# ПРОГРЕСИВНІ ІНФОРМАЦІЙНІ ТЕХНОЛОГІЇ

# ПРОГРЕССИВНЫЕ ИНФОРМАЦИОННЫЕ ТЕХНОЛОГИИ

# PROGRESSIVE INFORMATION TECHNOLOGIES

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## COMPARATIVE ANALYSIS OF THE COMPLEXITY OF CHAOTIC AND STOCHASTIC TIME SERIES

The new approach to the recognition mechanism of the time series generating process based on the results of the entropy and the recurrent analysis is proposed. The comparative analysis of the realizations properties of chaotic and stochastic processes with different correlation structure was carried out. It is shown that the derived set of information characteristics allows to distinguish the realizations of deterministic chaotic and fractal random processes. Depending on complexity measures of time series of process parameters were obtained. The information characteristics dependencies from the process parameters were obtained. The results of bioelectric signals and financial time series study are presented.

**Keywords:** time series, measures of complexity, approximate entropy, recurrence plot, pseudo-phase space, embedding dimension.

### NOMENCLATURE

$A$ is a control parameter;
$ApEn$ is aproximate entropy;
$C_m(\varepsilon)$ is correlation integral;
$Det$ is measure of determinism;
$F(t)$ is a $m$-dimensional pseudo-phase space;
$l_i$ is a length of the $i$-th diagonal line;
$K$ is a set of characteristics of recurrence and entropy analysis;
$N$ is a total number of points in the pseudo-phase space;
$N_l$ is a number of diagonal lines;
$n_{i\,m}(\varepsilon)$ is a number of vectors, that similar vector $P_m(i)$;
$P(l)$ is a frequency distribution of the diagonal lines lengths;
$RP_{i,j}$ is recurrence plot;
$RR$ is measure of recurrence;
$x(t)$ is a point of time series;
$x_i$ is a point in the reconstructed pseudo-phase space;
$\varepsilon$ is a neighborhood size;
$\phi$ is a autoregressive coefficient;
$\xi(t)$ is a uncorrelated white noise;
$\sigma_0$ is a diffusion coefficient;
$\tau$ is a delay period;
$\Theta(\cdot)$ is a Heaviside function.

### INTRODUCTION

Most dynamical systems are «complex systems», which implies the ladder structure with nonlinear feedback. These include the processes inherent in the human body and nature, informational, physical, technical and social processes. In practice, they are represented by time series, which are a projection of the internal and external relations of the dynamical system. One of the objectives of time series analysis is to extract information from the series and infer the properties and mechanism of the process that generates the series.



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Mathematical models of complex systems exhibiting irregular dynamics are both random and deterministic chaotic processes. Identification of the mechanism generating process based on characteristics obtained by time series is a daunting task. There are many approaches to the study of time series based on traditional statistical analysis, and the methods of nonlinear chaotic dynamics.

The object of study is the deterministic chaotic and stochastic fractal processes in the technical, economic and biological systems. The subject of study is the time series of a random type and the estimation methods of their characteristics. The purpose of the work is the following: based on the results recurrence and entropy analysis of fractal time series to identify the mechanism of generating process (deterministic or stochastic).

## 1 PROBLEM STATEMENT

Suppose given a time series of an irregular type $X = \{x(t)\}$, $t = 1,...,N$. Let this time series have fractal properties. Let we have obtained the set of qualitative and quantitative characteristics $K = \{K_i\}$, $i = 1,...,m$ of the resulting recurrence and entropy analysis. Need to find out whether the process of generating this series is chaotic deterministic or random. For this is necessary to conduct a comparative analysis of modelling time series of various types and determine the set of characteristics $K' \subset K$ for which the differences are significant

## 2 REVIEW OF THE LITERATURE

Most methods of chaotic dynamics used for time series analysis, based on the reconstruction space of single realization using the procedure Packard-Takens [1–4]. The reconstruction of the pseudo-phase space allows us to compute the embedding dimension, which is the main means of distinguishing chaotic and random processes [3, 5]. This approach allows us to well distinguish between chaotic dynamics and uncorrelated random noise, however, because this method is based on the estimation of the fractal dimension and detection autocorrelation relations, it has no effect for the fractal random processes having long dependence [6, 7].

In [8] proposed a method that extends the capabilities of nonlinear time series analysis, based on the fundamental property of dissipative dynamical systems – recurrence states. This method of analysis, based on the representation of process properties in the form of geometric structures, is a means for detection the hidden dependencies in the observed processes [9–12]. The method of recurrence plots is widely used for the analysis of stochastic time series of different nature [6, 13–16]. One of the characteristics of the complexity of the system behavior is entropy. Entropy methods of time series analysis are also used a reconstruction phase space [3, 7, 17, 18]. One of the characteristics that demonstrate the complexity of the time series dynamics is the approximate entropy of similarity introduced in [7].

## 3 MATERIALS AND METHODS

Consider the basic features of the recurrence and entropy analysis. The main idea of the application of nonlinear dynamics methods to the analysis of the realizations of a dynamical system is that the basic structure, which contains all the information about the system, namely, an attractor of a system, can be reconstructed by measuring only single component of this system [1, 3, 19]. Reconstruction phase space attractor is reduced to the construction of the pseudo-phase space. Widely used procedure Packard-Takens allows to restore the phase trajectory of a dynamical system from single realization:

$$F(t) = [x(t), x(t+\tau), ..., x(t+m\tau)]. \quad (1)$$

One of the most common methods used in practice to determine the existence of chaotic determinacy and estimate the fractal dimension of the attractor is to study the properties of the correlation integral $C_m(\varepsilon)$ and behavior of the correlation dimension $d_C(m)$ depending on the dimension $m$ of the pseudo-phase space. The correlation integral $C_m(\varepsilon)$ is a probability that a pair of points on the reconstructed attractor in $m$-dimensional space is within a distance of ε each other:

$$C_m(\varepsilon) = \frac{1}{N^2} \lim_{\varepsilon \to 0} \sum_{i,j=1}^{N} \Theta(\varepsilon - \|x_i - x_j\|). \quad (2)$$

Dependence the correlation integral on ε at small ε obeys a power law, i.e. $\lim_{\varepsilon \to 0} C_m(\varepsilon) = a\varepsilon^{d_C}$. By increasing the dimension of the pseudo-phase space $m$ correlation dimension $d_C(m)$ increases too. However, for deterministic chaotic time series correlation dimension will ultimately be saturate with its true value. Value $m$ at which $d_C(m)$ will stop changing, is the embedding dimension. For uncorrelated stochastic realizations embedding dimension increases with the dimension of the pseudo-phase space $m$.

Recurrence plot is a projection of the m-dimensional pseudo-phase space onto the plane [12, 14, 17]. Let point $x_i$ corresponds to the phase trajectory $x(t)$ describing the dynamic system in the $m$-dimensional space at a time $t = i$, for $i = 1, ..., N$, then the recurrence plot $RP$ is array of pixels, where a nonzero element of the coordinates $(i, j)$ corresponding to the case where the distance between $x_j$ and $x_i$ is smaller ε:

$$RP_{i,j} = \Theta(\varepsilon - \| x_i - x_j \|), \quad x_i, x_J \in R^m, \ i, j = 1,...N. \quad (3)$$

The states $x_j$ are recurrence if they are contained into the $m$-dimensional neighborhood of point $x_i$ with size ε. Arbitrarily chosen recurrence point does not contain useful information about the state of the system at time moments $i$ and $j$, only the totality of recurrence points allows you to restore the system properties. Analysis of the plot topology allows us to classify the observed processes: homogeneous processes with independent random values, processes with slowly varying parameters, periodic or oscillating processes corresponding to nonlinear systems, etc.

Numerical analysis of recurrence plots allows us to calculate the measure of complexity structures of recurrence plots, such as a measure of recurrence and determinism etc. The measure of recurrence $RR$ shows the density of recurrence points:

$$RR = \frac{1}{N^2}\sum_{i,j}^{N} RP_{i,j}. \quad (4)$$

Measure of determinism $Det$ is a characteristic of predictability process and equal to the ratio of the number of points in diagonal lines to the total number of recurrence points:

$$Det = \sum_{l=l_{\min}}^{N} P\ (l) / \sum_{i,j}^{N} RP_{i,j}. \quad (5)$$

Aproximate entropy $ApEn$ is the statistics of time series regularity that defines the possibility of its forecasting. Time series that contain a many of duplicate values, have a relatively small value, and for less predictable process $ApEn$ value is larger. Methods of estimating the approximate entropy $ApEn$ considered in [7, 17].

Consider a time series $\{x(t)\}$, $t = 1,...,N$. Let the vector $P_m(i)$ is subsequence values $\{x_i, x_{i+1},..., x_{i+m}\}$ length of $m$. Two vectors $P_m(i)$ и $P_m(j)$ will be similar, if the following condition:

$$\left|x_{i+k} - x_{j+k}\right| < \varepsilon,\ 0 \le k < m.$$

For each $i = 1,..., N - m + 1$ value $C_{im}(\varepsilon)$ is calculated:

$$C_{im}(\varepsilon) = \frac{n_{i\ m}(\varepsilon)}{N - m + 1}. \quad (6)$$

Approximate entropy $ApEn$ determined by the formula

$$ApEn(m, \varepsilon) = \ln \frac{C_m(\varepsilon)}{C_{m+1}(\varepsilon)}, \quad (7)$$

where $C_m(\varepsilon) = \frac{1}{N - m + 1}\sum_{i=1}^{N-m+1} C_{i\,m}(\varepsilon)$.

Consider the basic model types of data needed to conduct research and their statistical properties. As input data have been chosen realizations of deterministic chaotic systems and realizations of stochastic processes having different correlation structure: uncorrelated noise, autoregressive processes with short-term dependence and fractal processes with long-term memory.

Chaos is a complex dynamics of deterministic systems in steady state. The main feature of such systems is sensitive dependence to arbitrarily small changes in initial conditions. If $d_0$ is the initial distance between two points, then for short time $t$ later the distance between the trajectories, which start from these points, becomes $d(t) = d_0 e^{\lambda t}$, where the value of $\lambda$ is the Lyapunov exponent. This leads to the loss of deterministic predictability and the need to introduce probabilistic characteristics to describe the dynamics of chaotic systems.

Iterated maps $x_{n+1} = f(C, x_n)$, where $C$ is control parameter, are the most simple and intuitive mathematical chaotic models [1, 3]. For a wide class of nonlinear functions $f$ the sequence $\{x_n\}_{n=0}^{\infty}$ is chaotic. In the case of dissipative map the orbits $\{x_n\}_{n=0}^{\infty}$ lead to an attractor having a fractal structure.

Logistic map is the most famous example of chaotic maps. This one-dimensional quadratic map is defined as follows:

$$x_{n+1} = Ax_n(1 - x_n)\ \ A \in (0.4] \text{ and } x_n \in [0.1]; \quad (8)$$

Diagram of the Lyapunov exponent $\lambda$ is given in the upper part of fig. 1. Chaotic dynamics ($\lambda > 0$) is observed when the parameter $A > A^* = 3.569.....$ The regions of chaos alternate with «windows of stability» in which the dynamics becomes periodic. At the bottom of fig. 1 shows the time realizations of logistic map for parameter values $A = 3{,}7$ and $A = 3{,}9$. The corresponding Lyapunov exponents are equal $\lambda = 0{,}37$ and $\lambda = 0{,}5$.

Autoregressive process of 1st order was chosen as processes with short-term dependence [5]:

$$X(t) = \phi X(t-1) + \xi(t),\ |\phi| < 1 \quad (9)$$

Autoregressive coefficient value $\phi$ characterizes the degree of the autocorrelation process. Fig. 2 shows the realizations of the autoregressive process of the different values of coefficient $\phi$.

At present it has been generally accepted, that many stochastic processes in nature and in engineering exhibit a long-range dependence and fractal structure [20, 21]. Stochastic process $X(t)$ is self-similar with self-similarity parameter $H$, if the process $a^{-H}X(at)$ is described by the same finite-dimensional distributions that $X(t)$. One of the most famous and simple models of stochastic dynamics that have fractal properties, is the fractional Brownian motion (FBM).

Gaussian process $X(t)$ with a parameter $H$, $0 < H < 1$ called FBM if its increments $\Delta X(\tau) = X(t+\tau) - X(t)$ have a distribution of the form:

$$P(\Delta X < x) = \frac{1}{\sqrt{2\pi}\sigma_0\tau^H} \cdot \int_{-\infty}^{x} \mathrm{Exp}\left[-\frac{z^2}{2\sigma_0{}^2\tau^{2H}}\right] dz. \quad (10)$$

FBM with the parameter $H = 0{,}5$ coincides with the classical Brownian motion. Parameter $H$ called the Hurst exponent, is the degree of self-similarity. Along with this property, the index characterizes the measure of long-term dependence of a stochastic process, i.e. that autocorrelation function $r(k)$ decreases as a power law: $r(k) \sim k^{-\beta}$, $k \to \infty$, where $0 < \beta < 1$ and $H = 1 - (\beta/2)$. Fig. 3 shows the realizations of the FBM for the values $H$= 0.3, 0.5, 0.8.

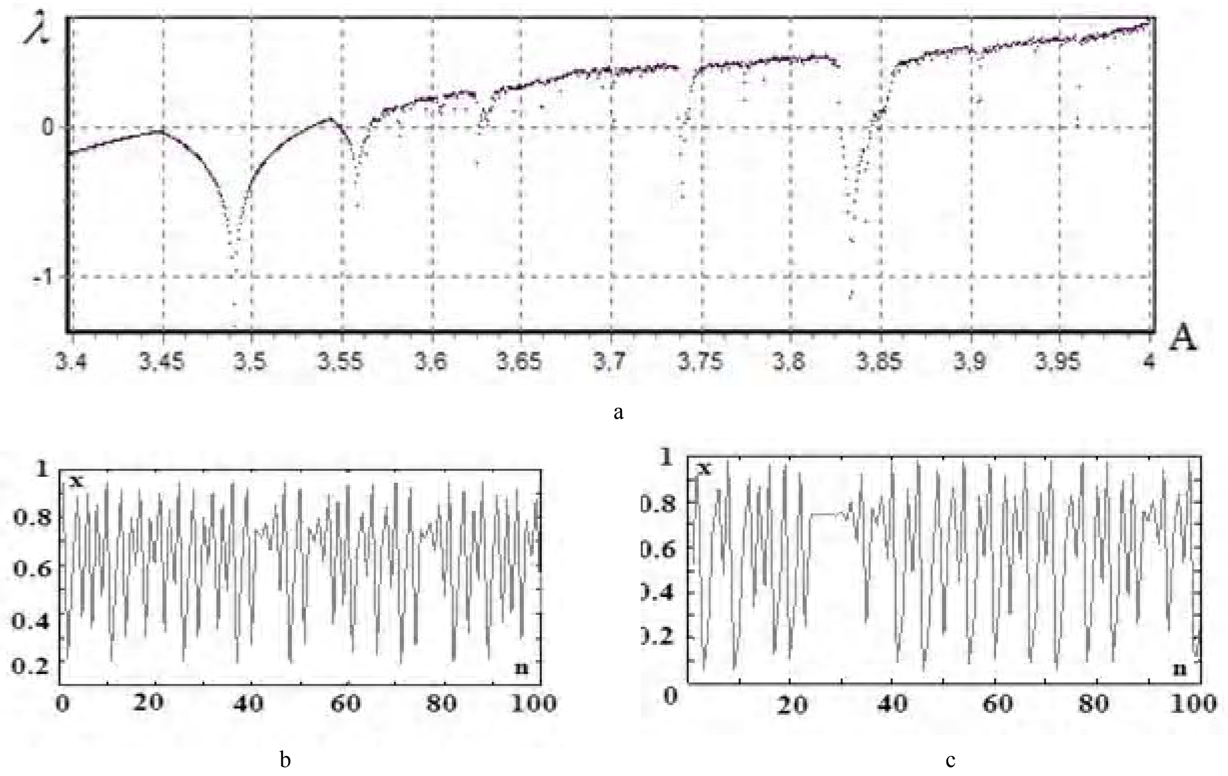


Figure 1 – Diagram of the Lyapunov exponent and realizations for logistic map: a – diagram of the Lyapunov exponent, b – realization with A=3.7, c – realization with A=3.9

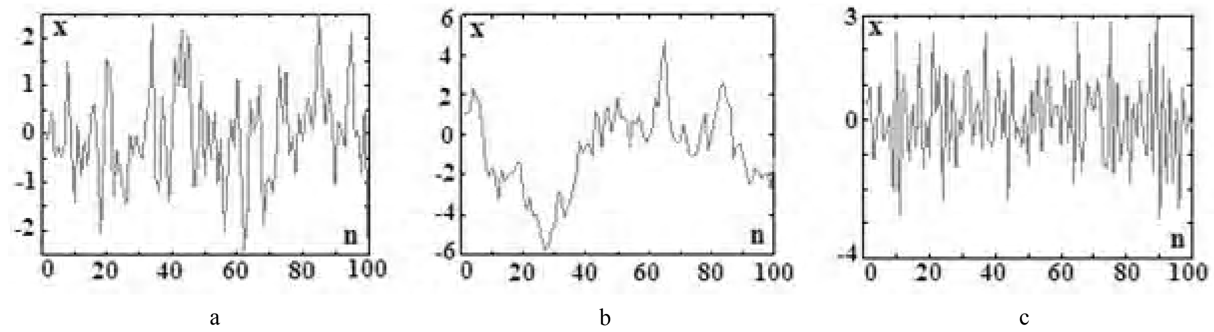


Figure 2 – The realizations of autoregressive process for different $\phi$: a – $\phi = 0.2$ , b – $\phi = 0.9$ ; c – $\phi = -0.7$

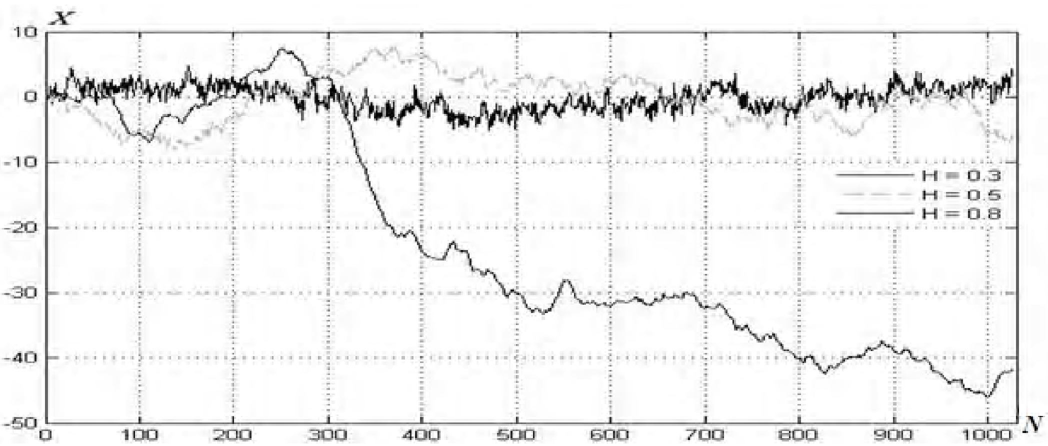


Figure 3 – FBM for different values $H$

## 4 EXPERIMENTS

For carrying out numerical experiments investigated time realizations $X = \{x_i\}$, $i = 1,..., N$, were generated according to (8–10) for various $N$. It was first performed the reconstruction of pseudo-phase space and estimation of embedding dimension. For this procedure were used time realizations of $N = 10000$. Smaller length of time series usually leads to large errors [1–3].

For carrying out entropy and recurrence analysis were used time realizations of $N = 1000$. This length is sufficient for good visualization of recurrence plots and small errors in the quantitative characteristics (about $\pm 0.005$). For each of the generated realization estimates of approximate entropy and a number of the recurrent characteristics were obtained. Estimation procedure was carried out for realizations of every type processes $M = 100$ times and each estimate was averaged over the $M$ values.

For clarity, we first considered the example of a completely different process on complexity: a periodic motion and uncorrelated white noise. For chaotic processes the realizations with different values of Lyapunov exponents were investigated. For autoregressive processes the autoregressive coefficient value was changed. The realizations of FBM were generated for different values of the Hurst exponent.

After examining the results of the analysis the modeling realizations the entropy and recurrence analysis of real time series such as bioelectrical signals and financial series was performed.

## 5 RESULTS

Consider the reconstruction of pseudo-phase space and estimation of embedding dimension. Fig. 4 shows the typical dependence of the correlation dimension $d_C(m)$ on the dimension $m$ of pseudo-phase space constructed in accordance with (1) for the realizations of an autoregressive process, the logistic map and FBM.

The comparative entropy and recurrence analysis of chaotic realizations and realizations of the stochastic processes having different correlation structure was carried out. Fig. 5 shows the recurrence plots for the sum of two sinusoids and independent values of a normal random variable. Table 1 shows the corresponding values of the measures of recurrence $RR$, determinism $Det$ and approximate entropy $ApEn$.

Recurrence plots for realizations of map (8), autoregression and FBM with different values of parameters are presented on fig. 6. In the case of logistic map the Lyapunov exponents are equal to $\lambda$= 0.37, 0.5, 0.69 according to parameter values.

Table 2 shows the means of recurrence $RR$, determinism $Det$ and approximate entropy $ApEn$ corresponding to the plots above.

Table 1 – Quantitative characteristics of complexity of sinusoid and uncorrelated noise

| | *RR* | *Det* | *ApEn* |
|---|---|---|---|
| Sinusoid | 0.18 | 0.998 | 0.03 |
| Uncorrelated noise | 0.0003 | 0.025 | 1.7 |

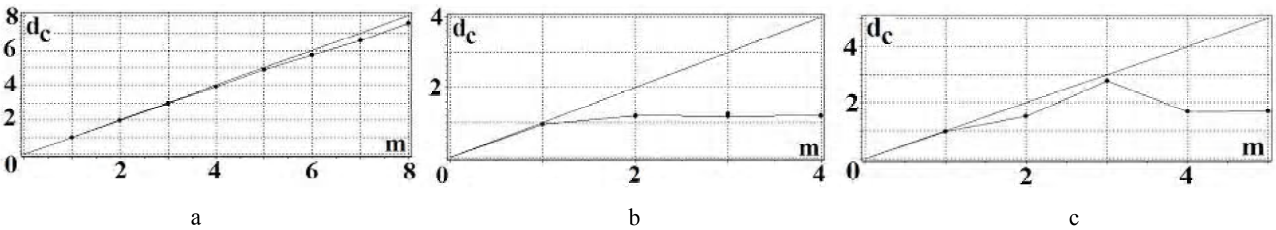


Figure 4 – Dependence of $d_C(m)$ on $m$ for different processes: a – autoregression, b – chaotic map, c – FBM

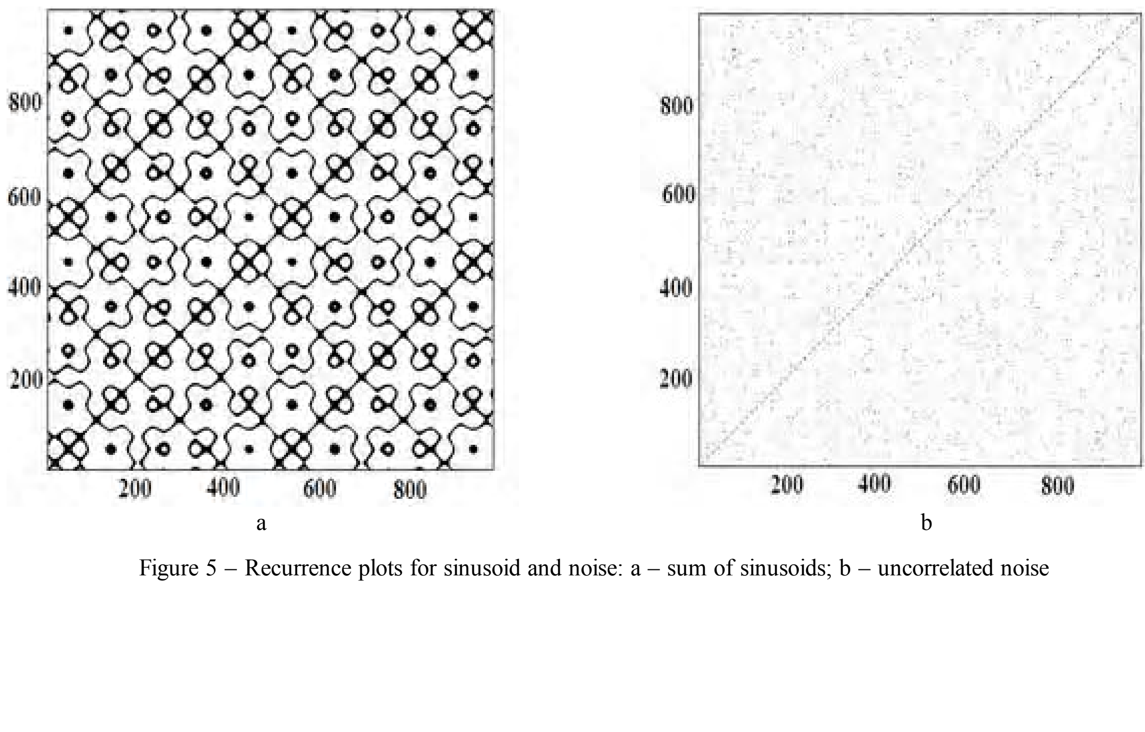


Figure 5 – Recurrence plots for sinusoid and noise: a – sum of sinusoids; b – uncorrelated noise

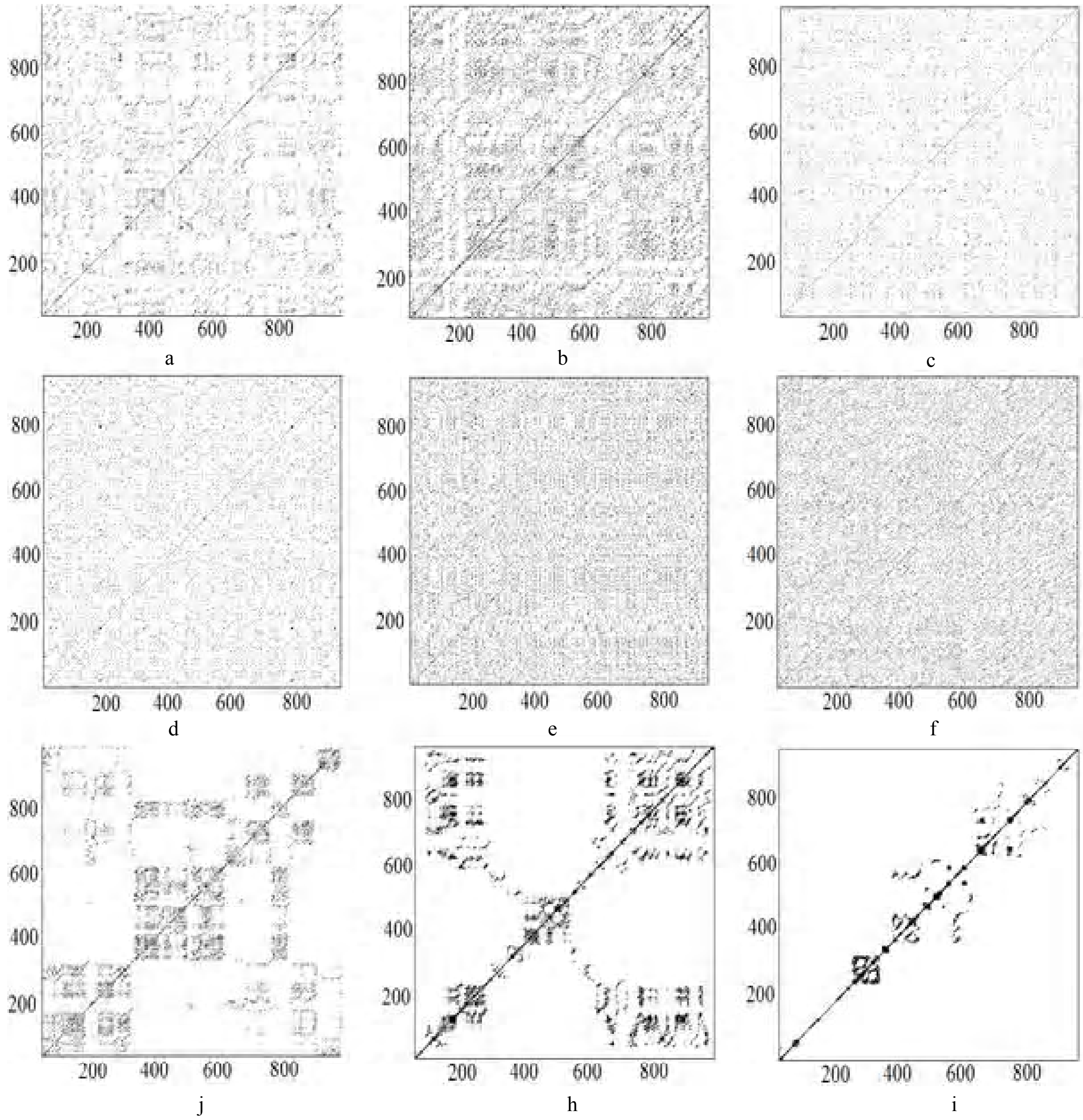

Figure 6 –Recurrence plots for under study realizations: a – logistic map with A=3.7, b –logistic map with A=3.9, c – logistic map with A=4, d – autoregression with $\phi = 0.3$, e –autoregression with $\phi = 0.6$, f – autoregression with $\phi = 0.9$, j – FBM with $H = 0.3$, h – FBM with $H = 0,6$, i – FBM with $H = 0.9$

Table 2 – Quantitative characteristics of complexity of realizations

| Logistic map | | | | Autoregression | | | | FBM | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *A* | *RR* | *Det* | *ApEn* | φ | *RR* | *Det* | *ApEn* | *H* | *RR* | *Det* | *ApEn* |
| 3.7 | 0.008 | 0.1 | 0.93 | 0.3 | 0.0003 | 0.03 | 1.72 | 0.3 | 0.02 | 0.55 | 0.47 |
| 3.9 | 0.004 | 0.07 | 1.2 | 0.6 | 0.0005 | 0.05 | 1.65 | 0.6 | 0.02 | 0.87 | 0.21 |
| 4 | 0.002 | 0.05 | 0.86 | 0.9 | 0.002 | 0.13 | 1.25 | 0.9 | 0.01 | 0.95 | 0.12 |

In this work the time series corresponding to a various complex dynamical systems: bioelectrical signals and financial series were considered. In particular, the RR-intervals series were investigated. RR-interval is the time interval between adjacent teeth of electrocardiogram and it equals to the duration of the cardiac cycle. Initial data for the research in this paper were obtained on a specialized website [24] containing an extensive medical database. As an example of financial series, the dynamics of change in the currency pair EUR/RUB for 2004–2006 was examined. Fig. 7 shows the time series and recurrence plots of data described above.

Quantitative recurrence and entropy characteristics obtained from the time series are presented in Table 3.

### 6 DISCUSSION

Numerical analysis shows that the realizations of the random and deterministic chaotic motion may have similar statistical characteristics [3, 22, 23]. Reconstruction of pseudo-phase space and estimations of embedding dimension detected essential differences in the structure of chaotic realizations and realizations autoregressive processes with short-term dependence. However, the embedding dimension, evaluated for the FBM realizations with a long-term dependence, is also limited [6, 7]. The estimation results presented in fig. 4 confirm that the construction of the pseudo-phase space and the estimation of embedding dimension cannot be a reliable tool for distinguishing between chaotic and stochastic fractal realizations and fitting of appropriate mathematical models.

Carried out recurrent analysis detected strong differences in visual topology and the numerical characteristics of realizations of the above processes. It is obvious that the characteristics of chaotic and random processes must be located within the range of characteristic values calculated for the periodic and completely random trajectories, see fig. 5 and tab. 1. It can be noted for chaotic realizations that greater value of Lyapunov exponent corresponds to a greater randomness of the system, which is clearly evident on recursive plots: the existence of some plot structure replaced uniform filling (top of Fig. 6). In the case of autoregressive process (middle Fig. 6) it is necessary to note the lack of plot structure and uniform filling regardless of the autocorrelation degree. The recurrent plots of FBM have the specific structure, which depends on the Hurst exponent value (bottom of Fig. 6). With the increasing exponent $H$ the range of values, i.e. plot filling, decreases.

Table 3 – Quantitative characteristics of complexity of time series

| | *RR* | *Det* | *ApEn* |
|---|---|---|---|
| RR-intervals | 0.05 | 0.61 | 1.07 |
| EUR/RUB | 0.08 | 0.85 | 0.17 |

As regards the quantitative characteristics, the research has shown that the most informative recurrent characteristics are the indexes of recurrence and determinism. The values *RR* and *Det* are measure of regularity, therefore in each case they decrease, when randomness or uncorrelation of realizations increase. The entropy *ApEn* is measure of unpredictability therefore it increases with uncorrelation. Value ranges of characteristics are quite different for various processes. This allows us to identify the generating process by the set of characteristics.

Based on the results of qualitative and quantitative analysis can be propose for modeling realizations RR-intervals to use deterministic chaotic systems, while the mathematical modeling of S&P500 series should be based on self-similar stochastic processes. For a correct choice of the model in the first case the estimation of such characteristics as the Lyapunov exponent, invariant measure

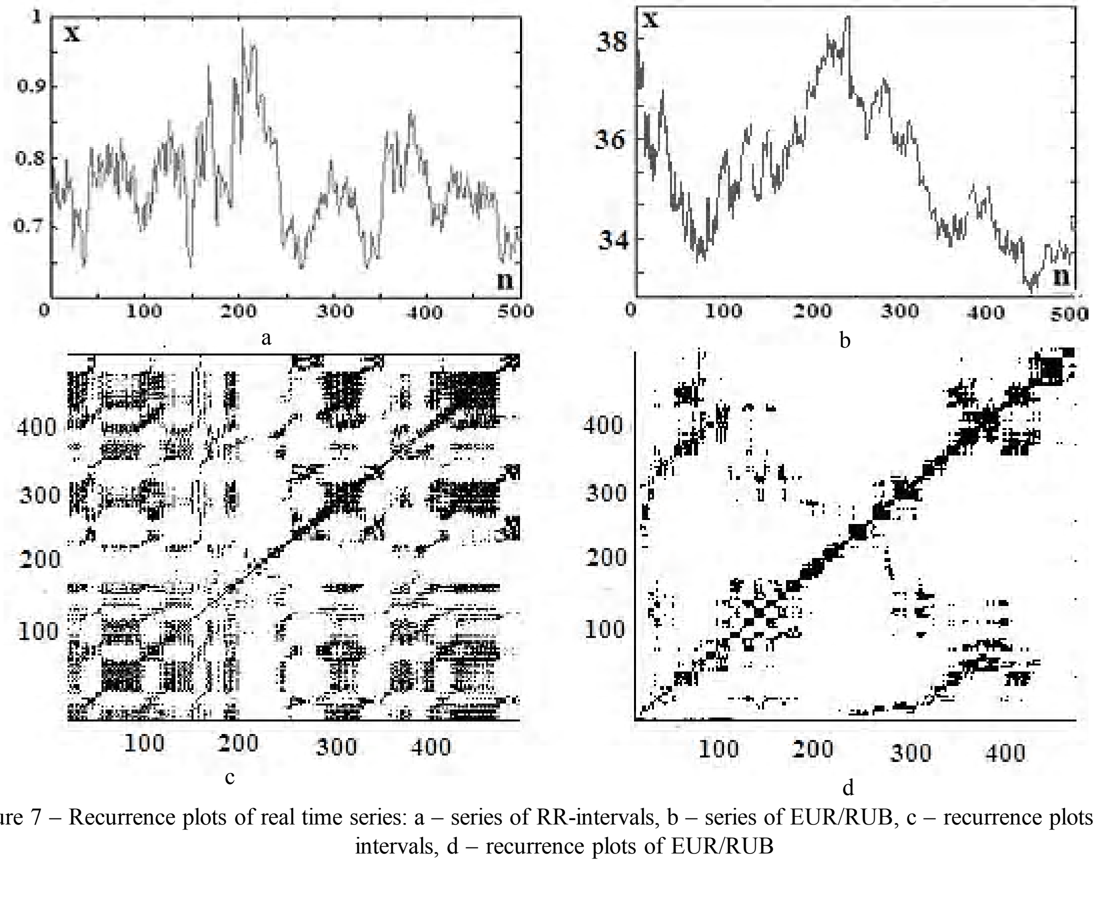


Figure 7 – Recurrence plots of real time series: a – series of RR-intervals, b – series of EUR/RUB, c – recurrence plots of RR-intervals, d – recurrence plots of EUR/RUB

distribution, etc. is necessary, and in the second case – the estimation of fractal characteristics.

## CONCLUSION

Using the results of the recurrent and entropy analysis to distinguish deterministic chaotic and fractal random processes was first proposed in this work. It is shown that the set of characteristics such as indexes of recurrence and determinism, approximate entropy and recurrence plot allows to identify the type of process that generated the time series. The dependences of information complexity measures of time series from the parameters of the processes were obtained. Thus it is possible to choose the mathematical model of process has a certain correlation and recursive structure for the simulation and forecasting. It is shown that series of RR-intervals corresponds to a chaotic process and S&P500 series has the structure corresponding to a fractal Brownian motion. Further studies propose the calculation of confidence intervals for estimates of the characteristics, the analysis of the short time series and investigation a large number of real time series a various complex dynamical systems.

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**СРАВНИТЕЛЬНЫЙ АНАЛИЗ СЛОЖНОСТИ ХАОТИЧЕСКИХ И СТОХАСТИЧЕСКИХ ВРЕМЕННЫХ РЯДОВ**

Предложен новый подход к распознаванию механизма процесса, породившего временной ряд, базирующийся на результатах энтропийного и рекуррентного анализа. Проведен сравнительный анализ свойств реализаций хаотических и стохастических процессов, имеющих различную корреляционную структуру. Показано, что полученное множество характеристик информационной сложности позволяет различать реализации детерминированных хаотических и фрактальных случайных процессов. Получены зависимости информационных характеристик от параметров процессов. Приведены результаты исследования биоэлектрических сигналов и финансовых рядов.

**Ключевые слова:** временной ряд, мера сложности, энтропия подобия, рекуррентная диаграмма, псевдо-фазовое пространство, размерность вложения.

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**ПОРІВНЯЛЬНИЙ АНАЛІЗ СКЛАДНОСТІ ХАОТИЧНИХ ТА СТОХАСТИЧНІХ ЧАСОВИХ РЯДІВ**

Запропоновано новий підхід до розпізнавання механізму процесу, що генерує часовий ряд, який базується на результатах ентропійного і рекурентного аналізу. Проведено порівняльний аналіз властивостей реалізацій хаотичних та стохастичних процесів, що мають різну кореляційну структуру. Показано, що отримана множина характеристик інформаційної складності дозволяє розрізняти реалізації детермінованих хаотичних і фрактальних випадкових процесів. Отримано залежності інформаційних характеристик від параметрів процесів. Наведено результати дослідження біоелектричних сигналів і фінансових рядів.

**Ключові слова:** часовий ряд, міра складності, ентропія подібності, рекурентна діаграма, псевдо-фазовий простір, розмірність вкладення.

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