві, і т. д.

КЛЮЧОВІ СЛОВА: вагова функція фільтра Колмогорова-Вінера, функції Уолша, телекомунікаційний трафік, MFSD модель, функції Бесселя, метод Галеркіна.

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