

# УПРАВЛІННЯ У ТЕХНІЧНИХ СИСТЕМАХ

## УПРАВЛЕНИЕ В ТЕХНИЧЕСКИХ СИСТЕМАХ

### CONTROL IN TECHNICAL SYSTEMS

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### RELIABILITY AND RISK OPTIMIZATION OF MULTISTATE SYSTEMS WITH APPLICATION TO PORT TRANSPORTATION SYSTEM

The complexity of technical systems' operation processes and its influence on the changing in time systems' structures and their components' reliability parameters poses a difficulty to first meet in real and then to fix and analyse those structures and reliability parameters. By constructing a joint model of reliability of complex technical systems at variable operation conditions, which links a semi-markov modelling of system operation processes with multi-state approach to system reliability analysis, we find the system's main reliability characteristics. Consequently, we use linear programming to build a model of complex technical systems reliability optimization. We investigate the model's application in marine transport, specifically in reliability and risk optimization of a bulk cargo transportation system. The tools we develop can be used in reliability evaluation and optimization of a very wide class of real technical systems operating at varying conditions that influence their reliability structures and the reliability parameters of their components. Consequently, the tools we developed can be implemented by reliability practitioners from both maritime transport industry and other industrial sectors.

**Keywords:** reliability function, risk function, operation process, optimization.

#### NOMENCLATURE

$Z(t), z_i$  is a system operation process; a system operation state;  $i = 1, 2, \dots, v$ ,

$\dot{p}_b$  is an optimal transient probability;

$U$  is a particular reliability state of the system;

$u = 1, 2, \dots, z$ ;

$T(u)$  is a lifetime of a system in the reliability state subset  $\{u, u + 1, \dots, z\}$ ;

$[R(t, u)]^{(b)}$  is a conditional reliability function of a system at the operational state  $z_b$ ;

$\dot{R}(t, u)$  is an optimal unconditional reliability function of a system;

$\mu(u)$  is a mean lifetime of the system in the reliability state subset  $\{u, u + 1, \dots, z\}$ ;

$\bar{\mu}(u)$  is a mean lifetime of the system in the reliability particular state  $u$ ;

$R$  is a critical state of the multi-state system;

$\dot{r}(t)$  is an optimal risk function of the multi-state system.

#### INTRODUCTION

Most real technical systems are very complex because they are composed of large numbers of components and subsystems and have high operating complexity. The complexity of the systems' operation processes and its influence on the changing in time systems' structures and their components' reliability parameters poses a difficulty to first meet in real and then to fix and analyse those structures and reliability parameters. A convenient tool to investigate this problem is a semi-markov [2] modelling of the system operation process linked with a multi-state approach for the system reliability analysis [1, 4, 9–10] and a linear programming for the system reliability optimization [3]. Using this approach, it is possible to find this complex system's main reliability characteristics including the system reliability function, the system mean lifetimes in the reliability states subsets and the system risk function [4, 6, 8]. Having those characteristics it is possible to optimize the system operation process to get optimal values [8]. To this end the linear programming [3] can be applied to maximize the mean value of the system lifetime in the subset of the system reliability states, which are not worse than the system critical reliability state.

## 1 SYSTEM RELIABILITY AT VARIABLE OPERATIONS PROCESS

We suppose that the system has  $v$  different operation states during its operation process. Thus, we can define the system operation process  $Z(t)$ ,  $t \in \langle 0, +\infty \rangle$ , as the process with discrete operation states from the set  $Z = \{z_1, z_2, \dots, z_v\}$ . Further, we assume that  $Z(t)$  is a semi-markov process [2] with its conditional sojourn times  $\theta_{bl}$  at the operation state  $z_b$  when its next operation state is  $z_l$ ,  $b, l = 1, 2, \dots, v$ ,  $b \neq l$ . In this case the process  $Z(t)$  may be described by:

- the vector of probabilities of the process initial operation states  $[p_b(0)]_{1 \times v}$ ,
- the matrix of probabilities of the process transitions between the operation states  $[p_{bl}]_{v \times v}$ ,
- the matrix of conditional distribution functions  $[H_{bl}(t)]_{v \times v}$  of the process sojourn times  $\theta_{bl}$ ,  $b \neq l$ , in the operation state  $z_b$  when the next operation state is  $z_l$ .

Under these assumptions, the sojourn times  $\theta_{bl}$  mean values are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^{\infty} t dH_{bl}(t), \quad b, l = 1, 2, \dots, v, \quad b \neq l. \quad (1)$$

The mean values  $E[\theta_b]$  of the unconditional sojourn times  $\theta_b$  are given by

$$M_b = E[\theta_b] = \sum_{l=1}^v p_{bl} M_{bl}, \quad b = 1, 2, \dots, v, \quad (2)$$

where  $M_{bl}$  are defined by (1).

Limit values of the transient probabilities at the operation states are given by

$$p_b = \lim_{t \rightarrow \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^v \pi_l M_l}, \quad b = 1, 2, \dots, v,$$

where the probabilities  $\pi_b$  of the vector  $[\pi_b]_{1 \times v}$  satisfy the system of equations  $[\pi_b] = [\pi_b][p_{bl}]$  and  $\sum_{l=1}^v \pi_l = 1$  and

$$p_b(t) = P(Z(t) = z_b), \quad t \in \langle 0, +\infty \rangle, \quad b = 1, 2, \dots, v.$$

We assume that the system is composed of  $n$  independent multistate components  $E_i$ ,  $i = 1, 2, \dots, n$ , and that the changes of the operation process  $Z(t)$  states have an influence on both the system components  $E_i$  reliability and on the system reliability structure. Consequently, we denote the component  $E_i$  lifetime in the reliability state subset  $\{u, u+1, \dots, z\}$  by  $T_i^{(b)}(u)$  and by

$$[R_i(t, \cdot)]^{(b)} = [1, [R_i(t, 1)]^{(b)}, [R_i(t, 2)]^{(b)}, \dots, [R_i(t, z)]^{(b)}],$$

where for  $t \in \langle 0, \infty \rangle$ ,  $b = 1, 2, \dots, v$ ,  $u = 1, 2, \dots, z$ ,

$$[R_i(t, u)]^{(b)} = P(T_i^{(b)}(u) > t | Z(t) = z_b),$$

is the conditional reliability function while the system is at the operational state  $z_b$ ,  $b = 1, 2, \dots, v$ .

Next, we denote the system lifetime in the reliability state subset  $\{u, u+1, \dots, z\}$  by  $T^{(b)}(u)$  and by

$$[\mathbf{R}(t, \cdot)]^{(b)} = [1, [\mathbf{R}(t, 1)]^{(b)}, [\mathbf{R}(t, 2)]^{(b)}, \dots, [\mathbf{R}(t, z)]^{(b)}],$$

where for  $t \in \langle 0, \infty \rangle$ ,  $b = 1, 2, \dots, v$ ,  $u = 1, 2, \dots, z$ ,

$$[\mathbf{R}(t, u)]^{(b)} = P(T^{(b)}(u) > t | Z(t) = z_b),$$

is the conditional reliability function of the system while the system is at the operational state  $z_b$ .

In the case when the system operation time is large enough, the unconditional reliability function of the system is given by

$$\mathbf{R}(t, \cdot) = [1, \mathbf{R}(t, 1), \mathbf{R}(t, 2), \dots, \mathbf{R}(t, z)], \quad t \geq 0,$$

where

$$\mathbf{R}(t, u) \cong \sum_{b=1}^v p_b [\mathbf{R}(t, u)]^{(b)}. \quad (3)$$

The mean values of the system lifetimes in the reliability state subset  $\{u, u+1, \dots, z\}$  are

$$\mu(u) = E[T(u)] \cong \sum_{b=1}^v p_b \mu_b(u), \quad u = 1, 2, \dots, z, \quad (4)$$

and the mean values of the system lifetimes in the particular reliability state  $u$ , are [4]

$$\bar{\mu}(u) = \mu(u) - \mu(u+1), \quad u = 1, 2, \dots, z-1, \quad \bar{\mu}(z) = \mu(z). \quad (5)$$

A probability  $r(t) = P(s(t) < r | R(0) = z) = P(T^{(b)}(r) \leq t)$ ,  $t \in \langle -\infty, \infty \rangle$ ,

that the system is in the subset of reliability states worse than the critical state  $r$ ,  $r \in \{1, \dots, z\}$  while it was in the state  $z$  at the moment  $t = 0$  is called a risk function of the multi-state system [4].

Under this definition, from (3), we have

$$r(t) = 1 - \mathbf{R}(t, r), \quad t \in \langle -\infty, \infty \rangle, \quad (6)$$

and if  $\tau$  is the moment when the risk exceeds a permitted level  $\delta$ , then

$$\tau = r^{-1}(\delta), \quad (7)$$

where  $r^{-1}(t)$ , if it exists, is the inverse function of the risk function  $r(t)$ .

## 2 OPTIMAL TRANSIENT PROBABILITIES MAXIMIZING SYSTEM LIFETIME

Considering the equation (3), it is natural to assume that the system operation process has a significant influence on the system reliability. This influence is also clearly expressed in the equation (4) for the mean values of the system

unconditional lifetimes in the reliability state subsets. From linear equation (4), we can see that the mean value of the system unconditional lifetime  $\mu(u)$ ,  $u = 1, 2, \dots, z$ , is determined by the limit values of transient probabilities  $p_b$ ,  $b = 1, 2, \dots, v$ , of the system operation states and the mean values  $\mu_b(u)$ ,  $b = 1, 2, \dots, v$ ,  $u = 1, 2, \dots, z$ , of the system conditional lifetimes in the reliability state subsets  $\{u, u + 1, \dots, z\}$ ,  $u = 1, 2, \dots, z$ . Therefore, the system lifetime optimization approach based on the linear programming can be proposed. Namely, we may look for the corresponding optimal values  $\dot{p}_b$  of the transient probabilities  $p_b$  in the system operation states to maximize the mean value  $\mu(u)$  of the unconditional system lifetimes in the reliability state subsets  $\{u, u + 1, \dots, z\}$  under the assumption that the mean values  $\mu_b(u)$  of the system conditional lifetimes in the reliability state subsets are fixed. As a special case of the above formulated system lifetime optimization problem: if  $r$ ,  $r = 1, 2, \dots, z$ , is a system critical reliability state, then we want to find the optimal values  $\dot{p}_b$  of the transient probabilities  $p_b$  in the system operation states to maximize the mean value  $\mu(r)$  of the unconditional system lifetime in the reliability state subset  $\{r, r + 1, \dots, z\}$  under the assumption that the mean values  $\mu_b(r)$ ,  $b = 1, 2, \dots, v$ , of the system conditional lifetimes in this reliability state subset are fixed. More exactly, we formulate the optimization problem as a linear programming model with the objective function of the following linear form

$$\mu(r) = \sum_{b=1}^v p_b \mu_b(r) \quad (8)$$

for a fixed  $r \in \{1, 2, \dots, z\}$  and with the following bound constraints

$$\sum_{b=1}^v p_b = 1, \quad \check{p}_b \leq p_b \leq \widehat{p}_b, \quad b = 1, 2, \dots, v, \quad (9)$$

where  $\mu_b(r)$ ,  $\mu_b(r) \geq 0$ ,  $b = 1, 2, \dots, v$ , are fixed mean values of the system conditional lifetimes in the reliability state subset  $\{r, r + 1, \dots, z\}$  and

$\check{p}_b$ ,  $0 \leq \check{p}_b \leq 1$  and  $\widehat{p}_b$ ,  $0 \leq \widehat{p}_b \leq 1$ ,  $\check{p}_b \leq \widehat{p}_b$ ,  $b = 1, 2, \dots, v$ , (10) are respectively the lower and upper bounds of the unknown transient probabilities  $p_b$ .

Now, we can obtain the optimal solution to the formulated by (8)–(10) the linear programming problem, i.e. we can find the optimal values  $\dot{p}_b$  of the transient probabilities  $p_b$ ,  $b = 1, 2, \dots, v$ , that maximize the objective function given by (8). First, we arrange the system conditional lifetime mean values  $\mu_b(r)$ ,  $b = 1, 2, \dots, v$ , in non-increasing order

$$\mu_{b_1}(r) \geq \mu_{b_2}(r) \geq \dots \geq \mu_{b_v}(r), \quad b_i \in \{1, 2, \dots, v\} \text{ for } i = 1, 2, \dots, v.$$

Next, we substitute

$$x_i = p_{b_i}, \quad \check{x}_i = \check{p}_{b_i}, \quad \widehat{x}_i = \widehat{p}_{b_i} \text{ for } i = 1, 2, \dots, v \quad (11)$$

and we maximize with respect to  $x_i$ ,  $i = 1, 2, \dots, v$ , the linear form (8) that takes the form

$$\mu(r) = \sum_{i=1}^v x_i \mu_{b_i}(r) \quad (12)$$

for a fixed  $r \in \{1, 2, \dots, z\}$  with the following bound constraints

$$\sum_{i=1}^v x_i = 1, \quad \check{x}_i \leq x_i \leq \widehat{x}_i, \quad i = 1, 2, \dots, v, \quad (13)$$

where  $\mu_{b_i}(r)$ ,  $\mu_{b_i}(r) \geq 0$ ,  $i = 1, 2, \dots, v$ , are fixed mean values of the system conditional lifetimes in the reliability state subset  $\{r, r + 1, \dots, z\}$  arranged in non-increasing order and

$$\check{x}_i, 0 \leq \check{x}_i \leq 1 \text{ and } \widehat{x}_i, 0 \leq \widehat{x}_i \leq 1, \quad \check{x}_i \leq \widehat{x}_i, \quad i = 1, 2, \dots, v, \quad (14)$$

are the lower and upper bounds of the unknown probabilities  $x_i$ ,  $i = 1, 2, \dots, v$ , respectively.

We define

$$\check{x} = \sum_{i=1}^v \check{x}_i, \quad \widehat{y} = 1 - \check{x} \quad (15)$$

and

$$\check{x}^0 = 0, \quad \widehat{x}^0 = 0 \text{ and } \check{x}^I = \sum_{i=1}^I \check{x}_i, \quad \widehat{x}^I = \sum_{i=1}^I \widehat{x}_i \text{ for } I = 1, 2, \dots, v. \quad (16)$$

Next, we find the largest value  $I \in \{0, 1, \dots, v\}$  such that

$$\widehat{x}^I - \check{x}^I < \widehat{y} \quad (17)$$

and we fix the optimal solution that maximize (12) in the following way:

i) if  $I = 0$ , the optimal solution is  $\dot{x}_1 = \widehat{y} + \check{x}_1$  and  $\dot{x}_i = \check{x}_i$  for  $i = 2, 3, \dots, v$ ; (18)

ii) if  $0 < I < v$ , the optimal solution is  $\dot{x}_i = \widehat{x}_i$  for  $i = 1, 2, \dots, I$ ,  $\dot{x}_{I+1} = \widehat{y} - \widehat{x}^I + \check{x}^I + \check{x}_{I+1}$  and  $\dot{x}_i = \check{x}_i$  for  $i = I + 2, I + 3, \dots, v$ ; (19)

iii) if  $I = v$ , the optimal solution is  $\dot{x}_i = \widehat{x}_i$  for  $i = 1, 2, 3, \dots, v$ . (20)

Finally, after making the inverse to (11) substitution, we get the optimal limit transient probabilities

$$\dot{p}_{b_i} = \dot{x}_i \text{ for } i = 1, 2, \dots, v, \quad (21)$$

that maximize the system mean lifetime  $\mu(r)$  in the reliability state subset  $\{r, r + 1, \dots, z\}$ , defined by the linear form (8) giving its maximum value in the following form

$$\dot{\mu}(r) = \sum_{b=1}^v \dot{p}_b \mu_b(r) \text{ for a fixed } r \in \{1, 2, \dots, z\}. \quad (22)$$

From the above, replacing  $r$  by  $u$ ,  $u = 1, 2, \dots, z$ , we obtain the corresponding optimal solutions for the mean values of the system unconditional lifetimes in the reliability state subsets  $\{u, u+1, \dots, z\}$  of the form

$$\dot{\mu}(u) = \sum_{b=1}^v \dot{p}_b \mu_b(u) \text{ for } u = 1, 2, \dots, z. \quad (23)$$

Further, according to (3), the corresponding optimal unconditional multistate reliability function of the system is

$$\dot{\mathbf{R}}(t, \cdot) = [1, \dot{\mathbf{R}}(t, 1), \dots, \dot{\mathbf{R}}(t, z)], \quad (24)$$

where

$$\dot{\mathbf{R}}_n(t, \cdot) \cong \sum_{b=1}^v \dot{p}_b [\mathbf{R}(t, u)]^{(b)} \text{ for } t \geq 0, u = 1, 2, \dots, z, \quad (25)$$

and by (5) the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states are of the form

$$\dot{\bar{\mu}}(u) = \dot{\mu}(u) - \dot{\mu}(u+1), \quad u = 0, 1, \dots, z-1, \quad \dot{\bar{\mu}}(z) = \dot{\mu}(z). \quad (26)$$

Moreover, considering (6) and (7), the corresponding optimal system risk function and the moment when the risk exceeds a permitted level  $\delta$ , respectively are given by

$$\dot{r}(t) = 1 - \dot{\mathbf{R}}(t, r) \text{ for } t \in (-\infty, \infty) \text{ and } \dot{\tau} = \dot{r}^{-1}(\delta). \quad (27)-(28)$$

### 3 OPTIMAL SOJOURN TIMES OF COMPLEX TECHNICAL SYSTEM OPERATION PROCESS

Replacing the limit transient probabilities  $p_b$  of the system operation process at the operation states by their optimal values  $\dot{p}_b$  and the mean values  $M_b$  of the unconditional sojourn times at the operation states by their corresponding unknown optimal values  $\dot{M}_b$  maximizing the mean value of the system lifetime in the reliability states subset  $\{r, r+1, \dots, z\}$ , we get the system of equations

$$\dot{p}_b = \frac{\pi_b \dot{M}_b}{\sum_{l=1}^v \pi_l \dot{M}_l}, \quad b = 1, 2, \dots, v. \quad (29)$$

After simple transformations the above system takes the form

$$\begin{cases} (\dot{p}_1 - 1)\pi_1 \dot{M}_1 + \dot{p}_1 \pi_2 \dot{M}_2 + \dots + \dot{p}_1 \pi_v \dot{M}_v = 0 \\ \dot{p}_2 \pi_1 \dot{M}_1 + (\dot{p}_2 - 1)\pi_2 \dot{M}_2 + \dots + \dot{p}_2 \pi_v \dot{M}_v = 0 \\ \dots \\ \dot{p}_v \pi_1 \dot{M}_1 + \dot{p}_v \pi_2 \dot{M}_2 + \dots + (\dot{p}_v - 1)\pi_v \dot{M}_v = 0, \end{cases} \quad (30)$$

where  $\dot{M}_b$  are unknown variables we want to find,  $\dot{p}_b$  are optimal transient probabilities and  $\pi_b$  are steady probabilities.

Since the system of equations is homogeneous and it can be proved that the determinant of its main matrix is equal to zero, then it has nonzero solutions and moreover,

these solutions are ambiguous. Thus, if we fix some of the optimal values  $\dot{M}_b$  of the mean values  $M_b$  of the unconditional sojourn times at the operation states, for instance by arbitrary fixing either one or multiple of them, we may find the values of the remaining ones and using this method arrive at the solution of this equation.

Another very useful and much easier applicable in practice tool that can help in planning the operation processes of complex technical systems are the system operation process optimal mean values of the total system operation process sojourn times  $\hat{\theta}_b$  at the particular operation states  $z_b$ ,  $b = 1, 2, \dots, v$ , during the fixed system operation time  $\theta$ . They can be obtained by replacing the transient probabilities  $p_b$  at the operation states  $z_b$  with their optimal values  $\dot{p}_b$ . This results in the following expression

$$\dot{M}_b = \dot{E}[\hat{\theta}_b] = \dot{p}_b \theta, \quad b = 1, 2, \dots, v. \quad (31)$$

The knowledge of the optimal values  $\dot{M}_b$  of the mean values of the unconditional sojourn times and the optimal mean values  $\dot{M}_b$  of the total sojourn times at the particular operation states during the fixed system operation time may be the basis for changing the complex technical systems operation processes in order to ensure that these systems operate both more reliably and more safely. This knowledge may also be useful in these systems operation cost analysis.

### 4 THE BULK CARGO TRANSPORTATION SYSTEM RELIABILITY AND RISK

The considered bulk cargo terminal placed at the Baltic seaside is designated for storage and reloading of bulk cargo, but its primary activity is loading bulk cargo on board the ships for export. There are two independent transportation systems: the system of reloading rail wagons and the system of loading vessels.

Cargo is brought to the terminal by trains consisting of self-discharging wagons, which are discharged to a hopper and then by means of conveyors transported into one of four storage tanks (silos). Loading of fertilizers from storage tanks on board the ship is done by means of special reloading system which consists of several belt conveyors and one bucket conveyor which allows the transfer of bulk cargo in a vertical direction. Researched system is a system of belt conveyors, referred to as the transport system.

In the conveyor reloading system we distinguish three bulk cargo transportation subsystems, the belt conveyors  $S_1$ ,  $S_2$  and  $S_3$ . The conveyor loading system is composed of six bulk cargo transportation subsystems, the dosage conveyor  $S_4$ , the horizontal conveyor  $S_5$ , the horizontal conveyor  $S_6$ , the sloping conveyors  $S_7$ , the dosage conveyor with buffer  $S_8$ , the loading system  $S_9$ .

The bulk cargo transportation subsystems are built, respectively:

- the subsystem  $S_1$ : 1 rubber belt, 2 drums, set of 121 bow rollers, set of 23 belt supporting rollers,
- the subsystem  $S_2$ : 1 rubber belt, 2 drums, set of 44 bow rollers, set of 14 belt supporting rollers,

- the subsystem  $S_3$ : 1 rubber belt, 2 drums, set of 185 bow rollers, set of 60 belt supporting rollers,
- the subsystem  $S_4$ : 3 identical belt conveyors, each composed of 1 rubber belt, 2 drums, set of 12 bow rollers, set of 3 belt supporting rollers,
- the subsystem  $S_5$ : 1 rubber belt, 2 drums, set of 125 bow rollers, set of 45 belt supporting rollers,
- the subsystem  $S_6$ : 1 rubber belt, 2 drums, set of 65 bow rollers, set of 20 belt supporting rollers,
- the subsystem  $S_7$ : 1 rubber belt, 2 drums, set of 12 bow rollers, set of 3 belt supporting rollers,
- the subsystem  $S_8$ : 1 rubber belt, 2 drums, set of 162 bow rollers, set of 53 belt supporting rollers,
- the subsystem  $S_9$ : 3 rubber belts, 6 drums, set of 64 bow rollers, set of 20 belt supporting rollers.

Taking into account the operation process of the considered system we distinguish the following as its three operation states:

- an operation state  $z_1$  – loading fertilizers from rail wagons on board the ship is done using  $S_1, S_2, S_3, S_6, S_7, S_8$  and  $S_9$  subsystems.
- an operation state  $z_2$  – discharging rail wagons to storage tanks or hall when subsystems  $S_1, S_2$  and  $S_3$ , are used,
- an operation state  $z_3$  – loading fertilizers from storage tanks or hall on board the ship is done by using  $S_4, S_5, S_6, S_7, S_8$  and  $S_9$ , subsystems.

The limit values of the bulk cargo transportation systems operation process transient probabilities  $p_b(t)$  at the operation states  $z_b, b = 1,2,3$ , determined in [5], on the basis of data coming from experts are

$$p_1 = 0.2376, p_2 = 0.6679, p_3 = 0.0945. \quad (32)$$

Further, assuming that the system is in the reliability state subset  $\{u, u+1, \dots, z\}$  if all its subsystems are in this subset of reliability states, we conclude that the bulk cargo transportation system is a series system [4] of subsystems  $S_1, S_2, S_3, S_6, S_7, S_8$  and  $S_9$ .

Under the assumption that changes of the bulk cargo transportation system operation states have an influence on both the subsystem  $S_i$  reliability and the entire reliability structure [8], on the basis of expert opinions and statistical data given in [9], [10], the bulk cargo transportation system reliability structures and their components reliability functions at different operation states can be determined.

Additionally, we assume that subsystems  $S_i, i = 1,2,3, \dots, 9$ , are composed of four-state exponential components, with the reliability functions

$$[R_i(t, \cdot)]^{(b)} = [1, [R_i(t, 1)]^{(b)}, [R_i(t, 2)]^{(b)}, [R_i(t, 3)]^{(b)}], \\ t \in < 0, \infty), b = 1,2,3, u = 1,2,3.$$

At the operation state  $z_1$ , at loading of fertilizers from rail wagons on board the ship, the system is composed of seven non-homogenous series subsystems  $S_1, S_2, S_3, S_6, S_7, S_8$ , and  $S_9$  forming a series structure.

The conditional reliability function of the system while it is at the operation state  $z_1$  is given by

$$[R(t, \cdot)]^{(1)} = [1, [R(t, 1)]^{(1)}, [R(t, 2)]^{(1)}, [R(t, 3)]^{(1)}],$$

where

$$[R(t, u)]^{(1)} = [R_{147}(t, u)]^{(1)} [R_{61}(t, u)]^{(1)} [R_{248}(t, u)]^{(1)} [R_{88}(t, u)]^{(1)}$$

$[R_{18}(t, u)]^{(1)} [R_{218}(t, u)]^{(1)} [R_{93}(t, u)]^{(1)}$  for  $t \in < 0, \infty), u = 1,2,3$ , i.e.

$$[R(t, 1)]^{(1)} = \exp[-74.426t], [R(t, 2)]^{(1)} = \exp[-93.472t],$$

$$[R(t, 3)]^{(1)} = \exp[-150.206t]. \quad (33)–(35)$$

The expected values of the conditional lifetimes in the reliability state subsets at the operation state  $z_1$ , calculated from the above result given by (33)–(35), are:

$$\mu_1(1) \cong 0.013, \mu_1(2) \cong 0.011, \mu_1(3) \cong 0.007 \text{ years}, \quad (36)$$

and further, using (5), it follows that the conditional lifetimes in the particular reliability states at the operation state  $z_1$  are:

$$\bar{\mu}_1(1) \cong 0.002, \bar{\mu}_1(2) \cong 0.004, \bar{\mu}_1(3) \cong 0.007 \text{ years}.$$

At the operation state  $z_2$ , i.e. at the state of discharging rail wagons to storage tanks or hall, the system is built of three subsystems  $S_1, S_2$  and  $S_3$  forming a series structure [4].

The conditional reliability function of the bulk cargo transportation system at the operation state  $z_2$  is given by

$$[R(t, \cdot)]^{(2)} = [1, [R(t, 1)]^{(2)}, [R(t, 2)]^{(2)}, [R(t, 3)]^{(2)}],$$

where

$$[R(t, u)]^{(2)} = [R_{147}(t, u)]^{(2)} [R_{61}(t, u)]^{(2)} [R_{248}(t, u)]^{(2)}$$

$$\text{for } t \in < 0, \infty), u = 1,2,3,$$

i.e.

$$[R(t, 1)]^{(2)} = \exp[-39.563t], [R(t, 2)]^{(2)} = \exp[-49.663t],$$

$$[R(t, 3)]^{(2)} = \exp[-64.280t]. \quad (37)–(39)$$

The expected values of the conditional lifetimes in the reliability state subsets at the operation state  $z_2$ , calculated from the above result given by (37)–(39), are:

$$\mu_2(1) \cong 0.025, \mu_2(2) \cong 0.020, 0.016 \text{ years}, \quad (40)$$

and further, using (5), it follows that the conditional lifetimes in the particular reliability states at the operation state  $z_2$  are:

$$\bar{\mu}_2(1) \cong 0.005, \bar{\mu}_2(2) \cong 0.004, \bar{\mu}_2(3) \cong 0.016 \text{ years}.$$

At the operation state  $z_3$ , i.e. at the loading of fertilizers from storage tanks or hall on board, the bulk cargo transportation system is built of six subsystems one series-parallel subsystem  $S_4$  and five series subsystems  $S_5, S_6, S_7, S_8, S_9$  forming a series structure [4].

The conditional reliability function of the system while it is at the operation state  $z_3$  is given by

$$[\mathbf{R}(t, \cdot)]^{(3)} = [1, [\mathbf{R}(t, 1)]^{(3)}, [\mathbf{R}(t, 2)]^{(3)}, [\mathbf{R}(t, 3)]^{(3)}],$$

where

$$[\mathbf{R}(t, u)]^{(3)} = [\mathbf{R}_{3,18}(t, u)]^{(3)} \cdot [\mathbf{R}_{1,73}(t, u)]^{(3)} \cdot [\mathbf{R}_{8,8}(t, u)]^{(3)} \cdot [\mathbf{R}_{1,8}(t, u)]^{(3)} \cdot [\mathbf{R}_{2,18}(t, u)]^{(3)} \cdot [\mathbf{R}_{9,3}(t, u)]^{(3)} \text{ for } t \in \langle 0, \infty \rangle, u = 1, 2, 3,$$

i.e.

$$[\mathbf{R}(t, 1)]^{(3)} = \exp[-57.758t] - 3 \exp[-55.007t] + 3 \exp[-52.256t] \quad (41)$$

$$[\mathbf{R}(t, 2)]^{(3)} = \exp[-70.974t] - 3 \exp[-68.018t] + 3 \exp[-65.062t] \quad (42)$$

$$[\mathbf{R}(t, 3)]^{(3)} = \exp[-89.416t] - 3 \exp[-86.140t] + 3 \exp[-82.864t] \quad (43)$$

The expected values of the conditional lifetimes in the reliability state subsets at the operation state  $z_3$ , calculated from the above result given by (41)–(43), are:

$$\mu_3(1) \cong 0.020, \mu_3(2) \cong 0.016, \mu_3(3) \cong 0.013 \text{ years,} \quad (44)$$

and further, using (5), it follows that the conditional lifetimes in the particular reliability states at the operational state  $z_3$  are:

$$\bar{\mu}_3(1) \cong 0.004, \bar{\mu}_3(2) \cong 0.003, \bar{\mu}_3(3) \cong 0.013 \text{ years.}$$

In the case when the system operation time is large enough, the unconditional reliability function of the bulk cargo transportation system is given by the vector

$$\mathbf{R}(t, \cdot) = [1, \mathbf{R}(t, 1), \mathbf{R}(t, 2), \mathbf{R}(t, 3)], t \geq 0,$$

where, according to (3) and after considering the values of  $p_b, b = 1, 2, 3$ , given by (32), its co-ordinates are as follows:

$$\mathbf{R}(t, u) = p_1 \cdot [\mathbf{R}(t, u)]^{(1)} + p_2 \cdot [\mathbf{R}(t, u)]^{(2)} + p_3 \cdot [\mathbf{R}(t, u)]^{(3)} \quad (45)$$

for  $t \geq 0, u = 1, 2, 3$ , where  $[\mathbf{R}(t, u)]^{(1)}$  and  $[\mathbf{R}(t, u)]^{(2)}$  and  $[\mathbf{R}(t, u)]^{(3)}$  are respectively given by (33)–(35) and (37)–(39) and (41)–(43), i.e.

$$\mathbf{R}(t, 1) = 0.6679 \exp[-39.563t] + 0.0945 \exp[-74.426t] + 0.2376 [\exp[-57.758t] - 3 \exp[-55.007t] + 3 \exp[-52.256t]] \quad (46)$$

$$\mathbf{R}(t, 2) = 0.6679 \exp[-93.472t] + 0.0945 \exp[-49.663t] + 0.2376 [\exp[-70.974t] - 3 \exp[-68.018t] + 3 \exp[-65.062t]] \quad (47)$$

$$\mathbf{R}(t, 3) = 0.6679 \exp[-150.206t] + 0.0945 \exp[-64.280t] + 0.0945 [\exp[-89.416t] - 3 \exp[-86.140t] + 3 \exp[-82.864t]] \quad (48)$$

The mean values of the system unconditional lifetimes in the reliability state subsets, according to (4) are respectively:

$$\mu(1) \cong 0.016, \mu(2) \cong 0.013, \mu(3) \cong 0.009. \quad (49)$$

The mean values of the system lifetimes in the particular reliability states, (5), are

$$\bar{\mu}(1) = \mu(1) - \mu(2) = 0.003, \bar{\mu}(2) = \mu(2) - \mu(3) = 0.004, \bar{\mu}(3) = \mu(3) = 0.009.$$

If the critical reliability state is  $r = 2$ , then the system risk function, according to (6), is given by

$$r(t) = 1 - [0.6679 \exp[-93.472t] + 0.0945 \exp[-49.663t] + 0.2376 (\exp[-70.974t] - 3 \exp[-68.018t] + 3 \exp[-65.062t])] \text{ for } t \geq 0. \quad (50)$$

Hence, the moment when the system risk function (Fig. 1) exceeds a permitted level, for instance  $\delta = 0.05$ , from (7), is

$$\tau = r^{-1}(\delta) \cong 0.000627 \text{ years.} \quad (51)$$

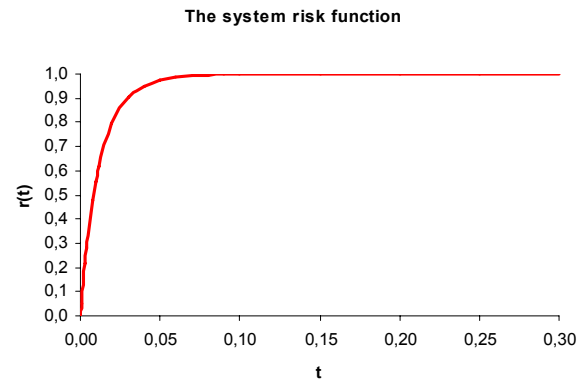


Figure 1 – The graph of the port bulk cargo transportation system risk function

## 5 OPTIMIZATION OF THE BULK CARGO TRANSPORTATION SYSTEM OPERATION PROCESS

In our case, as the critical state is  $r = 2$ , then considering the expression for  $\mu(2)$ , the objective function (8), takes the form

$$\mu(2) = p_1 \cdot 0.011 + p_2 \cdot 0.020 + p_3 \cdot 0.016 = 0.013 \text{ years.} \quad (52)$$

The lower  $\check{p}_b$  and upper  $\hat{p}_b$  bounds of the unknown transient probabilities  $p_b, b = 1, 2, 3$ , coming from experts, respectively are [6]:

$$\check{p}_1 = 0.150, \check{p}_2 = 0.005, \check{p}_3 = 0.015, \hat{p}_1 = 0.850, \hat{p}_2 = 0.120, \hat{p}_3 = 0.390.$$

Therefore, according to (9)–(10), we assume the following bound constraints

$$\sum_{b=1}^3 p_b = 1, 0.250 \leq p_1 \leq 0.850, 0.005 \leq p_2 \leq 0.150, 0.050 \leq p_3 \leq 0.550.$$

Now, before we find optimal values  $\check{p}_b$  of the transient probabilities  $p_b, b = 1, 2, 3$ , that maximize the objective function (53), we arrange the system conditional lifetimes mean values  $\mu_b(2), b = 1, 2, 3$ , in non-increasing order  $\mu_2(2) \geq \mu_3(2) \geq \mu_1(2)$ .

Next, according to (11), we substitute

$$x_1 = p_2 = 0.0945, x_2 = p_3 = 0.2376, x_3 = p_1 = 0.6679, \quad (53)$$

$$\begin{aligned} \tilde{x}_1 = 0.005, \quad \tilde{x}_2 = 0.050, \quad \tilde{x}_3 = 0.250, \quad \hat{x}_1 = 0.150, \\ \hat{x}_2 = 0.550, \quad \hat{x}_3 = 0.850, \end{aligned} \quad (54)$$

where  $\tilde{x}_i$  and  $\hat{x}_i$  are lower and upper bounds of the unknown limit transient probabilities  $x_i$ ,  $i = 1, 2, 3$ , respectively and we maximize with respect to  $x_i$ ,  $i = 1, 2, 3$ , the linear form (52) that according to (13) takes the form

$$\mu(2) = x_1 \cdot 0.011 + x_2 \cdot 0.020 + x_3 \cdot 0.016, \quad (55)$$

with the following bound constraints

$$\sum_{i=1}^3 x_i = 1,$$

$$0.005 \leq x_1 \leq 0.12, \quad 0.015 \leq x_2 \leq 0.390, \quad 0.150 \leq x_3 \leq 0.850. \quad (56)$$

According to (15)–(17), we calculate and fix the optimal solution that maximizes linear function (55) according to the rule (19). Namely, we get

$$\dot{x}_1 = \hat{x}_1 = 0.120, \quad \dot{x}^2 = \hat{x}^2 = 0.390,$$

$$\dot{x}_3 = 0.830 - 0.490 + 0.150 = 0.490.$$

Finally, according to (21) after making the inverse to (53) substitution, we get the optimal transient probabilities

$$\dot{p}_1 = \dot{x}_3 = 0.490, \quad \dot{p}_2 = \dot{x}_1 = 0.120, \quad \dot{p}_3 = \dot{x}_2 = 0.390, \quad (57)$$

that maximize the system mean lifetime in the reliability state subset  $\{2, 3\}$  expressed by the linear form (53) giving, according to (12) and (57), its optimal value

$$\begin{aligned} \dot{\mu}(2) = \dot{p}_1 \cdot 0.011 + \dot{p}_2 \cdot 0.020 + \dot{p}_3 \cdot 0.016 = 0.49 \cdot 0.011 + \\ + 0.12 \cdot 0.020 + 0.39 \cdot 0.016 = 0.014. \end{aligned} \quad (58)$$

### 6 OPTIMAL RELIABILITY CHARACTERISTICS OF THE BULK CARGO TRANSPORTATION SYSTEM

Further, substituting the optimal solution (57) according to (24), we obtain the optimal solution for the mean value of the system unconditional lifetime in the reliability state subset  $\{1, 2\}$ ,  $\{3\}$  that respectively amounts:

$$\dot{\mu}(1) \cong 0.0172, \quad \dot{\mu}(3) \cong 0.0104, \quad (59)$$

and according to (26), the optimal solutions for the mean values of the system unconditional lifetimes in the particular reliability states are

$$\dot{\bar{\mu}}(1) \cong 0.0032, \quad \dot{\bar{\mu}}(2) \cong 0.0036, \quad \dot{\bar{\mu}}(3) \cong 0.0104.$$

Moreover, according to (24)–(25), the corresponding optimal unconditional multistate reliability function of the system is of the form

$$\dot{R}(t, \cdot) = [1, \dot{R}(t, 1), \dot{R}(t, 2), \dot{R}(t, 3)] \text{ for } t \geq 0,$$

where according to (3) and after considering the values of  $\dot{p}_b$ , its co-ordinates are as follows:

$$\begin{aligned} \dot{R}(t, u) \cong 0.49 \cdot [R(t, u)]^{(1)} + 0.12 \cdot [R(t, u)]^{(2)} + 0.39 \cdot [R(t, u)]^{(3)} \\ \text{for } t \geq 0, \quad u = 1, 2, 3, \end{aligned} \quad (60)$$

where  $[R(t, u)]^{(1)}$ ,  $[R(t, u)]^{(2)}$ ,  $[R(t, u)]^{(3)}$  are respectively given by (33)–(35) and (37)–(39), (41)–(43).

If the critical reliability state is  $r = 2$ , then the system risk function, according to (27), is given by

$$\begin{aligned} \dot{r}(t) = 1 - \dot{R}(t, 2) = 1 - [0.49 \exp[-93.472t] + 0.12 \exp[-49.663t] + \\ + 0.39(\exp[-70.974t] - 3 \exp[-68.018t] + 3 \exp[-65.062t])] \quad (61) \end{aligned}$$

where  $\dot{R}(t, 2)$  is given by (60) for  $u = 2$ .

Hence, considering (28), the moment when the optimal system risk function (Fig. 2) exceeds a permitted level, for instance  $\delta = 0.05$ , is

$$\tau = \dot{r}^{-1}(\delta) \cong 0.000676 \text{ years.} \quad (62)$$

Comparing the bulk cargo transportation system reliability characteristics after its operation process optimization given by (58)–(62) with the corresponding characteristics before this optimization determined by (45)–(51) justifies this action.

The optimal system risk function

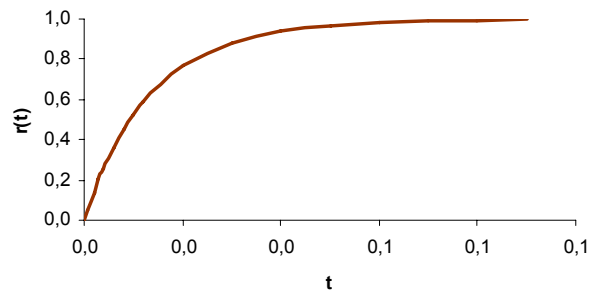


Figure 2 – The graph of the port bulk cargo transportation system optimal risk function

### 7 OPTIMAL SOJOURN TIMES OF BULK CARGO TRANSPORTATION SYSTEM OPERATION PROCESS AT OPERATION STATES

Having the values of the optimal transient probabilities determined by (57), it is possible to find the optimal conditional and unconditional mean values of the sojourn times of the bulk cargo transportation operation process at the operation states and the optimal mean values of the total unconditional sojourn times of the bulk cargo transportation system operation process at the operation states during the fixed operation time as well.

Substituting the optimal transient probabilities at operation states determined in (57) and the steady probabilities

$$\pi_1 = 0.315, \quad \pi_2 = 0.5, \quad \pi_3 = 0.185,$$

we get the following system of equations

$$\begin{cases} -0.16065\dot{M}_1 + 0.245\dot{M}_2 + 0.09065\dot{M}_3 = 0 \\ 0.0378\dot{M}_1 + (-0.44)\dot{M}_2 + 0.0222\dot{M}_3 = 0 \\ 0.12285\dot{M}_1 + 0.195\dot{M}_2 + (-0.11285)\dot{M}_3 = 0 \end{cases} \quad (63)$$

with the unknown optimal mean values  $\dot{M}_b$  of the system unconditional sojourn times in the operation states. Consequently, we get

$$\dot{M}_1, \dot{M}_2 = 0.154286\dot{M}_1, \quad \dot{M}_3 = 1.216216\dot{M}_1.$$

Thus, we may fix  $\hat{M}_1$  and determine the remaining ones. In our case, after considering expert opinion, we conclude that it is sensible to assume

$$\hat{M}_1 \cong 2.$$

This way the obtained solutions of the system of equations, are

$$\hat{M}_1 \cong 2, \hat{M}_2 = 0.308571, \hat{M}_3 = 2.432432. \quad (64)$$

It can be seen that these solutions differ substantially from the values  $M_1, M_2, M_3$ .

Other very useful and much easier to apply in practice tools that can help in planning the operation process of the technical system are the system operation process optimal mean values of the total sojourn times at the particular operation states during the fixed system operation time  $\theta$ . Assuming the system operation time  $\theta = 1$  year = 365 days, after applying (32), we get their values

$$\hat{M}_1 = \dot{E}[\hat{\theta}_1] = \dot{p}_1\theta = 0.49 \cdot 365 \cong 179 \text{ days},$$

$$\hat{M}_2 = \dot{E}[\hat{\theta}_2] = \dot{p}_2\theta = 0.12 \cdot 365 = 44 \text{ days},$$

$$\hat{M}_3 = \dot{E}[\hat{\theta}_3] = \dot{p}_3\theta = 0.39 \cdot 365 = 142 \text{ days}. \quad (65)$$

In practice, the knowledge of the optimal values of  $\hat{M}_b$  and  $\hat{M}_b$  given respectively by (64)–(65), is important and very helpful in planning and improving the operation process, as it allows for more reliable and safer system operation. From the performed analysis of the results of the bulk cargo transportation system operation process optimization it can be suggested to change the operation process characteristics that result in replacing (or the approaching/convergence to) the unconditional mean sojourn times  $M_b$  in the particular operation states before the optimization by their optimal values  $\hat{M}_b$  after the optimization. The easiest way of reorganizing the system operation process leads to replacing (or the approaching/convergence to) the total sojourn times,  $\hat{M}_b = E[\hat{\theta}_b]$ , of the bulk cargo transportation system operation process, and in particular operation states during the operation time  $\theta = 1$  year, with their optimal values  $\hat{M}_b = \dot{E}[\hat{\theta}_b]$ .

## CONCLUSIONS

The joint model of reliability of complex technical systems at variable operation conditions linking a semi-markov modelling of the system operation processes with a multi-state approach to system reliability analysis was constructed. Next, the final results obtained from this joint model and linear programming were used to build the model of complex technical systems reliability optimization. These tools can be used in reliability evaluation and optimization of a very wide class of real technical systems operating at

varying conditions that influence their reliability structures and the reliability parameters of their components. The practical application of these tools to reliability and risk evaluation and optimization of a technical system of a bulk cargo transportation system, operating in variable conditions, and the results achieved can be implemented by reliability practitioners from both maritime transport industry and other industrial sectors.

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## ОПТИМИЗАЦИЯ НАДЕЖНОСТИ И РИСКОВ СИСТЕМ С НЕСКОЛЬКИМИ УСТОЙЧИВЫМИ СОСТОЯНИЯМИ В ПРИЛОЖЕНИИ К ТРАНСПОРТНОЙ СИСТЕМЕ ПОРТА

Сложность процессов работы технических систем и их влияние на изменение во времени структур систем и параметров надежности их компонентов обуславливают сложности при первой встрече в реальности, а затем в фиксации и анализе этих структур и



параметров надійності. Путем построения объединенной модели надежности сложных технических систем в различных условиях эксплуатации, связывающей полумарковское моделирование процессов работы системы с подходом нескольких состояний в анализе надежности систем, мы находим основные характеристики надежности системы. Затем мы используем линейное программирование для того, чтобы построить модель оптимизации надежности сложных технических систем. Мы исследуем приложение модели в морском транспорте, в частности, в оптимизации надежности и рисков объемной системы грузоперевозок. Инструменты, разработанные нами, могут быть использованы для оценки надежности и оптимизации очень широкого класса реальных технических систем, работающих в различных условиях, которые влияют на их структуру надежности и параметры надежности их компонентов. Следовательно, разработанные нами инструменты могут быть использованы специалистами-практиками в области надежности как в отрасли морского транспорта, так и в других отраслях промышленности.

**Ключевые слова:** функция надежности, функция риска, рабочий процесс, оптимизация.

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#### ОПТИМІЗАЦІЯ НАДІЙНОСТІ І РИЗИКІВ СИСТЕМ З КІЛЬКОМА СТІЙКИМИ СТАНАМИ У ЗАСТОСУВАННІ ДО ТРАНСПОРТНОЇ СИСТЕМИ ПОРТУ

Складність процесів роботи технічних систем та їхній вплив на зміну в часі структур систем і параметрів надійності їхніх компонентів обумовлюють складнощі при першій зустрічі у реальності, а потім у фіксації і аналізі цих структур і параметрів надійності. Шляхом побудови об'єднаної моделі надійності складних технічних систем в різних умовах експлуатації, що зв'язує напівмарковське моделювання процесів роботи системи з підходом декількох станів в аналізі надійності систем, ми знаходимо основні характеристики надійності системи. Потім ми використовуємо лінійне програмування для того, щоб побудувати модель оптимізації надійності складних технічних систем. Ми досліджуємо застосування моделі в морському транспорті, зокрема в оптимізації надійності та ризиків об'ємної системи вантажоперевезень. Інструменти, розроблені нами, можуть бути використані для оцінки надійності та оптимізації дуже широкого класу реальних технічних систем, що працюють в різних умовах, які впливають на їх структуру надійності і параметри надійності їхніх компонентів. Отже, розроблені нами інструменти можуть бути використані фахівцями-практиками в галузі надійності як у галузі морського транспорту, так і в інших галузях промисловості.

**Ключові слова:** функція надійності, функція ризику, робочий процес, оптимізація.

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